THE COMPUTATION OF ELECTRODE SYSTEMS IN WHICH THE GRIDS ARE LINED UP

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Summary

Formulae are developed describing the paths of the electrons and the position of the focus in a system of electrodes in which the grids are lined up. These are then applied to the calculation of a plane arrangement such as to possess prescribed characteristics, and to have zero screen-grid current when the control grid is at zero potential.

Résumé

On développe les formules donnant les trajectoires des électrons et la position du foyer dans un système d'électrodes où les grilles sont alignées. Elles sont ensuite appliquées au calcul d'un système d'électrodes planes devant posséder des caractéristiques déterminées, et dont le courant de grille-écran doit être zéro quand la grille de commande est au potentiel zéro.

1. Introduction

A certain type of amplifier valve is so constructed that, viewed from the cathode, the grid wires of the screen grid are placed behind the grid wires of the control grid. This construction is chosen in order that the electrons travelling from cathode to anode shall pass between the grid wires of the second grid, so that the screen-grid current is zero or at least kept at a low value. It will be evident that this ideal condition can only be realized if the control grid bears a negative charge so that the electrons are repulsed away from the grid wires. They will then be concentrated in converging beams which possess a focus at some distance behind the control grid; and if the screen grid is located at a place where the width of the beam is less than the distance between two adjacent grid wires, the current going to the grid will be virtually zero. In this paper we shall develop a set of formulae for computing the correct dimensions of a valve of this type.

2. Basic principles

It is obviously necessary to know, at least to a fair approximation, the paths of the electrons, and we have also to bear in mind that in practical applications the varying potential of the first grid will produce corresponding variations in these paths.

This is illustrated in fig. 1 which reproduces a set of $I_a-V_a$ characteristics observed on a tetrode with a screen grid made of a fine-mesh gauze. By
this construction deflections of the electrons in passing the screen grid are kept at a very low value, so that the deflections actually observed are mainly due to the control grid. An increase in the average amount of deflection will entail a decrease in the slope of the steep part of the characteristic.

![Graph showing the $I_a-V_a$ characteristics of a tetrode plotted for various values of the control-grid potential.](image)

Fig. 1. The $I_a-V_a$ characteristics of a tetrode plotted for various values of the control-grid potential. Deflections in the screen grid have been kept low by constructing this grid of fine-meshed gauze. Deflections due to the control grid produce a decrease in the slope of the characteristic near the origin. The saturation current was kept at a very low value by using a low heating voltage.

It will be observed that the greatest deflections apparently occur when the grid is at zero potential, whereas one might be inclined to expect greater deflections at a lower grid potential owing to the greater negative charge on the grid. For a fixed point in the plane of the grid this will certainly be true. To explain fig. 1 we have to remember, however, that as the grid potential decreases a certain area surrounding the grid wires will become impenetrable to the electrons 1) 2) 3). Thereby the beam becomes narrower and the electrons that previously could pass the grid, though with a large deflection, are now completely reflected (fig. 2). In this way it can be understood that the largest deflections are observed at a grid potential in the neighbourhood of zero.

However, this being so, it will be evident that in a power valve, on the control grid of which an alternating voltage of considerable amplitude is impressed, it will be of foremost importance to construct the grids in such a way that the electron beams freely pass the screen grid when the control grid is at zero potential. As a basis for our calculation we shall therefore assume $V_{g1} = 0$. 
There will not as a rule be a very sharp focus, but we shall in the following be guided by considering the paths of those electrons which pass very close to the grid wires and which therefore obtain the largest deflections. The point where these paths cross each other will simply be taken as the focus of the beam, as this will be sufficiently correct for our purpose. The influence of the space charge between the grids will be neglected.

Fig. 2. The paths of the electrons in a plane triode with negatively charged grid. In the neighbourhood of the grid wires the potential is negative and the electron current on its way from cathode to anode has to pass between these negative regions.
It will be constructionally desirable to make the distance between the
two grids not too small, and it will for that reason be advisable to locate
the screen grid beyond the foci, where the electron beams are diverging.
If the space charge between the grids should have any influence at all, it
would, by the corresponding repulsion between the electrons, cause a shift
in the foci away from the control grid and towards the screen grid. This
will not invalidate the design of the valve.

So much for the principles on which the following calculations are based.

3. Formulae

The symbols to be used are:

\[ \alpha = \text{angle of deflection in the path of an electron while passing a grid}, \]
\[ V_{g1} = \text{potential of the control grid}, \]
\[ V_{eg1} = \text{average potential in the plane of the control grid}, \]
\[ V_{g2} = \text{potential of the screen grid}, \]
\[ d = \text{distance between two adjacent grid wires}, \]
\[ c = \text{radius of a grid wire}, \]
\[ x = \text{distance of a point in the plane of the grid from the centre of the nearest grid wire}, \]
\[ r_1 = \text{radius of the control grid}, \]
\[ r_2 = \text{radius of the screen grid}, \]
\[ 2z = \text{width of the beam (reckoned positive beyond the focus)}, \]
\[ l_1 = \text{distance between control grid and cathode}, \]
\[ l_2 = \text{distance between the two grids}, \]
\[ D = \text{"Durchgriff"} = 1/\mu, \text{ when } \mu \text{ designates the amplification factor}. \]

In an earlier paper \(^4\) it has been shown how the focus and the width
of the beam can be computed when the angles of deflection are small.
Since in power valves, for which the lined-up arrangement of the grids is
of special importance, this condition no longer holds good, a renewed and
more precise calculation will be required.

