Abstract
The basic mechanism of electron transport in vacuum through insulating structures is discussed. The transport is based on a self-regulating secondary electron emission process. A general description of the transport process is presented. Three methods to model steady-state transport are briefly reviewed. The features are discussed in the light of application in displays. Also, non-steady-state effects and the role of space charge are addressed.

Keywords: Zeus display, electron transport, secondary emission, low-hop approximation, long-duct model, Monte Carlo calculations, modes, stability, space charge, propagation.

1. Introduction

The Zeus display is based on electron transport in vacuum through insulating structures [1,2]. The interaction of electrons with the walls plays an essential role in the transport process. In this respect the Zeus transport principle is very distinct from conventional ballistic transport used in cathode ray tubes. The principle of electron transport over insulators and its application in displays have been discussed earlier [1]. In this paper a more detailed description of the transport process will be given.

In Sec. 2 we discuss the basic principle of electron transport over insulators. As an onset to a more quantitative description a simple example is discussed in Sec. 3. In Sec. 4 a general description of the transport process is given, resulting in a set of transport equations. In Sec. 5 three methods to model steady-state transport are discussed and some dedicated examples are given. In Sec. 6 we discuss some miscellaneous topics.
Fig. 1. Basic experiment showing the possibility of electron transport through insulating structures. Electrons are injected into an insulating round tube. If the voltage \( V_a \) at the anode is sufficiently high, electron transport occurs through a self-regulating secondary emission process.

Sections 2 and 3 are meant as an introduction, whereas Secs 4 to 6 are more elaborate. Since the field of electron transport over insulators is fairly comprehensive we cannot give a detailed treatment of all topics. This should be done in separate publications. In this paper four topics are briefly outlined in Appendices A to D.

2. Principle of electron transport

The basic experiment demonstrating the principle of electron transport over insulators is shown in Fig. 1. Electrons from a cathode are injected into a glass tube. A voltage difference \( V_a \) is applied between the cathode and the anode. The ratio of the anode current and the cathode current versus the anode voltage is shown in the graph. At low anode voltages the anode current is zero. Above a certain threshold voltage \( V_{thr} \) the anode current equals the cathode current. The transition region is rather narrow and often shows hysteresis and instabilities.

For anode voltages above \( V_{thr} \) the glass tube acts as an electron duct. The transport mechanism is based on secondary emission. If an electron hits the insulating wall, secondary electrons are generated. The average number of secondaries per incoming electron depends on the energy and angle of incidence of the primary electron, and is designated by the secondary emission coefficient \( \delta \). The secondary electrons travel downstream, gain energy from
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Fig. 2. a. The total number of secondary electrons $\delta$ versus the landing energy of the primary electrons. $E_1$ and $E_{II}$ are the first and second cross-over energies, respectively. b. The secondary electron energy distribution. The contribution at low energies is due to true secondaries. The contribution at higher energies is due to backscattered electrons.

The material properties of the insulator play an important role in the transport process. Figure 2 shows two characteristic parameters. In Fig. 2a the total number of secondary electrons per incoming electron, $\delta$, is plotted versus the energy of the primary electrons, $E_{in}$; $\delta$ equals one at $E_1$ and $E_{II}$, respectively.
Fig. 3. Hopping transport over an infinite plane. It is assumed that all secondary electrons leave the surface perpendicularly with energy $E_0$. The uniform electric field points away from the surface at a certain angle. The force on the electrons points towards the surface.

The first cross-over energy, $E_1$, is typically a few tens of eV, and the second cross-over energy, $E_{II}$, is typically hundreds or thousands of eV. Figure 2b shows the energy distribution, $N(E_{out})$, of the secondary electrons. Roughly, two contributions can be distinguished: the 'true secondaries' with energies around $E_0$, and backscattered electrons with energies close to the primary energy, $E_{in}$. $E_0$ is typically a few eV. The number of backscattered electrons depends on the primary energy and the angle of incidence. The typical starting energy of the secondary electrons, $E_0$, and the energy of the first cross-over point in the $\delta$ curve, $E_1$, are particularly important. In regular secondary emission transport the $\langle \delta \rangle = 1$ situation is accomplished by charge deposition on the walls, such that the electrons land with energies around $E_1$ (as opposed to $E_{II}$, see later).

Electron transport by secondary emission is not limited to circular tubes, but can be applied to a great variety of structures. Some examples have already been described [1], including long (addressable) ducts, funnels and current switches. All of these are applied in Zeus displays. Although the structures are quite different and each structure serves a specific application, the basic transport mechanism is the same. The current–voltage characteristic exhibits the same behaviour as shown in Fig. 1: if a sufficiently high voltage is applied.
between the entrance and exit(s) of the structure, electron transport through secondary emission occurs.

3. Transport over an insulating plane

In the previous section we have given a qualitative description of the transport process. As a first step to a more quantitative description we discuss the instructive example of transport over an ‘infinite’ insulating plane. This case is known from the field of high-voltage surface flashovers. It was first described by Boersch et al. [3]. An infinite plane is a rather poor representation of the actual geometries encountered in displays, but it is very useful to model and to estimate several transport effects. The geometry is schematically shown in Fig. 3. A transport field is applied using electrodes far outside the region of interest. The electrons enter from the left.

The aim is to find the potential distribution for which \( (\delta) = 1 \) everywhere on the plane. In this section we use the very simple ‘nominal particle’ approach: it is assumed that the secondary electrons depart in the direction perpendicular to the surface with one single energy equal to \( E_0 \). The energy distribution and the angular distribution of the secondary electrons will be taken into account in the next section. The \( (\delta) = 1 \) condition implies that the electrons must land with an energy of either \( E_I \) or \( E_{II} \). For the moment it is taken to be \( E_I \); we will return to this later.

For the potential distribution we assume a uniform electric field, pointing away from the surface at a certain angle. After departing, the secondary electrons are forced back to the surface. This results in a ‘hopping’ motion. During flight energy is gained from the transport field. All electrons follow the same trajectories. The equations of motion are readily solved. The \( x \) and \( z \) coordinates are given by:

\[
x = \left( \frac{qF_x}{2m} \right) t^2, \quad z = \left( \frac{2E_0}{m} \right)^{1/2} t + \left( \frac{qF_z}{2m} \right) t^2,
\]

with \( F_x \) and \( F_z \) the components of the electric field; \( q \) is the electron charge \((-e)\) and \( m \) is the electron mass. The trajectories are tilted parabolas. It is straightforward to deduce that the landing energy is given by:

\[
E_{\text{land}} = E_0 + 4 \left( \frac{F_x}{F_z} \right)^2 E_0.
\]

If we require that the landing energy equals \( E_I \), then the following relation
between the components of the electric field is obtained:

\[
\frac{F_z}{F_x} = 2 \left( \frac{E_t}{E_0} - 1 \right)^{-1/2}
\]

Hence, for stationary transport over an infinite plane the ratio of the perpendicular and the parallel (transport) field is determined by the ratio of the first cross-over energy \(E_t\) and the nominal secondary electron energy \(E_0\). The smaller \(E_t\) or the larger \(E_0\), the more the electric field is directed perpendicular to the surface, and vice versa.

