PULSE BROADENING IN MULTILAYERED FIBERS

by L. JACOMME

Abstract

The fabrication of overmoded fibers by chemical vapour deposition (CVD) requires many successive layers in order to approximate a given index profile. The more layers one deposits, the better the approximation to the index profile is, and the closer one gets to the performances of the smooth profile fiber. It is important to know, if one wants to approach the characteristics of a given smooth profile by a certain percentage, what is the minimum number of layers to be deposited. In this paper, we review the different methods published in the literature, that were used to solve this problem. Numerical results on the r.m.s. pulse width of two fibers with parameters of practical importance are also given. These results, obtained by a method derived from modified ray-optical techniques, help in gaining some insight into this problem.

1. Introduction

Early workers in optical fiber propagation 1) used stairlike approximations to the index profile in order to be able to solve the optical fiber propagation problem from the scalar wave equation. Exact mode solutions were found within each layer thus defined, and these were matched at the boundaries. This method has, since then, been called the stratification method.

Today the CVD procedure realizes in the laboratory the staircase approximation that had been, up to then, an analytical mathematical tool. This fact puts even more emphasis than before on the propagation problem in multilayered fibers. Thus different methods have evolved recently all of which start from the scalar wave equation.

Arnaud and Mammel 2) find the eigenmodes of the scalar wave equation by numerically solving an equivalent pair of coupled first-order differential equations (by Runge–Kutta techniques). They are thus able to determine the main propagation characteristics of the fiber (group velocity, propagation constant, etc.) in an order of magnitude less time 3) than by the stratification method.

Behm 4, 5) selects the parabolic profile which is near the optimum and for which exact eigenmodes can be found. The perturbation that is considered is not the staircase approximation but a “diffused” version of it whose analytical expression is sinusoidal. First-order perturbation theory is applied to find the pulse broadening due to the perturbation. As usually done, the perturbation of an eigenvalue corresponding to a given eigenmode is just the average of the perturbation in that eigenmode.
Olshansky \(^6\) uses essentially the same techniques as Behm. The calculation is carried out for the optimal profile (which in this paper means that the inhomogeneous dispersion has been included and an equal excitation of the modes is assumed). As the optimal index profile is very close to \(\alpha = 2\), the same eigenmodes as for \(\alpha = 2\) (see eq. (2)) are used to determine by first-order perturbation theory the pulse broadening due to a sinusoidal perturbation (and not a staircase approximation or a diffused version of it).

More will be said on these different aspects, later on, in the section on the discussion of the results.

Modified ray techniques are conceptually more difficult to use in order to solve the multilayered propagation problem; some of the difficulties inherent to the method have been pointed out by Arnaud and Mammei \(^2\); another difficulty comes from the fact that, for a non-negligible number of rays propagating within the cylindrical core region, the two caustics which are necessary to determine the transit time of the ray are not well defined. Thus, for such rays, a core-to-core tunnelling effect has to be taken into account, but then the problem gets to be rather intricate. In the ray sampling procedure that we chose, rays travelling into one or two adjacent layers are ignored on the basis that they are not well equalized by the gradient. Moreover, ignoring or taking into account the rays that exhibit a core-core tunnelling effect does not change appreciably the r.m.s. pulse width. Therefore, on the basis of computing time, they have been ignored. Thus, it is seen that the mode formulation of the problem is direct and simple especially if one assumes an equal excitation of the modes. The actual numerical solving of the problem may be more difficult especially if the source field has to be decomposed into the eigenmodes of the fiber, a problem that has been given little attention.

In the following, we shall describe the ray method that was used to compute the flight time of a ray based on results derived from the eikonal equation, which gives in many known cases a very good accuracy for the evaluation of the transit time even at low \(V\) values (\(V = 2\pi n_0 (2d)^{1/2} a/\lambda\)).

2. Theory and ray sampling procedure

Let us call \(K\) the total number of layers. We assume that the same amount of material is deposited in each layer and therefore the area of each layer seen in a cross-section of the fiber is constant.

