THE HEAT DISSIPATION
IN THE ANODE OF AN X-RAY TUBE*)

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I. INTRODUCTION; LOADS OF SHORT DURATION APPLIED TO STATIONARY ANODES

Summary
The life of an X-ray tube is often determined by the rate of evaporation of the target, and hence by the maximum temperature occurring during an exposure. For a given inflow of heat through the focus the temperature of the target may be computed from the equations of heat conduction. The requisite general equations are developed and are then applied to the problem of loads of short duration in tubes with stationary anodes. The problems connected with rotating anodes and with continuous loads will be treated in two further papers shortly to be published in this journal.

Résumé
La vie d’un tube à rayons X dépend d’ordinaire de la rapidité avec laquelle l’antécathode s’épuise et, par suite, du maximum de température atteint pendant son exposition au faisceau cathodique. Pour un apport donné de chaleur par le foyer, la température de l’antécathode peut être déduite de l’équation de la conduction calorifique. Les équations générales nécessaires sont développées et appliquées ensuite au problème de charges brèves dans des tubes à anode fixe. Les problèmes relatifs aux anodes tournantes et aux charges continues seront traités dans deux articles qui paraîtront d’ici peu de temps.

1. Introduction

The anode of an X-ray tube is shown diagrammatically in fig. 1; a tungsten target slightly larger than the focal spot is embedded in the front face of a copper anode, the cooling of which is effected by means of a large radiating body appended to it. The copper anode is also fitted with a ring for sealing to the glass envelope of the tube.

When a load is applied a beam of high-velocity electrons will strike the target, but only a small fraction of the total energy contained in this beam will be emitted as X-rays, the remainder being almost entirely converted into heat. The ensuing temperature increase limits the rating capacity, that is, the highest load at which the tube has an acceptable life.

As soon as its temperature rises above 2000 °K the target will be able to emit electrons in a measurable quantity, and when the tube is operated on

*) Slightly abridged and modified translation of Chapter II of the author’s thesis originally published in 1939 1).
A.C. an electronic current will be drawn from the anode to the cathode in the inverse phase. The consequent electron bombardment causes an increase in the temperature, and hence in the emission, of the cathode filament, which in turn leads to a further increase in the temperature of the target, and so on. This unstable condition, known as “inverse emission”, rapidly ends in the burning out of the filament. The temperature of the target in an A.C.-driven X-ray tube should therefore never rise above 2000 °K during the inverse phase.

![Fig. 1. Sectional view of an anode, with coordinate system; $F =$ focus, $T =$ tungsten target.]

However, most tubes are operated on D.C., and higher temperatures can then be permitted, until evaporation of the tungsten becomes appreciable. Owing to different orientations of the tungsten crystals and local differences in temperature, the rate of evaporation will not be uniform and the surface of the target will gradually become roughened. This causes absorption of part of the emitted X-rays in the anode itself, and hence a decrease in output.

In practice a total amount of evaporation of about 100 mg per cm², corresponding to a layer of 0·05 mm, can be permitted, and on this basis the life of an X-ray tube is represented in fig. 2 as a function of the target temperature. If we require a life-time of, say, 10,000 exposures of 0·1 sec each, the total time of exposure is not more than 16 minutes and the temperature may run up to 3200 °K; for 10,000 exposures of 10 sec the total time of exposure is 100 times as long and the maximum temperature lies in the neighbourhood of 2800 °K. For continuous loads, where a life of 1000 hours is usually demanded, the maximum temperature should not exceed 2600 °K.

Too high temperatures may limit the life of a tube in other ways; for instance, by causing gas eruptions, or by melting of the tungsten target or the copper behind it; these are accidents, however, that, with tubes of proper design, only occur as a result of faulty operation. Also, a heavy load may give rise to such pronounced temperature gradients that the target or the
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Copper backing gets cracked by the resultant stresses; by the use of suitable technological measures this can usually be avoided, and as a rule it is only observed in special types of tubes designed to withstand exceptionally heavy loads.

Fig. 2. Life-time of a tungsten target as a function of the temperature in °K for a total evaporation of 100 mg cm⁻².

In all the common types of tube the useful life as far as this is affected by temperature effects, is determined either by inverse emission or by the rate of evaporation, and we shall direct our attention mainly to the temperature limits set by these phenomena.

