PROPOSALS AND RECOMMENDATIONS CONCERNING THE DEFINITIONS AND UNITS OF ELECTROMAGNETIC QUANTITIES

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Summary

As shown in a previous article, the fundamental concepts and equations of electromagnetic theory can be developed from a simple and unified point of view if we accept the concepts of current and voltage as our point of departure; a line of arguments that automatically presents itself, once rationalized Giorgi units have been adopted. The present paper brings a set of definitions of electromagnetic quantities and of the corresponding units which follow in a natural way from our previous arguments. To these some recommendations have been added that may be of use in passing over from one of the older cgs systems to the rationalized Giorgi system.

Résumé

Comme il a été montré dans un précédent article, les notions fondamentales et les équations de la théorie électromagnétique peuvent être développées d’un point de vue simple et unifié, si nous admettons les notions de courant et de tension comme point de départ, suite de discussions qui se présente d’elle-même, une fois que les unités rationalisées Giorgi ont été adoptées. Le présent article offre une série de définitions de quantités électromagnétiques et des unités correspondantes qui découlent d’une façon naturelle de nos discussions précédentes. Quelques recommandations pouvant être utiles lorsqu’on passe de l’un des anciens systèmes cgs au système rationalisé de Giorgi, y ont été ajoutées.

To complete the arguments of a previous article \(^1\) we venture to bring forward a set of proposals by which, in our opinion, many questions still in dispute may be finally settled. In these proposals various suggestions by Messrs Casimir, Gevers, De Groot and Tellegen of this laboratory have been incorporated.

Proposals \(^*)\)

(1) The Giorgi system is introduced in its rationalized form; \(\mu_0\) in particular is given by

\[
\mu_0 = \frac{4\pi}{10^7} \ldots \text{(H/m)}.
\]

(2) The “electric polarization”, \(P\), expressible in \(\text{C/m}^2\), is defined by

\[
D = \varepsilon_0 E + P \ldots \text{(C/m}^2\text{)},
\]

where \(D\) is the displacement or electric induction; \(\varepsilon_0 = 10^7/4\pi c^2 \ldots \text{(F/m)}\); and \(E = \text{electric field strength} \ldots \text{(V/m)}\).

\(^*)\) For the proposals (2), (4), (7), (9), (10), (11), (12), (13), (15) see, for instance, ref. \(^2\).
For the electric moment, $p$, (see 11) of a polarized body we then have

$$p = PV \ldots \text{(C} \cdot \text{m)} ,$$

where $V$ is the volume in m$^3$. The last equation can also be used for defining $P$.

(3) If required, we define an “electrization”, $F$, expressible in V/m, by

$$D = \varepsilon_0 (E + F) \ldots \text{(C/m}^2) .$$

By (2) we then have

$$P = \varepsilon_0 F \ldots \text{(C/m}^2) .$$

(4) The “magnetic polarization”, $J$, with the unit Wb/m$^2$, is defined by

$$B = \mu_0 H + J \ldots \text{(Wh/m}^2) ,$$

where $B = \text{magnetic induction} \ldots \text{(Wh/m}^2) ; \mu_0 = 4\pi/10^7 \ldots \text{(H/m)} ;$ and $H = \text{magnetic field strength} \ldots \text{(A/m)} .$

The magnetic moment of a magnetized body, $m$, in Wb·m (see 12) will then be given by

$$m = JV \ldots \text{(Wh} \cdot \text{m)} ,$$

an equation that also may be used for defining $J; V \ldots \text{(m}^3) .$

(5) The “magnetization”, $M$, in A/m, is defined by

$$B = \mu_0 (H + M) \ldots \text{(Wh/m}^2) .$$

By (4) we then have

$$J = \mu_0 M \ldots \text{(Wb/m}^2) .$$

(6) A quantity $\chi$, with the unit F/m, called the “absolute electric susceptibility”, is defined by

$$\varepsilon = \varepsilon_0 + \chi \ldots \text{(F/m)} ,$$

so that

$$\chi = P/E \ldots \text{(F/m)} .$$

(7) The quantity $\chi_r$, called the “relative electric susceptibility”, is defined by

$$\varepsilon_r = 1 + \chi_r ,$$

where $\varepsilon_r$ is the relative dielectric constant. This gives

$$\chi = \chi_r \varepsilon_0 \ldots \text{(F/m)} .$$

(8) Analogous to (6) we define the “absolute magnetic susceptibility”, $\mu$, by

$$\mu = \mu_0 + \chi \ldots \text{(H/m)} ,$$

so that

$$\chi = J/H \ldots \text{(H/m)} .$$
(9) In analogy with (7) the "relative magnetic susceptibility" is defined by
\[ \mu_r = 1 + \chi_r, \]
where \( \mu_r \) denotes the relative permeability. We then have
\[ \chi = \chi_r \mu_0 \ldots (\text{H/m}). \]