For the geometry of an infinite plane it can be made plausible that the transport ‘stabilizes’ on \(E_t\) and not on \(E_{\Pi}\) \([3]\). For this purpose the transport field, i.e. the field parallel to the surface, is assumed to be fixed. Now suppose that the perpendicular field is somewhat too large, because the plane is (somehow) positively charged. Consequently, the secondary electrons land too soon and do not gain enough energy from the transport field to establish the \(\delta = 1\) condition. If the nominal landing energy is around \(E_t\), a shift to lower energy implies \(\delta < 1\) (see Fig. 2a) and results in the deposition of negative charge and a decrease of the perpendicular field. Hence the system ‘stabilizes’. If the nominal landing energy were around \(E_{\Pi}\) a shift to lower energy would imply \(\delta > 1\), resulting in the deposition of positive charge. In this case the charge on the surface and the perpendicular field would continuously increase. Although it is evident that the transport on \(E_{\Pi}\) is not stable, this reasoning is not a proof that transport on \(E_t\) is actually stable. We will return to this in Sec. 6.2.

4. General description

4.1. Transport equations

In the previous section we treated a simple example which served as a qualitative introduction. For more complex geometries and secondary emission properties a more sophisticated description is required. The modelling of electron transport through secondary emission is, however, complicated. The basic equations are readily written down, but they are generally difficult to solve. Still it is worthwhile to start with a fairly general description which may become more condensed depending on the application and on the assumptions made.

A distinction is made between the following three processes: secondary emission, the deposition of charge on the wall, and the flight of the electrons in the vacuum. The central quantity in the modelling is the phase space density.
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of electrons \( f(r,v,t) \), which is the number of electrons per unit volume in \( r,v \) space at time \( t \). The phase space density should obey the following equations:

**Secondary emission:**

\[
\text{Secondary emission: } f_{\text{out}}(r,v_{\text{out}},t) = -\int \gamma(v_{\text{in}},v_{\text{out}})f_{\text{in}}(r,v_{\text{in}},t)v_{\perp\text{in}}dv_{\text{in}}. \tag{4}
\]

**Wall charging:**

\[
\text{Wall charging: } -\frac{1}{q} \frac{d\sigma(r,t)}{dt} = \int f_{\text{out}}(r,v_{\text{out}},t)v_{\perp\text{out}}dv_{\text{out}} + \int f_{\text{in}}(r,v_{\text{in}},t)v_{\perp\text{in}}dv_{\text{in}}. \tag{5}
\]

**Flight:**

\[
f_{\text{out}}(r_{\text{start}},v_{\text{start}},t_{\text{start}}) = f_{\text{in}}(r_{\text{land}},v_{\text{land}},t_{\text{land}}). \tag{6}
\]

The positions \( r, r_{\text{start}} \) and \( r_{\text{land}} \) are on the wall of the duct. The subscripts ‘out’ and ‘in’, c.q. ‘start’ and ‘land’, refer to particles moving ‘towards’ and ‘away from’ the surface, respectively. The outgoing and incoming velocity components perpendicular to the wall, \( v_{\perp\text{out}} \) and \( v_{\perp\text{in}} \), are positive and negative, respectively; the integrations should be carried out accordingly. The quantity \( \sigma \) denotes the charge density on the walls.

Equation (4) describes the secondary emission process. The function \( \gamma \) represents the secondary emission properties of the wall. Any time delay in the secondary emission process itself is neglected. The \( \delta \) curve and the \( N(E) \) distribution are both contained in \( \gamma \).

The process of charge deposition on the wall is expressed in eq. (5). The temporal change in the charge on the wall is proportional to the difference between the incoming and the outgoing particle fluxes.

In eq. (6) the incoming particle density is related to the outgoing particle density through Liouville’s theorem [4]. The starting coordinates \( (r_{\text{start}},v_{\text{start}},t_{\text{start}}) \) are related to the landing coordinates \( (r_{\text{land}},v_{\text{land}},t_{\text{land}}) \) through the electron trajectories, which are determined by the potential distribution.

The potential distribution plays an important role. For many applications it is of prime interest. It is implicitly contained in eq. (6) and it is determined by the wall charge appearing in eq. (5). Taking into account the proper boundary conditions (potential and/or charge on the wall) the potential must obey the Poisson equation:

\[
\Delta V(r) = -\frac{\rho(r)}{\varepsilon_0}, \tag{7}
\]

where \( V \) denotes the potential, \( \rho \) is the charge distribution in the vacuum and \( \varepsilon_0 \) is the dielectric constant. Usually the space charge of the electrons in the vacuum is neglected. This means that \( \rho \) is set to zero. Equation (7) then becomes the Laplace equation.
4.2. Time-dependence

There are two typical time scales in the electron transport process. The shortest time scale is due to the travelling time of the electron in the vacuum, which shows up in eq. (6). The travelling time can be estimated by considering the fact that during flight the electron must gain an energy of approximately $E_1$ from the transport field. In a transport duct of the Zeus display the time between start and impact is of the order of $10^{-9}$ s, and the distance travelled is about $10^{-3}$ m. This short time scale plays a role in propagation phenomena. The flight time must be accounted for if the electron density varies very rapidly in space and/or time. An example is the propagation and broadening of a short current pulse in a duct. However, for the majority of phenomena occurring in displays the travelling time is of little importance and can be neglected. This is done by setting $t_{\text{start}} = t_{\text{land}}$ in eq. (6).

A second time scale is related to the process of charge deposition as described by eq. (5). If the system is not in the steady state, e.g. shortly after the potential of an electrode in or close to the duct has been changed, charge is deposited. The time scale depends on the (local) current density and on the typical capacitances involved. In normally operated Zeus panels a time scale of typically $10^{-7}$ s is encountered. This is, however, a very crude figure. Throughout a duct, the current density may vary by some orders of magnitude. Particularly if somehow negative charging has occurred, the local current density can be very low and the time to re-establish the proper charge distribution may be orders of magnitude longer.