Let us call \(a\) the radius of the core; then the area of each successive layer is

\[
\pi (r_{k+1}^2 - r_k^2) = \frac{\pi a^2}{K}, \quad k = 1, 2, \ldots, K, K + 1
\]

with

\[
r_1 = 0 \quad \text{and} \quad r_{K+1} = a.
\]
We thus define the sequence

\[ r_k = \left( \frac{k-1}{K} \right)^{\frac{1}{4}}, \quad k = 1, 2, \ldots, K, K+1. \]  

(1)

We assume that the staircase approximation is superimposed on a power law profile (fig. 1):

\[ n^2(r) = n_0^2 \left[ 1 - 2\Delta \left( \frac{r}{a} \right)^\alpha \right]. \]

(2)

Fig. 1. Stairlike approximation to an \( \alpha \)-profile. The two caustics are well defined for a ray \( (E, l) \), which are the two constants of the motion of a ray.

Thus for each \( r_k \) as previously defined, we have a corresponding \( n(r_k) = n_k \):

\[ n(r_k) = n_k = n_0 \left[ 1 - 2\Delta \left( \frac{k-1}{K} \right)^{\alpha/2} \right]^{\frac{1}{4}}, \quad k = 1, 2, \ldots, K, K+1. \]  

(2)

In order to simulate an equal excitation of the modes, the parallel beam technique was used. The response of the fiber is computed for a series of parallel beams (\( \approx 1000 \)) of rays impinging on the input face of the fiber at increasing angles \( \gamma_0 \). The overall response is then obtained by adding up the elementary responses properly weighted.

The point of impact on the entrance face of the fiber \( (r_0, \theta_0) \) is selected by a pseudo-random generator.
The constants of the motion of a ray are

\[ l = r_0 \cos \theta_0 \tan \gamma_0, \]
\[ E = n_k \cos \gamma_0 \quad \text{if} \quad r_k < r_0 < r_{k+1}. \]

As is well known, \( E \) and \( l \) stay invariant in the broken line trajectory of the ray. In order to determine the flight time of a ray, the two caustics of the trajectory need to be known. It turns out that for a large percentage of the bound rays (core-cladding tunnelling leaky rays and refracted rays are not taken into account) these two caustics can be determined without ambiguity (more than two-thirds of the rays enter into this category).

We will briefly indicate how this transit time can be evaluated.

Once the constants of the motion of a ray are determined (this requires a comparison of \( r_0 \) with the sequence \( \{r_k\} \)), the two caustics of a "well-behaved" ray are determined by first computing the two caustics of the \( \alpha \)-profile upon which the staircase approximation is superimposed. This determination is carried out numerically in the general case. We call \( r_{\text{min}}^\alpha, r_{\text{max}}^\alpha \) these two caustics.

The caustics of the multilayered fiber are then defined by ordering \( r_{\text{min}}^\alpha, r_{\text{max}}^\alpha \) within the sequence \( \{r_k\} \).

For example, if

\[ r_p < r_{\text{min}}^\alpha < r_{p+1}^\alpha, \]
\[ r_{m-1} < r_{\text{max}}^\alpha < r_m, \]

then

\[ r_{\text{min}} = \frac{EL}{(n_p^2 - E^2)^{\frac{1}{2}}}, \quad r_{\text{max}} = r_m. \]

The transit time is computed by first evaluating the \( z \) propagation period and its corresponding time period

\[(r, z)-\text{period} = 2 \int_{r_{\text{min}}}^{r_m} \left[ \frac{n^2(r)}{E^2} - \frac{l^2}{r^2} - 1 \right]^{-\frac{1}{2}} \, dr = U - W \quad (3)\]

with

\[ U = 2E \sum_{\nu=p}^{m-1} \frac{1}{n^2_\nu - E^2} [(n_\nu^2 - E^2) r_{\nu+1}^2 - E^2 l^2]^{\frac{1}{2}}, \]
Pulse broadening in multilayered fibers

\[ W = 2E \sum_{v=p+1}^{m-1} \frac{1}{n_v^2 - E^2} [(n_v^2 - E^2) r_v^2 - E^2 l^2]^\perp. \]