In a previous investigation \(^6\) we have found the angle of deflection for
an electron passing a plane grid at a positive potential to be

\[ \alpha = \frac{V_{eg} - V_g}{V_{eg}} \cdot \frac{\pi}{2} \cdot \frac{d - x}{d \ln(d/2\pi c)}, \]

provided \( |V_g - V_{eg}| \ll V_{eg}. \) When \( V_g = 0 \) this condition cannot be satisfied,
but from observation with the rubber membrane it may be inferred \(^6\) that
the above equation provides a sufficient approximation, even in this
region, if \( \alpha \) is replaced by \( \tan \alpha. \) The maximum angle of deflection \( (x = 0) \)
at \( V_g = 0 \) will then be given by

\[ \tan \alpha_m = \frac{\pi}{2 \ln(d/2\pi c)} \quad (1) \]
It will be noted that the angle of deflection becomes independent of the potential applied to the next grid, as might have been expected. For, neglecting the initial velocities of the electrons, their paths are known to depend only on the ratio of the two grid potentials, and this ratio will invariably be infinite when $V_g = 0$. Since the maximum angle of deflection by the control grid only depends on the construction of the control grid itself ($c$ and $d$), equation (1) may also be applied to cylindrical grids in a concentric arrangement with the grid wires at right angles to the cylinder axis.

We now proceed to trace the paths of the electrons after they have passed the grid and have been deflected through an angle $a_m$ (fig. 3). The total velocity of an electron between the control grid and the screen grid will be given by

$$v^2 = \frac{2e}{m} \left\{ V_{eg1} + (V_{g2} - V_{eg1}) \frac{\ln(r_2/r_1)}{\ln(r_2/r_1)} \right\}, \tag{2}$$

assuming for simplicity that we may put $V_{eg2} \approx V_{g2}$.

Since the electron considered has been deflected through an angle $a_m$, the square of its velocity parallel to the axis of the cylinder will be

$$v_1^2 = \frac{2e}{m} V_{eg1} \cdot \sin^2 a_m, \tag{3}$$

and consequently its velocity in a radial direction becomes

![Diagram](image_url)

Fig. 3: Illustrating the path of an electron while passing a negatively charged grid, and the splitting-up of its velocity vector into two components, one in a radial direction and the other at right angles to it.
\[
    v_{II} = \sqrt{v^2 - v_1^2} = \sqrt{\frac{2e}{m} \left\{ V_{eg1} + (V_{g2} - V_{eg1}) \frac{\ln(r/r_1)}{\ln(r_2/r_1)} - V_{eg1} \sin^2 a_m \right\}}. \tag{4}
\]

From this we obtain for the time, \( t \), required to travel from the first grid (radius \( r_1 \)) to a point with radius \( r \),

\[
    t = \int_1^r \frac{dr}{\sqrt{\frac{2e}{m} \left\{ (V_{g2} - V_{eg1}) \frac{\ln(r/r_1)}{\ln(r_2/r_1)} + V_{eg1} \cos^2 a_m \right\}}}. \tag{5}
\]

This is an integral of the form

\[
    t = \int_1^r \frac{dr}{\sqrt{a \ln r + b}},
\]

which may be solved by the substitution

\[
    a \ln r + b = u^2.
\]

In this manner we obtain the final solution

\[
    t = \frac{2\sqrt{r_1} e^{-C/r_1}}{2e \left( V_{g2} - V_{eg1} \right) / \ln(r_2/r_1)} \int e^{u^2} du \tag{6}
\]

where

\[
    C = \cos^2 a_m \frac{V_{eg1}}{V_{g2} - V_{eg1}} r_1 \ln \frac{r_2}{r_1}
\]

and

\[
    P(w) = \int_0^w e^{u^2} du \quad ?
\]

The time \( t_z \), required by an electron to travel from the plane of the control grid to a point where the beam has a width \( 2z \), is

\[
    t_z = \frac{1}{2} d + z = \frac{d + 2z}{2 \sin a_m \sqrt{2e \frac{V_{eg1}}{m}}}. \tag{7}
\]

Inserting this in (6) we find an expression for the width of the beam:

\[
    2z = 4e^{-C/r_1} \sin a_m \sqrt{r_1} \frac{V_{eg1} \cdot r_1 \cdot \ln(r_2/r_1)}{V_{g2} - V_{eg1}} \int P \left( \sqrt{\frac{\ln(r/r_1) + C/r_1}{\sqrt{C/r_1}}} \right) - d. \tag{8}
\]

With the aid of these equations we may compute the width of the beam at a given point or the position of the focus \( (z = 0) \) when the geometrical
dimensions of the valve are known. If, however, we intend to use these equations for designing a valve, further data are necessary. For a cylindrical arrangement I have not succeeded in obtaining a satisfactory solution of this problem in a closed form, but for plane electrodes such a solution is possible as will be shown in the following sections.