For the purpose of transporting electrons it is desirable to have a steady-state situation in which no wall charging occurs, i.e. $d\sigma/dt = 0$. In that case, the current entering the duct exactly equals the current leaving the duct. Many characteristics of the display can be described in terms of steady-state effects. Hence most effort is focused on investigating the steady-state situation where no dynamic charging occurs. However, for a number of phenomena (dynamic) charge deposition is actually important. In a display the electron current is continuously redirected using switching electrodes. At every switching event charge is deposited or removed causing 'charge transfer' effects [5,6]. Moreover, the charge deposition process plays an important role in the stability of the system. A steady state (if it exists) is not necessarily stable. In a stable system perturbations from outside and intrinsic stochastic variations are (continuously) counteracted by charge deposition. To investigate stability, dynamic charging can be incorporated using perturbation theory. A full dynamic description is very complicated and beyond the scope of this paper.
4.3. Material properties

As regards secondary emission, the material properties play an important role in the transport process. A detailed discussion of the secondary emission process can be found in the literature [e.g. 7–9]. In eq. (4) the material properties are represented by the function $\gamma$. In its general form $\gamma$ is of limited use. In practice it is split up into the more familiar total yield curve, $\delta(E_{\text{in}})$ and the secondary electron energy distribution, $N(E_{\text{out}})$. Roughly, this boils down to a separation of variables: ideally $\delta(E_{\text{in}})$ only depends on the incoming velocity or energy and $N(E_{\text{out}})$ only depends on the outgoing velocity or energy. In this case, the electrons have no 'memory' and modelling of the transport process is significantly simplified. However, in the actual transport process typically 20% of the electrons are backscattered [9]. Hence, the incoming and outgoing energy distributions are not fully independent.

Although much work has been done in the field of secondary emission, detailed and 'complete' sets of data are scarce. Generalizing, the secondary emission properties show the following features. The shapes of the $\delta$ curves for various materials are similar. They can be well mapped onto a universal reduced yield curve, describing $\delta/\delta_{\text{max}}$ versus $E/E_{\text{max}}$ [7–9]. At the energies encountered in electron transport, the $\delta$ curve depends only weakly on the angle of incidence [7,10]. The energy distribution of true secondaries can be approximated by a Maxwell–Boltzmann flux distribution [9,11]. The angular distribution is a cosine distribution [8,11]. The fraction of elastically backscattered electrons versus the primary energy follows a single curve for several materials [9]. The fraction increases at low primary energies. The angular distribution of the elastically backscattered electrons is generally more complicated than cosine [7].

If backscattering is neglected and if it is assumed that $\delta(E_{\text{in}})$ and $N(E_{\text{out}})$ only depend on the incoming and outgoing energy respectively, the function $\gamma$ is given by:

$$\gamma(v_{\text{in}}, v_{\text{out}}) = \delta(E_{\text{in}}) \frac{m^2}{2\pi E_{\text{out}}} N(E_{\text{out}}). \quad (8)$$

The factor $m^2/2\pi E_{\text{out}}$ accounts for normalization: $\int N(E) dE = 1$. In accordance with experimental data this representation implies that $\delta$ does not depend on the angle of incidence and that the secondary electron flux has a cosine distribution. It is readily verified that in this case the phase space density of outgoing electrons is given by:

$$f_{\text{out}}(r_{\text{out}}, v_{\text{out}}) = J_{\text{out}}(r_{\text{out}}) \frac{m^2}{2\pi q E_{\text{out}}} N(E_{\text{out}}), \quad (9)$$
with $J_{out}$ the current density due to electrons leaving the surface in A/m². In the steady state it is counterbalanced by an equal current density due to electrons coming towards the surface. The density $f_{out}$ is now the product of a position-dependent function and a velocity-dependent function; this simplifies calculations considerably.

There are several levels of sophistication in which the material properties can be implemented. If the proper data are available a sophisticated model for the detailed material properties can be implemented in e.g. Monte Carlo simulations. In analytical modelling the detailed shape of the $\delta$ curve and backscattering are complicating factors. For this purpose we use a Maxwell–Boltzmann energy distribution and a linear $\delta$ curve according to:

$$N(E_{out}) = \frac{E_{out}}{E_0^2} \exp\left(-\frac{E_{out}}{E_0}\right), \quad \delta(E_{in}) = \frac{E_{in}}{E_1},$$  \hspace{1cm} (10)$$

with $E_1$ the first cross-over point. Backscattering is neglected. For large primary energies the linear $\delta$ curve is a poor representation. However, since the energies of the impinging electrons are around $E_1$, the linearization can be justified. The advantage of this ‘model-emitter’ is that only two parameters are involved: $E_0$ and $E_1$. For the materials used in displays, even these two parameters (or parameters that can be reduced to an apparent $E_0$ or $E_1$) are often difficult to determine experimentally.
4.4. The insulating plane revisited

In Sec. 3 we discussed transport over an insulating plane. The energy distribution and the angular distribution of the secondary electrons were neglected. When these distributions are taken into account somewhat different results are obtained. The geometry is schematically represented in Fig. 4. As with the simple example of Sec. 3 a uniform electric field at a fixed angle with the surface is applied. It is obvious that for quite general materials parameters a field ratio $F_z/F_x$ can always be found such that a steady-state solution of eqs (4), (5) and (6) is obtained. The value of the field ratio is determined by an implicit expression containing the material properties. For the model-emitter it can be solved explicitly [12]. Inserting eqs (8), (9) and (10) into the transport equations (eqs (4), (5), (6)) yields:

$$\left| \frac{F_z}{F_x} \right| = \alpha,$$

with

$$\alpha = 2 \left( \frac{E_1}{E_0} - 2 \right)^{-1/2}. \tag{12}$$

Since it is regularly encountered we introduce $\alpha$ as a material parameter. Note the difference from eq. (3). The electron density is constant in the lateral direction. In the direction perpendicular to the surface the electron density drops exponentially according to $\exp(qF_zz/E_0)$ (see Sec. 5.3). The larger the transport field, the more the electrons are forced back to the surface. This phenomenon also occurs in the channels of displays, causing a sharp drop of the 'leakage' from the ducts with increasing transport field [6,13].

For completeness we mention that the solution discussed above is the most obvious one, but it is not unique. It is part of a broader class of solutions, in which the lateral electron density depends exponentially on the position according to $\exp(q(\lambda_xx + \lambda_yy)/E_0)$. This situation is slightly more complicated. The electric field is still uniform, but now there are two field ratios to be matched (see Appendix A). Moreover, the electron current and the electric field parallel to the surface are not in line. The reason is that the current consists of a field-driven component and a diffusion-driven component. The latter is caused by the fact that the electron density is not uniform. As such, the general solution is of limited practical importance: except for $\lambda_x = \lambda_y = 0$ it 'explodes' towards infinity. However, it can be used as a building block in the 'low-hop approximation', which will be discussed later.

Due to its simplicity, the example of transport over a plane is illustrative. In
Fig. 5. Schematic drawing, representing two regimes in the electron transport process. In the crossing mode (a) the electrons cross the volume of the duct. This mode usually occurs with relatively low transport fields. In the hopping mode (b) the electrons return to the wall by making small hops. This mode is associated with high transport fields.

fact, for most other geometries analytic calculations of the exact solution are prohibitively involved. A serious impediment is the calculation of trajectories. In the example given above, the field is uniform and the trajectories are simple tilted parabolas. The geometry of an infinite plane can be used to model and estimate several transport effects. Examples are: space charge, stability, propagation, etc. Furthermore, the concept of constant field ratio is encountered in many parts of the Zeus panel and forms the basis of the low-hop approximation.