The corresponding time period is given by

\[ \tau\text{-period} = \frac{2}{cE} \int_{r_{\min}}^{r_m} \left[ \frac{n^2(r)}{E^2} - \frac{l^2}{r^2} - 1 \right]^{-\frac{1}{2}} n^2(r) \, dr = P - Q \quad (4) \]

where \( c \) is the vacuum light velocity.

\[ P = \frac{2}{c} \sum_{v=p}^{m-1} \frac{n_v^2}{n_v^2 - E^2} [(n_v^2 - E^2) r_{v+1}^2 - E^2 l^2]^\perp, \]

\[ Q = \frac{2}{c} \sum_{v=p+1}^{m-1} \frac{n_v^2}{n_v^2 - E^2} [(n_v^2 - E^2) r_v^2 - E^2 l^2]^\perp. \]

The transit time is then simply given by

\[ TV = AM \frac{\tau\text{-period}}{(r, z)\text{-period}}, \quad (5) \]

where \( AM \) is the length of the fiber.

The impulse response \( P_0(t) \) for each injection angle \( \gamma_0 \) is obtained as a transit time histogram. The synthesized impulse response corresponding to an equal excitation of the modes is obtained as

\[ P(t) = \sum_{\gamma_0} \gamma_0 P_0(t). \quad (6) \]

From this evaluation, the r.m.s. width of the impulse response is computed through the formula

\[ \sigma = [\langle TV \rangle^2 - \langle TV^2 \rangle]^\perp, \quad (7) \]
where

$$\langle \xi \rangle = \int_{-\infty}^{+\infty} \xi P(t) \, dt$$

$$\int_{-\infty}^{+\infty} P(t) \, dt.$$  (8)

3. Results

The calculations have been carried out for two different sets of parameters corresponding to fiber no. 1 and fiber no. 2.

Fiber no. 1: $AM = 1$ km; $a = 20$ μm; $2\delta = 0.02; n_0 = 1.5$.
Fiber no. 2: $AM = 1$ km; $a = 40$ μm; $2\delta = 0.01; n_0 = 1.5$.

For each of these two fibers the exponent $\alpha$ was given different values namely $\alpha = 1; 1.8; 1.9; 2(1 - 2\delta)^{1/2}; 2; 2.1; 3; 4$ and for each of these $\alpha$-values, the number of layers $K$ was given the values $K = 10; 20; 40; 80; 160$.

The curves of figs 2 and 4 have been obtained from formulas (6), (7), (8) for the injection angles $\gamma_0 = 1^\circ; 2^\circ; 3^\circ; 4^\circ; 5^\circ$. They correspond to fiber 1 and 2 respectively with the smooth profiles. Core-cladding tunnelling leaky rays and refracted rays have been neglected.

The curves of figs 3 and 5 have been obtained from formulas (6), (7), (8) in the same conditions as for figs 2 and 4. They give the departure in time

![Fig. 2. R.M.S. width = SIG (∞) for smooth α-profiles as a function of α, for fiber no. 1; $a = 20$ μm; $n_0 = 1.5$; length = 1 km; $2\delta = 0.02$.](image-url)

162  Philips Journal of Research Vol. 35 No. 2 1980
Pulse broadening in multilayered fibers

Fig. 3. R.M.S. width (k layers) divided by R.M.S. width for the smooth-profile as a function of the number of layers; \( \alpha \) is taken as a parameter. Results are for fiber no. 1 (\( \alpha = 20 \mu m; n_0 = 1.5; \) length = 1 km; \( 2\Delta = 0.02 \)).