2. The equation of heat conduction

The temperature distribution in the anode during an exposure will be described by the equation of Fourier:

\[ a^2 \Delta T - \frac{\partial T}{\partial t} + \frac{a^2 q}{k} = 0, \]  

in combination with the appropriate boundary conditions, where:

- \( T \) = temperature,
- \( t \) = time,
- \( a^2 = k/c \) = thermometric conductivity,
- \( k \) = thermal conductivity,
\[ c \text{ = thermal capacity per unit volume,} \]
\[ q \text{ = internal heat gained or lost per unit volume in unit time,} \]
\[ \Delta \text{ = Laplace's operator} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \]

As heat is not generated within the anode but only dissipated at the boundary, the last term on the left-hand side is zero and the equation reduces to
\[ a^2 \Delta T - \frac{\partial T}{\partial t} = 0. \] (2)

In solving this equation we shall make two essential assumptions, viz:
(a) That the inflow of heat into the anode takes place only through the focal spot. This is not quite correct since the greater part of the secondary electrons dislodged in the focus returns to the anode outside the focal area, carrying with them an energy that, according to Seeman and Schotzky, may amount to 20% of the primary energy; the actual temperatures will then be lower than the calculated temperatures so that we err on the safe side in making this assumption.
(b) That the thermal properties of the anode materials are independent of temperature. This assumption may give errors of about 10% in the results of our calculations.

We shall discuss solutions of equation (2) in two particular cases, viz:
*Loads of short duration*, such that all the heat generated in the focus during one exposure is stored up in the anode, practically no heat being dissipated. This condition holds in medical radiography.
*Continuous loads*, leading to a state of equilibrium between the heat supplied and the heat dissipated. This is a condition that nearly always holds in therapeutical and technical X-ray applications.

Intermediate conditions, where energy is partly stored in the anode and partly dissipated at its boundary, are encountered in medical fluoroscopy and in some technical applications. Under these circumstances the solution of (2) grows too complex for simple treatment, but insight into the temperature distribution can be obtained from the solutions for the two cases mentioned above.

It may incidentally be noted that the dissipation of heat by the anode can be brought about in various ways. For small loads this is generally effected by using a cooling body, as shown in fig. 1, mounted in air or immersed in oil; for heavy loads it is, however, necessary to cool the anode by a forced stream of air or by a flow of liquid (water or oil). In addition to this, cooling by radiation is applied in medical tubes with rotating anodes and occasionally in other tubes too.
In the remainder of the present article we shall discuss the problems arising when loads of short duration are applied to stationary anodes. A considerable increase in output can be achieved by using rotating anodes, as the inflow of heat is then spread out over a larger area; the problems of short loads applied to a rotating anode will be treated in a second article, whilst in a third paper we shall discuss continuous loads in combination with stationary and with rotating anodes.

3. Loads of short duration applied to stationary anodes

A. General solution of equation (2)

As the dimensions of the focal spot are considerably smaller than the diameter of the anode, we may safely assume that the latter extends to infinity in both the $x$- and the $y$-direction $^*$. Moreover, our definition of "loads of short duration" implies that during one exposure no heat is conducted away at the boundary of the anode or the cooling body appended to it. Consequently the temperature of the anode at a certain distance from the focal spot remains substantially constant, so that in the $z$-direction, too, the anode can be assumed to extend to infinity.

The load $W$ may be inhomogeneously distributed over the focal area and may vary with time; hence we shall suppose that the specific load $W$, which will be expressed in watts/cm$^2$, is a function of $x$, $y$ and $t$. We shall also suppose that at time $t = 0$ the anode has a temperature $T = 0$.

Since the amount of energy lost as thermal radiation or in the form of X-rays may be ignored in comparison with the energy supplied, the appropriate boundary conditions will be as follows:

$$
\begin{align*}
\frac{\partial T}{\partial z} &= -\frac{W}{k} \quad \text{in the focus,} \\
\frac{\partial T}{\partial z} &= 0 \quad \text{outside the focus,}
\end{align*}
$$

and

$$
t = 0: \quad T = 0.
$$

Let us first solve equation (2) for a homogeneous isotropic anode. This may be done by a consistent application of the superposition principle, which holds in virtue of the linearity of the differential equation: the temperature field under the influence of any number of sources of energy is equal to the sum of the temperature fields brought about by each source

$^*$ This does not hold for a line focus but in that case only a negligible amount of heat will be conveyed in the $y$-direction. Hence $T$ will be independent of $y$, and the problem reduces from three to two dimensions.
separately. From this it follows at once that the temperature at any point and at any time will be proportional to the load $W$. In solving (2) two different cases will be considered.

a) The focal spot has finite dimensions both in the $x$- and in the $y$-direction.