5. Steady-state transport

5.1. General

In this section we discuss steady-state transport. Space charge is neglected. There are a few strategies for attacking the problem. The one to be followed depends on the geometry and on the boundary conditions. An important criterion is the applied transport field. Figure 5 shows an arbitrary geometry. If the transport voltage is relatively low the electrons will cross 'from one wall to the other'. Because they have to gain an energy of approximately $E_I$ from the transport field, they must be in free flight for a sufficiently long time. If the transport voltage is high, the electrons readily gain energy and the average flight time must be short. This can simply be achieved by hopping along the
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wall instead of crossing between the walls. For hopping transport the force due to the electric field must point towards the surface, so the perpendicular field itself must point away from the surface. The larger the transport voltage, the larger the perpendicular field and the lower the hop height.

The distinction between the hopping mode and the crossing mode is important. The former is much easier to describe than the latter. This is due to the fact that in hopping transport the electrons travel in a relatively thin sheet close to the wall. They only 'feel' the local field at the wall, and not the field distribution in the volume of the duct. Hopping transport can often be described by using the low-hop approximation, which amounts to solving an electrostatic boundary value problem.

In the crossing mode the trajectories extend over the whole volume of the duct. The equations of motion in combination with the Laplace equation hamper a solution in closed form. For long ducts, however, a model is available which simplifies the problem considerably. This long-duct model is instructive since it relates the electron density to the potential in a simple and almost obvious manner. In the end, however, the equations of motion still come into play.

A third — and for many geometries the only remaining — approach to obtain an 'exact' solution is to perform Monte Carlo simulations [12,14]. The advantage of these simulations is that they offer great flexibility with respect to the choice of the geometry and the material parameters. A drawback is that the calculations are time-consuming.

In the following we briefly outline the different methods for describing the transport process. Each method is illustrated with an example. For consistency we have chosen one single geometry that can be modelled with all methods described: two infinite parallel planes. This geometry is not actually encountered in displays, but it is an instructive example.

5.2. The low-hop approximation (LHA)

In the low-hop approximation it is assumed that the electrons travel in a thin sheet close to the wall by hopping. The force due to the electric field must point towards the surface. Furthermore, the field must be sufficiently large, so that the hop height is small compared to the typical dimensions of the system. Then, locally, the electric field can be considered uniform and the wall can be considered flat. Hence, as with transport over an infinitely large plane, the field ratio $F_z/F_x$ must satisfy:

$$\frac{|F_z|}{F_x} = \alpha. \quad (13)$$
The subscripts $x$ and $z$ refer to a local coordinate system. $F_z$ is the field perpendicular to the surface and $F_x$ is the field in the direction of the current flow.

For simple systems, where the symmetry requires that the direction of the field parallel to the surface coincides with the current flow, the definition of $F_x$ is obvious; it is simply the local transport field. Examples are two-dimensional systems or rotationally symmetric systems such as a funnel or a screen spacer [1]. This '2D' LHA is frequently encountered in practice; it boils down to solving the Laplace equation with oblique boundary conditions [15]. This concept has also been applied in the field of surface flashovers [16]. In more complicated systems the current flow generally does not coincide with the direction of the field. The electron density varies along the surface, creating a diffusion current. Then a '3D' description must be used, based on the more
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general solution of hopping over a plane (see Sec. 4.4). The result is a set of non-linear differential equations; these are briefly outlined in Appendix A. In the general case eq. (13) still holds.

Simple boundary conditions are also obtained for the 3D (or quasi 2D) problem of transport through a long translationally invariant duct. Disregarding the transport field, the potential is two-dimensional (in the plane perpendicular to the axis of the duct); the direction of current flow is parallel to the axis of the duct. The externally applied uniform transport field also points along the axis. Now, eq. (13) implies that the perpendicular field should be constant on the wall of the duct.

Hence, the LHA appears in two simple forms: for problems which are effectively 2D, it yields the concept of constant field angle, and for a long duct it yields the concept of constant perpendicular field.

As an example, Fig. 6 shows electron transport between two infinite planes. In Fig. 6a the transport occurs from bottom to top. There is no field or current perpendicular to the plane of the drawing. The potential\(^1\) is determined by solving the Laplace equation (eq. (7), with \(\rho = 0\)) with boundary conditions according to eq. (13) (constant field angle). In the example the parameter \(\alpha\) equals one. The potential in the transport direction decays exponentially. The scatter plot visualizes the electron density. Initially the electrons travel close to the surface. Going downstream, the perpendicular field decreases and hop height increases. Eventually the electrons will cross the space between the plates and the LHA is no longer valid. The field at the entrance of the duct is enhanced due to electron transport. This phenomenon is frequently encountered in Zeus displays: if a high (transport) voltage is applied, current-induced field emission and surface flashovers can occur at the entrance. Field enhancement is particularly important in the screen spacer [17] and in the constriction spacer.

As already mentioned, the concept of constant field angle is no longer valid if the hop height is of the order of the distance between the planes. With long ducts it is to be expected that the initial exponential decay of the transport field will level off and reach a constant value. In this case the transport field is evenly distributed over the length of the duct. If this transport field is sufficiently large, the LHA with constant perpendicular field can be applied. The potential\(^2\) is determined by solving the Laplace equation with the boundary

\[ V = a + b \cos(\mu x) \exp(-\mu y), \]

\[ a = \frac{\alpha F_T}{d}. \]

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\(^1\) The potential distribution is given by: \(V = a + b \cos(\mu x) \exp(-\mu y)\), with \(\mu = (2/d) \arctan(1/\alpha)\); \(x\) and \(y\) are the horizontal and vertical coordinates in the plane of the drawing, and \(a\) and \(b\) are constants (which are determined by externally applied boundary conditions). The plates are at \(x = d/2\) and \(x = d/2\), respectively.

\(^2\) The potential distribution in the plane of the drawing is given by: \(V = ax^2 - ay^2\), with \(a = \alpha F_T/d. F_T\) is the transport field.
condition of constant perpendicular field (eq. (13), with $F_x$ equal to the transport field $F_{tr}$). The result is shown in Fig. 6b. The transport occurs perpendicular to the plane of the drawing. The potential varies parabolically in both directions. The scatter plot visualizes the electron density. In this mode the transport is more localized, whereas in the constant angle mode the transport occurs over the whole (infinite) cross-section.

An intrinsic feature of the LHA is that the shape of the potential distribution is determined by the hopping process, but that the absolute scale is determined by externally applied boundary conditions. For instance, in the example of Fig. 6a the scale is determined by the potential difference applied between the bottom (the entrance) and the top (the exit) of the structure. If a twice as large potential difference is applied, the shape of the potential distribution remains unaltered, but the absolute values change by a factor of two. Since we have not specified the applied potentials or the $E_0$ of the secondary electrons, the scatter plots are somewhat arbitrary; they serve for visualization only. In the crossing mode the shape and the absolute scale of the potential distribution cannot be uncoupled.