(10) The factor \( N \) of, respectively, de-electrization and de-magnetization is defined in such a manner that
(a) for a sphere we have \( N = 1/3 \),
(b) for an air-gap \( d \) in, respectively, a dielectric and a magnetic path of total length \( s \) and with a homogeneous distribution of electric or magnetic induction we have \( N = d/s \).
Then:
\[
\begin{align*}
D_m &= \varepsilon_0 E + P(1-N) & B_m &= \mu_0 H + J(1-N) \\
E_m &= E - NP/\varepsilon_0 & H_m &= H - NJ/\mu_0 \\
D_m &= \varepsilon_0 (E + F(1-N))' & B_m &= \mu_0 (H + M(1-N))' \\
E_m &= E - NF & H_m &= H - NM
\end{align*}
\]
(the subscript \( m \) signifies "in matter").

(11) The electric moment \( p \) of a dipole, with the unit C\cdot m, is defined by
\[ p = Qs \ldots (\text{C}\cdot\text{m}). \]
This dipole may be conceived as consisting of two point-charges \( Q \) equal in magnitude, opposite in sign, and a distance \( s \ldots (\text{m}) \) apart.

The maximum mechanical moment exerted by a homogeneous electric field with field strength \( E \ldots (\text{V/m}) \) on an electric moment \( p \) will then be
\[ M = pE \ldots (\text{N}\cdot\text{m}). \]

(12) The magnetic moment \( m \) with the unit Wb\cdot m is in the special case of a long bar-magnet, or a long coil, of length \( s \ldots (\text{m}) \) likewise defined by
\[ m = \Phi s \ldots (\text{Wb}\cdot\text{m}), \]
where \( \Phi \ldots (\text{Wb}) \) measures the flux emerging at one and re-entering at the other end.

For a coil with \( n \) turns, a cross-sectional area of \( A \ldots (\text{m}^2) \) and carrying a current \( I \ldots (\text{A}) \) we have in vacuum
\[ m = \mu_0 n AI \ldots (\text{Wb}\cdot\text{m}). \]

The maximum mechanical moment exerted by a homogeneous magnetic field with field strength \( H \ldots (\text{A/m}) \) on a magnetic moment \( m \) will then be
\[ M = mH \ldots (\text{N}\cdot\text{m}). \]
(13) The “electric polarizability”, \( \alpha \), is defined as
\[
\alpha = \frac{p'}{E} \quad (\text{C} \cdot \text{m}^2/\text{V}),
\]
where \( p' \) is the induced electric moment of one molecule \((\text{C} \cdot \text{m})\), and \( E \) the electric field strength \((\text{V}/\text{m})\). As indicated, the unit of electric polarizability is \( \text{C} \cdot \text{m}^2/\text{V} = (\text{C} \cdot \text{m}) : (\text{V}/\text{m}) \). Using the polarizability, the equation of Clausius and Mosotti with the Debye term added becomes
\[
\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{1}{3\varepsilon_0} \left( n \left( \alpha + \frac{p''^2}{3kT} \right) \right),
\]
where \( n \) is the number of molecules per \( \text{m}^3 \); and \( p'' \) (in the literature commonly denoted by \( \mu \)) is the permanent electric moment of one molecule.

(14) If desired, a quantity called the “electrizability”, \( \alpha_v \), may be introduced. Its unit is \( \text{m}^3 = (\text{C} \cdot \text{m}) : (\text{C}/\text{m})^2 \) and it is defined by
\[
\alpha_v = \frac{p'}{\varepsilon_0 E} \quad (\text{m}^3),
\]
so that
\[
\alpha = \varepsilon_0 \alpha_v \quad (\text{C} \cdot \text{m}^2/\text{V}).
\]
The electrizability conforms to the analogous concept in the system of Gauss, which has the unit of \( \text{cm}^3 \). Using the electrizability the equation of Clausius and Mosotti given under (13) reads
\[
\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{1}{3\varepsilon_0} \left( \alpha_v + \frac{p''^2}{3\varepsilon_0 kT} \right).
\]

(15) The unit of “magnetic polarizability”, \( \beta \), shall be \( \text{Wb} \cdot \text{m}^2/\text{A} = (\text{Wb} \cdot \text{m}) : (\text{A}/\text{m})^2 \), while we have
\[
\beta = \frac{m'}{H} \quad (\text{Wb} \cdot \text{m}^2/\text{A}),
\]
where \( m' \) is the magnetic moment of a single molecule \((\text{Wb} \cdot \text{m})\), and \( H \) the magnetic field strength \((\text{A}/\text{m})\).