If at time $t = 0$ an amount of energy $dE$ is supplied to a point inside a homogeneous and isotropic body the consequent temperature rise at time $t$ and at distance $r$ is known to be $3)\]

$$dT = \frac{dE}{8k\alpha\pi^2r^2} e^{-\frac{r^2}{4\alpha^2\tau}}.$$ \hspace{1cm} (4)

The temperature distribution will be symmetrical with respect to a plane through the point source, and there will be no heat transport through such a plane. Hence it is possible to apply expression (4) to our present problem; to that purpose we have only to replace $dE$ by $2W dF d\tau$, where $dF$ is a surface element of the focus, and to integrate over the focal area and over the interval during which the load is applied. This yields

$$T = \frac{1}{4\pi^2 \alpha k\alpha} \int_{\delta}^{+\delta} \int_{\delta}^{+\delta} \frac{W}{r} e^{-\frac{r^2}{4\alpha^2\tau}} d\tau dF,$$ \hspace{1cm} (5)

where $r$ is now the distance between the point for which the temperature is computed and the focus element $dF$. This expression gives the temperature for any point in the anode and after any time of exposure.

b) The focus extends to infinity in the $y$-direction and has a width of $2\delta$ in the $x$-direction; we shall call this a line focus of width $2\delta$.

The temperature will now be independent of $x$, and the solution of (2) may be derived from the two-dimensional analogue of (4), viz:

$$dT = \frac{dE}{4\pi k\tau} e^{-\frac{r^2}{4\alpha^2\tau}}.$$ \hspace{1cm} (6)

This equation expresses the relation between the energy $dE$ supplied to a line focus per unit length at time $t = 0$, and the rise in temperature thereby produced in a point at a distance $r$ and the time $\tau$.

Substituting $2W dx d\tau$ for $dE$ as before, and integrating over the focal width and the time of exposure, we obtain

$$T = \frac{1}{2\pi k} \int_{-\delta}^{+\delta} \int_{-\delta}^{+\delta} \frac{W}{r} e^{-\frac{r^2}{4\alpha^2\tau}} d\tau dx,$$ \hspace{1cm} (7)

which gives the temperature in any point of the anode at the end of an exposure. This equation may also be deduced from (5) by a single integration.
We now proceed to calculate the temperature field from equations (5) and (7) in a few practical cases. The temperature will of course be highest at the centre of the focal spot, and to this point we shall mainly direct our attention.

B. A circular focus of radius $R$ and homogeneously distributed load

From the general equation (5) we find for the temperature $T_m$ at the centre of the focus

$$T_m = \frac{W}{2ka\sqrt{\pi}} \int_0^1 \int_0^R e^{-\frac{r^2}{4\alpha t}} r \, dr,$$

or

$$T_m = \frac{2W}{\sqrt{k\pi}} \sqrt{t} \left[1 - e^{-\alpha t} + \alpha \sqrt{\pi} \right] 1 - \Phi(a),$$

where

$$a = \frac{R}{2a\sqrt{t}}$$

and $\Phi$ denotes the error integral *) which may be taken from tables 3). Putting

$$\Psi(a) = 1 - e^{-\alpha t} + \alpha \sqrt{\pi} \left[1 - \Phi(a)\right],$$

this equation becomes

$$T_m = \frac{WR \sqrt{t}}{k\alpha \sqrt{\pi}} \Psi(a).$$

(8)

Two particular cases may be distinguished:

1) Exposures of such short duration that

$$a = \frac{R}{2a\sqrt{t}} > 1.5.$$

Then $\Psi(a) = 1$, and consequently

$$T_m = 2W \sqrt{\frac{t}{\pi kc}}.$$

(9)

from which we see that the temperature at the centre of the focal spot is proportional to the square root of the time of exposure and independent of the diameter of the focus.