In the LHA the field should point away from the surface everywhere on the insulating wall. This is not always possible. According to Gauss' law the total flux emanating from a volume is proportional to the charge enclosed. For a duct this means that flux out of the system is balanced by an equal amount of flux into the system (if space charge is not taken into account). Hence, there must be areas where the field points towards the surface. In most cases the flux emanating from the insulating walls is carried off by conductors at the entrance (or the exit) of the system. For an infinitely long duct a problem arises. In the example of Fig. 6b the flux is carried off at infinity. For a closed insulating geometry, however, this cannot occur and the LHA does not comply with Gauss' law. For the example of the long round tube this is an issue. In displays the ducts usually contain sufficient metallization to carry off the flux; the channels, for instance, contain many row extraction electrodes [12].

In Zeus displays the typical hop height is different in the various parts of the panel and also depends on the material properties. The glass walls are usually coated with MgO, which has good secondary emission properties and is very stable under electron bombardment. $E_0$ and $E_1$ are about 2 eV and 17 eV, respectively [9]. The value of $\alpha$ is close to one.

In normal display conditions the typical hop height of the (true) secondaries in the transport channels is about 25% of the channel width. Backscattered electrons have higher energies ($\sim E_1$). Hence, for the channels the LHA is at the edge of validity. For the constriction spacer and the screen spacer the
Fig. 7. Example of transport between parallel plates. The potentials are calculated using the long-duct model. The electrons travel perpendicularly to the plane of the drawing. In this example four modes can be distinguished: two symmetric modes (a and b) and two asymmetric modes (c and d).

The scatter plots indicate the electron density. Only one of the symmetric modes is confined.

hop height is quite small with respect to the spacer dimensions: of the order of 1% or less. In the spacers of the selection plates the hop height is estimated to be of the order of 10% of the spacer dimensions.

5.3. The long-duct model

The long-duct model applies to a long duct which is translationally invariant in the transport direction. It is not restricted to large transport fields and applies to both the hopping mode and the crossing mode. The secondary emission properties of the walls are taken according to eq. (10). The transport field is assumed to be uniform and directed along the axis of the duct. The aim is to find the potential distribution, $V_{cs}(r)$, in the cross-section of the duct. As is briefly outlined in Appendix B, the electron density in the duct is related to
In fact, the electron density and the velocity distribution perpendicular to the direction of current flow behave according to a free electron gas. If the potential is known, the charge density follows immediately. Knowing this in advance, the next problem is to calculate the potential. The transport equations imply that the average square time of flight of the electrons, \((\tau^2)\), is subject to certain conditions (see Appendix B). At this point trajectories must be calculated. This can be done in closed form for a limited number of geometries only.

As an example we again discuss the case of two infinite parallel plates. It can be shown that solutions exist if the potential is parabolic in the coordinates perpendicular to the transport direction. The example exhibits a very interesting feature of electron transport: several solutions or modes can exist. Figure 7 shows the potential distributions and scatter plots of four modes. In contrast to the LHA the potential distribution does not simply scale with the transport field; hence the plots represent the situation for one particular transport field. There are two symmetrical modes \((a,b)\) and two asymmetrical modes \((c,d)\). In one of the symmetrical modes \((b)\) the electron density is not confined; it explodes towards infinity and is therefore physically less relevant. The modes change as the transport field is varied. At a low field only the two (symmetric) modes exist. If the transport field is increased, the confined symmetric mode branches into one symmetric and two asymmetric confined modes. Beyond a certain transport field the asymmetric modes are no longer confined, so at high fields the only remaining physically relevant mode is the symmetric confined mode. This hopping mode is basically the same as that found using the LHA. The other, exploding, solutions are crossing modes.

The mathematical existence of modes does not necessarily mean that they occur in practice. This depends on the stability against perturbations. For two parallel plates the stability is still an open question. In this context it is worth noting that even at an arbitrary small (‘zero’) transport field two steady-state modes are found. Hence, a transmission curve like that in Fig. 1

\[ V_{cs}(r) \text{ by:} \]
\[ \rho(r) \propto \exp\left(-\frac{q V_{cs}(r)}{E_0}\right). \]  

In the example this quantity is set at 4, yielding four solutions with \((a,b) = (1.21,0), (-0.81,0), (0.61,1.28)\) and \((0.61,-1.28)\), respectively.

---

3) The general form of the solutions is: \( V_{cs} = \frac{E_0}{q}(ax^2 + bx - ay^2) \). Since the potential is the sum of a function of \(x\) and a function of \(y\), the average square time of flight associated with each starting point only depends on the starting position \(x\) and not on \(y\). The parameters \(a\) and \(b\) must be chosen such as to match eq. B.3 at both plates. \(a\) and \(b\) are functions of the dimensionless quantity \( qdxF_r/\gamma_0 \). In the example this quantity is set at 4, yielding four solutions with \((a,b) = (1.21,0), (-0.81,0), (0.61,1.28)\) and \((0.61,-1.28)\), respectively.
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Fig. 8. Example of a (2D) Monte Carlo simulation of transport between two parallel plates. Figure 8a shows the electron trajectories. Figure 8b shows the potential distribution. Figure 8c shows the potential distribution according to the low-hop approximation, which ignores crossing electrons.

cannot be obtained on the basis of the existence of modes alone. Presumably the system does actually exhibit a transmission curve behaviour, but then the blocking at low transport field would be a matter of stability instead of the mere existence of modes.

Although the example of two parallel plates is somewhat academic, the geometry does show a resemblance to a channel in a Zeus display. The aspect ratio of the channels is such that the side walls of the duct play an important role. In experiments [13] and in Monte Carlo simulations the potential distribution along the side wall shows a parabolic behaviour. For this reason the potential difference between the selection electrodes and the bottom of the duct is usually significantly larger than is to be expected without side walls.

5.4. Monte Carlo calculations

In Monte Carlo calculations the transport equations are solved numerically by simulating the transport process. Briefly, starting with a certain potential distribution electrons are injected into the duct. The landing position and the velocity are determined by ray tracing. Secondary electrons are generated according to the specified material properties. The generation occurs using random numbers. After tracing (part of) the cascade, the potential is adjusted according to the deposited charge. Then tracing is resumed and the cycle.
repeats until a steady state is reached [12]. The Monte Carlo simulation method is very flexible with respect to the geometries and the material properties. A number of examples found in displays are presented in a separate paper [14].

In this section we briefly present a Monte Carlo simulation of transport between two (semi-) infinite planes. The calculation is performed in two dimensions. The material properties are described in ref. [14]. Figure 8a shows the electron trajectories. The electrons travel from bottom to top. The corresponding potential distribution is shown in Fig. 8b. For comparison the result of the low-hop approximation is shown in Fig. 8c; the field ratio (the local $F_z/F_x$) is 0.77.