Fig. 4. R.M.S. width = \( \text{SIG}(\infty) \) for smooth \( \alpha \)-profiles as a function of \( \alpha \), for fiber no. 2; \( \alpha = 40 \mu m; n_0 = 1.5; \) length = 1 km; \( 2\Delta = 0.01 \).
dispersion of the multilayered structure as compared to the smooth structure as a function of the number of layers (the inhomogeneous dispersion is not taken into account). As an example, we note from figs 2 and 3 for fiber no. 1:

\[
\begin{align*}
\sigma_{\text{r.m.s.}} (10 \text{ layers}) & \approx 4 \times 10^{-9} \text{ s/km}, \\
\sigma_{\text{r.m.s.}} (160 \text{ layers}) & \approx 1 \times 10^{-9} \text{ s/km}, \\
\sigma_{\text{r.m.s.}} (\infty \text{ layers}) & \approx 1.8 \times 10^{-11} \text{ s/km}.
\end{align*}
\]

In a similar way, we note from figures 4 and 5 for fiber no. 2:

\[
\begin{align*}
\sigma_{\text{r.m.s.}} (10 \text{ layers}) & \approx 2 \times 10^{-9} \text{ s/km}, \\
\sigma_{\text{r.m.s.}} (160 \text{ layers}) & \approx 6 \times 10^{-10} \text{ s/km}, \\
\sigma_{\text{r.m.s.}} (\infty \text{ layers}) & \approx 5 \times 10^{-12} \text{ s/km}.
\end{align*}
\]

Half-height widths and total time excursion of the impulse response have
also been obtained but will not be presented in this paper, as we aim mainly at getting the major trends.

It was pointed out that the staircase approximation was superimposed on the $\alpha$-profile as drawn in fig. 1. Other types of stairlike approximations have been carried out as well (least square, equal increments in radius): they do not lead to significantly different results, within this framework, from the ones already described.

4. Discussion

As pointed out earlier in the introduction, several workers have published results on the multilayered propagation problem.

Arnaud and Mammel\textsuperscript{2}) for the stairlike profile make the following predictions.

For a fiber with the following parameters: $2\Delta = 0.01$, $\alpha = 1.988$ (optimum profile), $a = 40$ $\mu$m, $\lambda = 1$ $\mu$m one gets

\begin{align*}
\sigma_{\text{r.m.s.}} \text{ (10 layers)} & \approx 3 \times 10^{-10} \text{ s/km}, \\
\sigma_{\text{r.m.s.}} \text{ (160 layers)} & \approx 6 \times 10^{-12} \text{ s/km}, \\
\sigma_{\text{r.m.s.}} \text{ (\infty layers)} & \approx 4 \times 10^{-12} \text{ s/km}.
\end{align*}

This fiber can be compared to fiber no. 2 described previously, and we see that according to Arnaud and Mammel very good time dispersions can be obtained even for 10 layers, and results comparable to the smooth profile can be obtained for less than 100 layers. Another example is given for $2\Delta = 0.04$, $a = 40$ $\mu$m, $\lambda = 1$ $\mu$m, $\alpha = 1.951$ (optimum profile). However, no direct comparison can be carried out with fiber no. 1 as previously described.

Behm\textsuperscript{4}) uses a sinusoidal perturbation of the form

\[ cn_1^2 \frac{\Delta}{N} \sin \left( 2\pi N \frac{r^2}{r_c^2} \right), \]

where $N$ is the number of layers, $r_c$ the core radius, $n_1$ corresponds to the on-axis refractive index. This is actually a smoothed-out version of the staircase approximation, where the factor $c$ simulates the effect of the diffusion between adjacent layers. The results show that for all the cases considered by Behm, i.e. $10 < r_c/\lambda < 50$ and $0.005 < \Delta < 0.01$, the number of layers required to reach the performances of the smooth profile is not greater than 60 (the results are given in terms of the total time excursion rather than the r.m.s.).

As an example for $r_c/\lambda \approx 40$ and $\Delta = 0.005$ one can infer from the curves of Behm that
These results are to be compared with the results of fiber no. 2 and to the results of Arnaud and Mammel.