*)

$$\Phi(a) = \frac{2}{\sqrt{\pi}} \int_0^a e^{-\beta^2} d\beta.$$
If we assume that the focal spot extends to infinity both in the \( x \)- and in the \( y \)-direction, heat conduction will take place solely in the \( z \)-direction, and equation (7) transforms into

\[
T = \frac{1}{\sqrt{\pi}kc} \int_0^t \frac{W}{\sqrt{\tau}} e^{-\frac{x^2}{4a\tau}} \, d\tau,
\]

which for \( z = 0 \) yields

\[
T = \frac{W}{\sqrt{\pi}kc} \int_0^t \frac{1}{\sqrt{\tau}} \, d\tau = \frac{2W}{\sqrt{\pi}kc} \sqrt{t},
\]

an expression identical with (9). It may be noted that equation (9) applies to focal spots of any shape provided \( f/4a\sqrt{t} > 1.5 \), where \( f \) is the smallest dimension of the focus; under these conditions an appreciable transfer of heat takes place in the \( z \)-direction only, as during one exposure no heat penetrates into the anode to a depth of the same order of magnitude as the dimensions of the focus, so that lateral dissipation of heat does not yet play a part.

Such conditions prevail in tubes with a homogeneous copper or tungsten anode when the exposures last 0·005 sec or less. With a bimetallic anode this holds for even longer exposures, up to 0·05 sec, if the thermal conductivity of the target material (f.i. tungsten) is considerably lower than that of the basis metal (copper), as will be explained more fully in part D of this section.

2) Long exposures for which

\[
\frac{R}{2a\sqrt{t}} < 0.5;
\]

then \( \Psi(a) = a\sqrt{\pi} \), and we obtain:

\[
T_m = \frac{WR}{k},
\]

Thus we see that for these long exposures the maximum temperature in the focus decreases — and consequently the specific rating capacity per \( \text{mm}^2 \) increases — with decreasing focal dimensions; this results from the influence of lateral heat dissipation. This equation will be deduced in a different and very simple way in our third paper, where we shall deal with continuous loads.

It should be noted that the long exposures we are here considering still belong to the class of loads of short duration as defined in section 2. Equation (12) shows that under these conditions the maximum temperature is independent of the time of exposure.
For a copper anode and a circular focus of 6 mm diameter the quantity $kT_m/W$ computed from equations (12) and (9) has been plotted in fig. 3 against the time of exposure, curves I and II respectively. By comparing these curves it becomes apparent at what time of exposure the lateral heat dissipation begins to take effect.

![Graph showing $kT_m/W$ as a function of time for different focus shapes.](image)

Fig. 3. $kT_m/W$ as a function of the time of exposure for an isotropic copper anode and a homogeneous distribution of the load over the focal spot.

Curve I: a circular focus of 6 mm diameter.
Curve II: a focus of arbitrary shape if the time of exposure is very short (equation (9)).
Curve III: an infinite line focus of a width of 3 mm.
Curve IV: a finite line focus of $3 \times 9$ mm$^2$.

C. A homogeneously loaded line focus of width $2\delta$.

In this case equation (7) yields for the maximum temperature at the centre of the focus:

$$T_m = \frac{W\delta}{k\sqrt{\pi}} \int_{-\delta}^{\delta} \frac{1}{\alpha^2} \Phi(\alpha) d\alpha,$$

where $\Phi(\alpha)$ is the error integral as before; $kT_m/W$ computed from this expression for a copper anode and 3 mm focal width is represented by curve III of fig. 3.

For a finite line focus of $3 \times 9$ mm$^2$, having about the same area as the circular focus of 6 mm diameter, the maximum temperature at the centre has been computed in an analogous manner and is represented in fig. 3,
curve IV. For long exposures the difference in rating capacity is approximately 10% in favour of the line focus, but this is only a secondary advantage. As is well known, the major advantage is that a focus of $3 \times 9 \, \text{mm}^2$ when projected at an angle of $19^\circ$ into a square of $3 \times 3 \, \text{mm}^2$ gives a brightness about three times that in a perpendicular direction, owing to the fact that the X-rays are emitted with uniform intensity in all directions.

**D. Discussion of practical data**

We have seen above that the permissible maximum specific load is proportional to $T \sqrt{\kappa c}$ and to $T \kappa$ for short and for long exposures respectively, where $T$ is the permissible maximum temperature of the target material. The ratio of the energy radiated as X-rays to the heat dissipated in the anode is proportional to the atomic number $Z$, and consequently the maximum X-ray output obtainable per unit area will be determined by the product $ZT \sqrt{\kappa c}$ for short and $ZT \kappa$ for long exposures. For the metals generally used as target materials the data determining the maximum output have been collected in table I.