In the entrance region close to the wall the secondary electrons make small hops and the potential clearly shows the 'fixed angle' behaviour. This takes a considerable part of the transport voltage and results in a high electric field. In this region the Monte Carlo results and the results of the low-hop approximation are in good agreement. Beyond the entrance region a crossing mode sets in. This results in a more gradual distribution of the transport voltage. The potential distribution differs markedly from the result obtained with the low-hop approximation. One might expect that far downstream a situation with a perfectly homogeneous transport field would be reached. However, with the imposed invariance in the direction perpendicular to the plane of the drawing this is not possible. In the Monte Carlo simulations a true steady-state situation far downstream is not reached: details of the potential distribution continuously change. For parallel plates a steady-state mode which is invariant in the transport direction is only found if the potential and particle density vary in both perpendicular directions (see the previous section).

The strength of the Monte Carlo simulations is that they can be used to describe the transport in realistic three-dimensional geometries used in displays. The results, however, depend on the material properties used as input. If these are not (accurately) available, only a qualitative description can be obtained; by comparison with experimental data, material parameters may be inferred. If data on the material parameters are available a fair agreement with experiment is usually obtained.

6. Miscellaneous topics

6.1. General

So far we have dealt with 'regular' steady-state transport. In this section we discuss some topics that do not fit into this description: stability, propagation and space charge. The first two have in common that the time-dependence
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must be taken into account. In the last topic we address the role of the spatial charge distribution in the hopping process at high current densities.

6.2. Stability

The stability of electron transport is an important issue. The transport is stable if small perturbations in the potential distribution are corrected by charge deposition, in such a way that the original ($\langle \delta \rangle = 1$) potential distribution is re-established. In displays, the transport through the channels generally becomes unstable below a certain critical transport field. This can be observed through the 'leakage' current [13] which then varies in time and space. At higher fields the transport is usually stable. Stability is one of the criteria determining the minimum transport field used in displays.

In the example of transport over an infinite plane we have already addressed the question of stability. It has been argued that the transport 'stabilizes' on $E_1$ by considering the response to uniform charge deposition. This is not sufficient to prove stability, however. For that purpose it is necessary to consider the response to a localized excess amount of charge. Then, simple reasoning easily leads to the wrong conclusion. It can be argued that an excess amount of positive charge on a certain spot causes secondary electrons to land on that spot with higher than nominal energies. Since then $\langle \delta \rangle > 1$, more positive charge will be generated, leading to 'amplification' of the excess charge. On this basis it might be concluded that transport over a plane is unstable. However, electrons 'flying over' the spot and landing in the neighbourhood also feel the attraction of the positive charge. As a consequence they land somewhat too early and cause negative charging. The actual feedback process is a keen balance between these two processes and requires careful description.

To judge the stability of a system one may depart from a steady-state situation and introduce a small (time-dependent) perturbation. If the system is stable the perturbation 'decays' with time, if not it 'explodes'. A small surface charge changes the potential distribution, which in turn alters the phase space density. Through secondary emission this causes a change in surface charge. The process can be captured in the somewhat abstract equation $\partial \sigma / \partial t = A\sigma$. The operator $A$ is generally very complicated. Thus far the only case which has allowed a rigorous calculation is the transport over an infinite plane. This calculation is fairly elaborate and an outline is outside the scope of this paper. For the infinite plane the eigenfunctions of $A$ are plane waves. For the model-emitter it can be shown that the real part of the eigenvalues is always negative. This means that every perturbation decays, which implies that the transport is stable. With reference to the preceding paragraph, the
stabilizing feedback due to electrons flying over a (localized) perturbation apparently surpasses the amplifying feedback of the electrons landing on the perturbation itself.

For the example of the infinite plane the stability does not depend on the transport field. However, for a more realistic geometry a characteristic distance such as the width of the duct is involved. Then the transport changes from the hopping mode to the crossing mode and the invariance with the transport field is lost. Presumably, for the channels used in displays, the transition to the crossing regime causes instability.

The geometry of the round tube, mentioned in Sec. 2, shows a different behaviour. In experiments a stable mode is not observed, irrespective of the transport field. It is not yet clear whether this is due to the fact that geometry simply has no 'preferences' or that it is essentially unstable. In any case, the round tube is not an outstanding example of 'regular' transport. In an unstable system the local average secondary emission coefficient is not one; charge is continuously deposited and removed. Still, 'on average', a quasi-steady transport can occur.

6.3. Propagation

A current pulse released at the entrance of a duct will appear at the exit after a certain time delay. Furthermore, the stochastic nature of the secondary emission process causes a broadening of the pulse. The propagation time is related to the average velocity of the electrons in the duct. A characteristic property of the transport process is that the propagation velocity only weakly depends on the transport field because the impact energy of the electrons must always be around $E_i$. For hopping transport over a plane (and in the LHA), the propagation velocity is essentially independent of the transport field.

To describe the propagation process, the time-dependence of the 'flight' equation (eq. (6)) must be taken into account. The time-dependence of eq. (5) can be neglected (at low currents). For transport over an infinite plane expressions for the propagation velocity and the broadening can be derived; this is briefly outlined in Appendix C. In a round tube a propagation velocity of the order of $10^6$ m/s has been measured. This complies with an estimate based on flat plane transport. For a realistic display size the resulting delay is of the order of $10^{-7}$ s. The broadening is found to be proportional to the square root of the propagation time. For displays the broadening (in time) is of the order of $10^{-8}$ s. Since the addressing time is typically a few microse-
conds, the delay and the broadening are no obstacle for proper operation. At high field repetition rates the delay might be accounted for in the timing of the addressing pulses.

6.4. Space charge

At large transport currents the spatial charge distribution significantly affects the potential distribution and cannot be neglected. The question is at which value of the current space charge becomes important. As with previous topics, we consider transport over an infinite plane. For the infinite plane a solution can be obtained using the long-duct model; this is briefly outlined in Appendix D. Close to the surface the field lines bend due to space charge. Far from the surface the current density is low and the field is uniform. An interesting phenomenon is that the ratio of the perpendicular field (far from the plane) and the transport field depends on the current. With increasing current the field ratio decreases, meaning that the field is directed more parallel to the surface. From a distance it seems as if the secondary emission properties deteriorate with increasing current. The characteristic current at which space charge starts to play a role is designated \( i_0 \); it is defined in Appendix D. \( i_0 \) is the amount of charge per unit of time that passes an imaginary plane normal to the direction of current flow; it is expressed in A/m. Taking \( E_0 \) and \( F_{ir} \) as encountered in the channels of displays, \( i_0 \) amounts to a few times \( 10^{-2} \) A/m. The peak currents used in large displays are actually about this value. Hence, in these circumstances the transport occurs at the edge of space charge effects. In practice, the consequences are limited. It is only if the current is raised well beyond \( i_0 \), that the leakage from the channels [13] increases rapidly. This is consistent with a decreasing field ratio \( \alpha \). If it occurs, the space charge effect can be (largely) compensated by increasing the transport voltage.