Another example to be compared to the results of fiber no. 1 is related to the following set of parameters:

\[ \frac{r_e}{\lambda} \approx 20, \quad \Delta = 0.01. \]

Then from Behm’s curves one deducts

\[ \Delta \tau (10 \text{ layers}) \approx 9 \times 10^{-10} \text{ s/km}, \]
\[ \Delta \tau (60 \text{ layers}) \approx 2.2 \times 10^{-10} \text{ s/km}, \]
\[ \Delta \tau (\infty \text{ layers}) \approx 2.2 \times 10^{-10} \text{ s/km}. \]

It should be noticed that the sinusoidal perturbation considered by Behm is much less detrimental to the time dispersion than the staircase approximation.

In any case, the results displayed by Behm would incline one to think that a relatively low number of layers would suffice to approximate a parabolic profile.

In a subsequent paper, Behm introduced an additional perturbation exponentially decreasing inside of each layer, and is able to predict by using first-order perturbation that the number of layers for optimum performance could go as high as 200, for the fiber parameters introduced previously. This additional term, in fact, together with the sinusoidal term tend to better approximate the stairlike profile as long as few oscillations occur within each layer.

Olshansky evaluates the pulse broadening of optimum profile fibers (very near to the parabolic profile) and thus uses first-order perturbation theory as Behm did. The perturbation considered by Olshansky is

\[ 2\Delta n_1^2 g \sin (2\pi n g) \]

with \( n \) = number of layers, \( n_1 \) the on-axis index of refraction and \( g \) the strength of the perturbation, \( g = r/a \). From the expression of the perturbation, one sees that it does not respect the equal incremental area condition. Olshansky finds a variation of the pulse broadening that is maximum for \( n_{\text{max}} = 0.15 \nu \). Thus, for fiber no. 1, \( \nu \approx 27 \), so \( n_{\text{max}} = 4 \), and for fiber no. 2, \( \nu = 38 \) so \( n_{\text{max}} = 6 \); thus the maximum pulse broadening should occur for 4 layers in the case of fiber no. 1 and 6 layers in the case of fiber no. 2. It also means from Olshansky derivation that beyond 12 layers for fiber no. 1, or 18 layers for fiber no. 2, the pulse broadening is practically negligible. Although this result is somewhat surprising, strictly speaking it does not apply to the case at hand, because as \( n \)}
Pulse broadening in multilayered fibers

increases, the strength $g$ should be decreasing as well in order to simulate properly the multilayered structure of the fiber, as Behm did.

5. Conclusions

We have reviewed the theoretically possible approaches to the multilayered propagation problem, as well as the main results.

Perturbation methods (first-order) have been used for the parabolic or near parabolic profile cases, where eigenmodes are well-known functions. Possibly, the same techniques could be extended to power-law profiles by using asymptotic expressions for the eigenfunctions; however, such methods have not been used up until now.

Numerical techniques have been used for the staircase approximation of the $\alpha$-profiles near $\alpha = 2$, by determining numerically the eigenfunctions of the scalar wave equation.

We have used ray techniques which, if they are conceptually not very well adapted to the multilayered problem, require very little computing effort. Moreover, the results obtained by this method, as shown in figs 3 and 5, do not go against what one may call an educated common sense.

In fact, whatever the theoretical tool used to study the problem, one should always reach the result that a larger number of layers (a few hundred at least with the help of our theory) are necessary in order to approach the performances of the smooth profile for near optimum fibers than for the far from optimum fibers for which a few tens of layers at most are quite sufficient in order to approach the smooth profile fiber performances.

A simple glance at figs 2 and 4 shows that the slightest perturbation of a near optimum fiber results in a dramatic loss of performance whereas the same perturbation will be hardly noticed for the far-from-optimum fibers.

Laboratoires d'Electronique et de Physique appliquée
Limeil-Brévannes, December 1979

REFERENCES


2) J. A. Arnaud and W. Mammel, Dispersion in optical fibers with stairlike refractive index profiles, Electron. Letters 12, 6-8, 1976.