(16) If desired, a “magnetizability”, \( \beta_v \), may be introduced, measured in \( \text{m}^3 = (\text{Wb} \cdot \text{m}) : (\text{Wb}/\text{m}^2) \), and defined by
\[
\beta_v = \frac{m'}{\mu_0 H} \quad (\text{m}^3),
\]
so that
\[
\beta = \mu_0 \beta_v \quad (\text{Wb} \cdot \text{m}^2/\text{A}).
\]

Some physicists prefer definitions of relative magnetic susceptibility (9) and of magnetic moment (12) differing from those given above, and which are based on the \( E-B \) analogy instead of on the \( E-H \) analogy which we have used. In that case it will be desirable also to introduce a different definition of magnetic polarizability. We are then led to what King 3 has called the \( B \)-definitions and which read as follows (see also ref. 4):
The "relative magnetic B-susceptibility" $\kappa_{rB}$ is defined by

$$1/\mu_r = 1 + \kappa_{rB}.$$  

By (9) we have

$$\kappa_{rB} = -\kappa_r/(1 + \kappa_r).$$

The "magnetic B-moment" is measured in units of A·m² and defined by

$$m_B = nIA \ldots (A\cdot m^2).$$

By (12) we have $m_B = m/\mu_0$.

The maximum mechanical moment of force exerted by a homogeneous magnetic field with an induction $B \ldots (Wb/m^2)$ on a magnetic moment $m_B$ will then be $M = m_B B \ldots (N\cdot m)$.

The "magnetic B-polarizability", $\beta_B$, is defined by

$$\beta_B = m'/B,$$

where $m'$ is the magnetic B-moment of one molecule \ldots (A·m²).

Its unit is A·m⁴/Wb = (A·m²) : (Wb/m²). We have

$$\beta_B = \beta/\mu_0^2.$$  

We are, however, of the opinion that the definitions (9a), (12a) and (15a) are not to be recommended for technical or practical purposes, and they should be used for theoretical purposes only when this is unavoidable.

To the previous proposals we should finally like to add some further recommendations:

(17) In the electromagnetic cgs system and in the system of Gauss it has so far been common practice to represent the magnetic properties of iron respectively by a $(B, H)$- or a $(J, H)$-curve, where $B$ is expressed in gausses, $H$ in oersteds or strictly also in gausses; $J$, too, is expressed in gausses, but the value of $J$ has to be multiplied by $4\pi$ before it can be added to $H$, according to $B = H + 4\pi J$. In these units the $(B, H)$-curve for vacuum is a straight line through the origin and making an angle of $45^\circ$ with the horizontal axis. For iron, $\mu_r$ equals the quotient of the ordinate and the abscissa.

In order to retain these convenient characteristics of this $(B, H)$-curve we recommend the use of respectively a $(B, \mu_0 H)$-curve and a $(J, \mu_0 H)$-curve in the rationalized Giorgi system.

The ordinate and the abscissa are then both expressed in the same unit, Wb/m². From the $(J, \mu_0 H)$-curve the induction $B$ can be deduced by adding corresponding values of $J$ and $\mu_0 H$, according to $B = \mu_0 H + J$. Besides, since 1 Wb/m² = 10 000 gausses, the values of the ordinates in the old $(B, H)$-curve system can be rapidly converted into those of the new $(B, H)$-curve by dividing by $10^4$. From the values of the ordinates
thus reduced we may compute the new values of the abscissa by dividing by \( \mu_r \) (abscissa = ordinate/\( \mu_r \)), the value of \( \mu_r \) being unaltered.

(18) We further recommend not to omit the multiplication sign "\( \cdot \)" in the expression of units defined as products of other units; we should write for instance

\[ V \cdot \text{sec} \] and not \( V \text{sec} \).

This is advisable in order to avoid ambiguities between \( m = \) milli- and \( m = \) metre; for example \( mV = \) millivolt, but \( m \cdot V = \) meter \( \times \) volt.

(19) The physical nature of a quantity should no longer be characterized by its dimension but simply by its rationalized Giorgi unit. Likewise the dimensional compatibility of an equation should always be checked by means of the rationalized Giorgi units and not by means of separate dimensional symbols.

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REFERENCES