7. Summary and conclusions

The basic mechanism of electron transport through insulating structures has been discussed. A general description of the transport process has been presented. Three methods for modelling steady-state transport have been reviewed: the low-hop approximation, the long-duct model and Monte Carlo simulations. Non-steady-state effects and the role of space charge have also been addressed.

The physics of electron transport through insulating structures is a novel and exciting field. Although in this issue the main interest is in displays, electron transport over insulators may have several other applications,
e.g. as an electron fibre, a bright current source, etc. [1]. Many different aspects of electron transport meet in displays. A detailed knowledge of the transport process is crucial for a proper design. In this respect modelling plays an important role. Obviously, the modelling of electron transport over insulators is only scarcely explored and deserves to be the subject of further investigations.

REFERENCES


Appendix A. The 3D low-hop approximation

The 3D low-hop approximation applies to three-dimensional geometries. The material properties are described by eq. (10). The force due to the electric field must be directed towards the surface. The hop height must be small compared to the dimensions of the system.

We first consider the general solution of hopping over a plane in a uniform field. The geometry is shown in Fig. 4. The density $f_{\text{out}}$ is written according to
eq. (9). For $J_{\text{out}}$ we assert a solution of the form:

$$J_{\text{out}}(r) = J_0 \exp \left( \frac{q}{E_0} (L_x x + L_y y) \right). \quad (A.1)$$

The factor $J_0$ (in $\text{A/m}^2$) is a constant. The quantities $L_x$ and $L_y$ (in $\text{V/m}$) are constants that should be matched in order to satisfy eqs (4), (5) and (6). The phase space density of incoming electrons $f_{\text{in}}$ can be expressed in $f_{\text{out}}$ using eq. (6) and the equations of motion. The latter are readily solved (e.g. eq. (1)). Inserting this into eqs (4) and (5) results in two expressions containing $f_{\text{out}}$. Taking $J_{\text{out}}$ according to eq. (A.1), a straightforward evaluation of these expressions yields:

$$L_x^2 + L_y^2 - L_x F_x - L_y F_y = 0, \quad L_x^2 + L_y^2 - F_x^2 - F_y^2 + F_z^2/\alpha^2 = 0. \quad (A.2)$$

Hence, eq. (A.1) is actually a solution of the transport equations. In the general solution the current is not uniformly distributed and it is not in line with the electric field parallel to the surface. It can be shown that the current density parallel to the surface (integrated over the $z$-direction and expressed in $\text{A/m}$) is given by:

$$i_x = J(r) \frac{4E_0}{qF_z^2} (F_x - L_x), \quad i_y = J(r) \frac{4E_0}{qF_z^2} (F_y - L_y). \quad (A.3)$$

In the low-hop approximation the surface is locally considered as a plane. The local solutions behave according to eqs (A.1) and (A.2). The local solutions can be 'connected' by setting:

$$J_{\text{out}}(r) = J_0 \exp \left( \frac{q}{E_0} L(r) \right). \quad (A.4)$$

A power series expansion of $L$ can be made in terms of an order parameter equal to the ratio of the typical hop height and the typical dimensions of the duct. Taking the lowest order terms again yields eqs (A.2), but now $L_x$ and $L_y$ denote the derivatives of $L$ with respect to the local coordinates $x$ and $y$; hence eqs (A.2) become a set of non-linear partial differential equations for the quantity $L$. With the same provisions to the meaning of $L_x$ and $L_y$, eq. (A.3) can also be applied to calculate the 'surface' current. If the local $x$-direction is chosen to coincide with the direction of current flow, eq. (13) is obtained.

Appendix B. The long-duct model

The long-duct model applies to long translationally invariant ducts. The materials properties are described by eq. (10). First, we consider a duct with
no transport field applied along the axis. Hence, the potential \( V(r) \) only depends on the coordinates in the plane of the cross-section of the duct. We now set:

\[
J_{\text{out}}(r) = J_0 \exp\left( -\frac{qV(r)}{E_0} \right), \tag{B.1}
\]

with \( J_0 \) a constant. The phase space density of electrons leaving the wall is:

\[
f(r, v) = \frac{J_0 m^2}{2\pi qE_0^2} \exp\left( -\frac{E + qV(r)}{E_0} \right).	ag{B.2}
\]

The phase space density only depends on the sum of the kinetic energy \( E \) and the potential energy \( qV(r) \). This total energy is conserved along an electron trajectory. According to Liouville's theorem [4] the phase space density at an arbitrary point in phase space (in the volume of the duct) equals the phase space density at the corresponding departure point on the wall. Hence, provided that the particle can be traced back to the wall, the density at every point in phase space is given by eq. (B.2); \( r \) may be in the volume of the duct and \( v \) may have any direction. For simplicity it is assumed that each point can be traced back to the wall. (If not, this can easily be corrected for afterwards.)

With a phase space density according to eq. (B.2) the outgoing particle flux balances the incoming particle flux at every point on the wall. The average landing energy is \( 2E_0 \). To get a situation where the average landing energy is \( E_l \), such that \( \langle \delta \rangle = 1 \), a transport field \( F_{\text{tr}} \) is applied. By applying the field, only the velocities parallel to the axis are affected. Energy is gained as the electrons move downstream. The 'add-on' energy equals 'force times the average distance travelled downstream'. Hence, the following relation must hold:

\[
qF_{\text{tr}} \frac{1}{2} \left( \frac{qF_{\text{tr}}}{m} \right) \langle \tau^2 \rangle = E_l - 2E_0, \tag{B.3}
\]

with \( \langle \tau^2 \rangle \) the average square time of flight. The averaging should occur over the incoming particle flux while keeping the point of impact fixed (or over the outgoing flux while keeping the starting point fixed).\(^4\) In the steady state the average square time of flight associated with each point should obey eq. (B.3). By applying the transport field the spatial density in the vacuum

\(^4\) \( \langle \tau^2 \rangle \) should be calculated according to: \( \langle \tau^2 \rangle = \int \tau^2(r_{\text{in}}, v_{\text{in}}) f_{\text{in}}(r_{\text{in}}, v_{\text{in}}) u_{\perp_{\text{in}}} d\nu_{\perp_{\text{in}}} / \int f_{\text{in}}(r_{\text{in}}, v_{\text{in}}) u_{\perp_{\text{in}}} d\nu_{\perp_{\text{in}}}. \) The subscript 'in' may be replaced by the subscript 'out'. \( \tau(r, v) \) is the time of flight associated with each point of impact (or starting point) \( r, v \); it only depends on the coordinates and velocities in the plane of \( V_0 \).
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ρ(r) remains unchanged and, according to eq. (B.2), is given by:

\[ \rho(r) = J_0 \left( \frac{2\pi m}{E_0} \right)^{1/2} \exp \left( -\frac{qV_{cs}(r)}{E_0} \right), \]  

(B.4)

with \( V_{cs} \) the potential distribution in the cross-section of the duct (the transport voltage is 'subtracted') and \( \rho_0 \) is a constant. For completeness we mention that the total current through the duct, \( I \), is related to \( J_0 \) by:

\[ I = J_0 \frac{4E_0}{qF_{tr} \alpha^2} \int_C \exp \left( -\frac{qV_{cs}}{E_0} \right) dl. \]  

(B.5)

The integral extends over the contour of the wall in the plane determining \( V_{cs} \).