**TABLE I**

<table>
<thead>
<tr>
<th>Metal</th>
<th>$Z$</th>
<th>$k$</th>
<th>$c$</th>
<th>Melting point $^\circ\text{C}$</th>
<th>$T \sqrt{\kappa c}$</th>
<th>$ZT \sqrt{\kappa c}$</th>
<th>$T \kappa$</th>
<th>$ZT \kappa$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu</td>
<td>29</td>
<td>3.9</td>
<td>3.5</td>
<td>1080</td>
<td>$4.0 \times 10^3$</td>
<td>$1.2 \times 10^6$</td>
<td>$4.2 \times 10^3$</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>Ag</td>
<td>47</td>
<td>4.2</td>
<td>2.5</td>
<td>960</td>
<td>$3.1 \times 10^4$</td>
<td>$1.5 \times 10^6$</td>
<td>$4.0 \times 10^3$</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>Ta</td>
<td>73</td>
<td>0.53</td>
<td>2.4</td>
<td>3000</td>
<td>$3.4 \times 10^4$</td>
<td>$2.5 \times 10^6$</td>
<td>$1.6 \times 10^6$</td>
<td>$1.2 \times 10^6$</td>
</tr>
<tr>
<td>W</td>
<td>74</td>
<td>1.60</td>
<td>2.7</td>
<td>3350</td>
<td>$7.0 \times 10^4$</td>
<td>$5.2 \times 10^6$</td>
<td>$5.4 \times 10^3$</td>
<td>$4.0 \times 10^5$</td>
</tr>
<tr>
<td>Pt</td>
<td>78</td>
<td>0.70</td>
<td>2.8</td>
<td>1770</td>
<td>$2.5 \times 10^4$</td>
<td>$1.9 \times 10^6$</td>
<td>$1.2 \times 10^5$</td>
<td>$0.9 \times 10^5$</td>
</tr>
<tr>
<td>Au</td>
<td>79</td>
<td>2.96</td>
<td>2.5</td>
<td>1060</td>
<td>$2.9 \times 10^4$</td>
<td>$2.3 \times 10^5$</td>
<td>$3.1 \times 10^3$</td>
<td>$2.4 \times 10^5$</td>
</tr>
</tbody>
</table>

$k = \text{thermal conductivity in watts cm}^{-1} \, ^\circ\text{C}^{-1}$;

$c = \text{heat capacity per unit volume in watts sec cm}^{-3} \, ^\circ\text{C}^{-1}$;

$T \sqrt{\kappa c}$ is expressed in watts sec$^{1/2}$ cm$^{-2}$;

$T \kappa$ is expressed in watts cm$^{-1}$.

Since the vapour pressures of these metals are not known with sufficient accuracy, the melting points have been used to indicate the maximum temperatures that can be permitted; the temperature limit set by evaporation lies as a rule from 20 to 30% below the melting point; this suffices since in drawing conclusions from these data a high accuracy will not be required.

Tungsten is by far the best target metal owing to its high atomic number, its high maximum temperature, and its comparatively good
thermal conductivity. As the thermal conductivity of copper is far higher still, a considerable improvement can be effected by embedding a thin tungsten target in a solid copper anode. The optimum thickness of the tungsten will then be that thickness for which the tungsten and the copper simultaneously reach their permissible maximum temperatures.

For short exposures and a fixed maximum temperature the specific load is inversely proportional to the square root of the time of exposure, and the temperature gradient in the z-direction will vary in the same way, whereas the depth of thermal penetration will be directly proportional to the square root of the time of exposure. For long exposures the rate of decrease of the specific load with time will be slower owing to the lateral dissipation of heat. It will be evident that the optimum thickness of the tungsten target will vary with the time of exposure in a similar way. For exposures of 1 sec — an average value in medical diagnostics — the optimum thickness for stationary anodes is 2 mm according to Bouwers 4).

The temperature distribution in such a bimetallic anode can be computed in much the same way as for a homogeneous anode. Since the lines of flow of heat will be refracted from the normal in the ratio of the thermal conductivities, the divergence in the tungsten will be less than in a homogeneous copper anode. Hence the rating capacity remains independent of the focal dimensions up to higher values of the time of exposure than in the case of a homogeneous copper anode.

Eindhoven, January 1948

REFERENCES
4) A. Bouwers, Z. tech. Phys. 8, 272-277, 1927.