Appendix C. Propagation

The propagation of a bunch of electrons over an infinite plane is described. The material properties are taken according to eq. (10). The transport occurs in the x-direction. The current density is invariant in the y-direction. A change in the potential due to the non-stationary character of the transport is neglected. Hence the potential remains fixed (with a field ratio \( \alpha \)). It is assumed that the spatial electron distribution varies slowly on the scale of a hop length. In contrast to ordinary steady-state transport the time of flight must be taken into account (eq. (6)). By inserting eq. (6) into eq. (4), an expression containing only \( J_{\text{out}} \) remains. Using eq. (9) this is turned into an expression containing \( J_{\text{out}}(r_{\text{land}},t_{\text{land}}) \) at the left-hand side and \( J_{\text{out}}(r_{\text{start}},t_{\text{start}}) \) at the right-hand side. By a Taylor expansion of the latter around \( (r_{\text{land}},t_{\text{land}}) \), sorting to order and integrating, the propagation equations are obtained. In first order one obtains:

\[ \left( \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \right) J_{\text{out}} = 0 \]  

(C.1)

This equation describes pure propagation. The current density profile simply propagates with velocity \( u \). In second order, and in a frame moving with velocity \( u \) (which means replacing \( x - ut \) by \( x \)), a diffusion-type equation is obtained:

\[ \left( \frac{\partial}{\partial t} - D \frac{\partial^2}{\partial x^2} - E \frac{\partial^2}{\partial x \partial t} - F \frac{\partial^2}{\partial t^2} \right) J_{\text{out}} = 0. \]  

(C.2)

This expression describes broadening due to 'diffusion'. It can be shown that for large \( t \) the last two terms between brackets become negligible. The remaining terms form a familiar diffusion equation containing only the parameter \( D \).
The propagation velocity $u$ (in m/s) and the diffusion coefficient $D$ (in m$^2$/s) can be expressed as follows:

$$u = \left(\frac{E_0}{m}\right)^{1/2} g(\alpha), \quad D = \frac{m}{qF_x} \left(\frac{E_0}{m}\right)^{3/2} h(\alpha).$$

(C.3)

$g$ and $h$ are (untractable) functions of the field ratio $\alpha$. They are shown in Fig. C.1. If a sharp current pulse is released at $t = 0$, it propagates with velocity $u$. In the moving frame it broadens and becomes Gaussian-shaped with a full 1/e width of $4(Dt)^{1/2}$, corresponding to a spread in time of $4(Dt)^{1/2}/u$.

Appendix D. Space charge

Electron transport over an infinite plane in the presence of space charge is described. The geometry is shown in Fig. 4. The material properties are taken according to eq. (10). The transport occurs in the x-direction. The transport field $F_x$ is fixed. The lateral current density $i$ (A/m) is uniform over the plane. The potential is calculated using the long-duct model.

We take $V(r) = V_{cs}(z) - qF_xx$. The charge density $\rho$ depends on $V_{cs}$ according to eq. (B.4). Inserting this into the Poisson equation (eq. (7)) yields:

$$\frac{\partial^2 V_{cs}}{\partial z^2} = -\frac{J_0}{\varepsilon_0} \left(\frac{2\pi m}{E_0}\right)^{1/2} \exp\left(-\frac{qV_{cs}}{E_0}\right).$$

(D.1)

5) The explicit expressions for $g$ and $h$ are: $g(\alpha) = \{4\sqrt{2}(8 + 5\alpha^2)\} / \{\alpha_\pi(12 + 5\alpha^2)\}$ and $h(\alpha) = \{8\sqrt{2}(8192 - 2304\pi + (13312 - 3408\pi)\alpha^2 + (7040 - 1512\pi)\alpha^4 + (1200 - 85\pi)\alpha^6 + 50\pi \alpha^8)\} / \{\alpha^3(12 + 5\alpha^2)^3\}$. 

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Fig. C.1. The propagation quantities $g$ and $h$ versus the field angle parameter $\alpha$. 

---

Fig. D.1. The relative field ratio, $\alpha^*/\alpha$, versus the relative current density, $i/i_0$. The quantity $\alpha^*$ is the field ratio $F_z/F_x$ at a large distance from the surface and $i_0$ is a characteristic current (see text).

$J_0$ is related to $i$ according to eq. (B.5). It is straightforward to show that eq. (D.1) is satisfied by:

$$V_{cs}(z) = \frac{2E_0}{q} \ln \left\{ \frac{\alpha^*}{\alpha} \left( \frac{i}{i_0} \right)^{1/2} \sinh \left\{ -\frac{qF_x\alpha^*}{2E_0} z + \sinh \left( \frac{\alpha^*}{\alpha} \left( \frac{i_0}{i} \right)^{1/2} \right) \right\} \right\}$$  (D.2)

with

$$i_0 = \varepsilon_0 F_x \left( \frac{2E_0}{\pi m} \right)^{1/2}, \quad \alpha^* = \left[ \frac{F_z(\infty)}{F_x} \right].$$  (D.3)

$i_0$ is expressed in terms of known quantities. The only 'free' parameter in eq. (B.2) is $\alpha^*$, which is the ratio of the perpendicular field and the transport field far above the surface. This field ratio determines the shape of the potential and the particle density. The parameter $\alpha^*$ is determined by eq. (B.3) (which follows from the $\langle \delta \rangle = 1$ condition). The flight time $\tau$ is given by:

$$\tau(v_{z0}) = 2 \int_0^{z_{\text{max}}} \left( v_{z0}^2 - \frac{2q}{m} V_{cs}(z) \right)^{-1/2} \, dz,$$  (D.4)

where $v_{z0}$ is the initial velocity in the $z$-direction and $z_{\text{max}}$ is the turning point. By inserting eq. (D.4) into B.3 a relation between $\alpha^*/\alpha$ and $i/i_0$ can be derived, which must be evaluated numerically. The result is shown in Fig. D.1. The field ratio at infinity decreases with increasing current. When compared to the case without space charge, the charge density far from the surface increases, whereas the density in the layer close to the surface decreases.