To arrive at formulae valid for a plane arrangement we have in (8) to replace \( r_2 \) by \( r_1 + l_2 \) and \( r \) by \( r_1 + y \), and then take the limit \( r_1 \to \infty \). This gives the width of the beam in the plane of the screen grid \( (y = l_a) \) as

\[
2z = \frac{2l_a V_{eg1} \sin 2a_m}{V_{eg1} - V_{g2}} \left\{ 1 - \sqrt{1 + \frac{V_{g2} - V_{eg1}}{V_{eg1}} \cos^{-2} a_m} \right\} - d ,
\]

and the focal distance as

\[
A = \frac{d}{2 \tan a_m} + \frac{d^2 (V_{g2} - V_{eg1})}{16 l_a V_{eg1} \sin^2 a_m} .
\]

4. The design of a valve with plane electrodes

We shall suppose that the following data have been prescribed:

(1) the characteristic specified by
   (a) the current \( I_{a0} \) at \( V_{g1} = 0 \), and
   (b) the cut-off potential \( V_{c0} \), that is, that value of \( V_{g1} \) at which \( I_a \) is zero;

(2) the effective cathode area \( O \);

(3) the effective screen-grid potential \( V_{g2} \).

Our purpose will then be to compute the dimensions of the control grid \((c \text{ and } d)\) and the distances between the cathode, the control grid, and the screen grid \((l_1 \text{ and } l_2)\) in such a manner that at \( V_{g1} = 0 \) we have \( I_{g2} = 0 \).

In addition to the various equations developed in the previous section we shall have to make use of Langmuir’s equation:

\[
I_k = 2.33 \times 10^{-6} \times O \times \frac{(V_{eg1} - V_m)^{n/2}}{(l_1 - x_m)^2} \ldots \text{(amp)},
\]

where \( I_k \) is the cathode current and \( V_m \) and \( x_m \) designate the potential and the position of the potential minimum in the space-charge region. In power valves it will often be permissible to neglect \( V_m \) and \( x_m \) in comparison with \( V_{eg1} \) and \( l_1 \), whilst in virtue of the low screen-grid current we may take \( I_k \approx I_a \); we then have

\[
I_a = A.0. \frac{V_{eg1}^{n/2}}{l_1} \ldots \text{amp},
\]

with

\[
A = 2.33 \times 10^{-6} \ldots \text{(amp.volt}^{-n/2}\text{)}.
\]
As our next step we have to express the average potential \( V_{eg1} \) in terms of the prescribed screen-grid potential \( V_{g2} \) and the dimensions of the grids. To this purpose we may use

\[
V_{eg1} = \frac{D_2 V_{g2}}{1 + \frac{D_1}{D_2}},
\]

and

\[
D_2 l_2 = \frac{3}{4} D_1 l_1,
\]

where \( D_1 \) and \( D_2 \) designate the "Durchgriff", respectively of the cathode and of the screen grid relative to the control grid. The distance \( l_1 \) between cathode and control grid has been multiplied by \( \frac{3}{4} \) in order, to account for the influence of the space charge on the potential distribution. \(^8\)

Provided

\[
\frac{2c}{d} < 0.1, \quad \frac{l_1}{d} > 0.7, \quad \text{and} \quad \frac{l_2}{d} > 2,
\]

we further have

\[
D_1 = \frac{d \ln(d/2\pi c)}{\frac{3}{4} \pi l_1} \quad \text{and} \quad D_2 = \frac{d \ln(d/2\pi c)}{2\pi l_2}.
\]

Or, alternatively, up to \( 2c/d < 0.3 \),

\[
D = \frac{\ln \coth (2\pi c/d)}{2\pi l/d - \ln \cosh (2\pi c/d)}
\]

will furnish a more accurate approximation. \(^9\)

Finally, we have for the cut-off potential

\[
V_{c0} = -D_2 V_{g2}.
\]

Assuming that the use of (12) and (15) or (16) is permissible we then have seven equations from which to compute nine unknown quantities, namely \( l_1, l_2, d, c, D_1, D_2, V_{eg1}, \sigma_m, \) and \( z \). To arrive at a definite result we have therefore to introduce two further conditions, for which we shall choose

(1) that the width of the beam in the plane of the screen grid shall be a prescribed fraction of the distance between the grid wires; this is necessary to ensure a low screen-grid current, and will be expressed in the form

\[
\frac{2z}{d} = S - 1,
\]

where \( S \) has a prescribed value;

(2) that the ratio of the diameter of the grid wires to \( d \) has a prescribed value; this condition can most conveniently be used as

\[
\ln \frac{d}{2\pi c} = Bz,
\]

where \( B \) is supposed known at the outset.
The problem is now completely specified, though the various equations to be used for its solution have not yet been arranged in an order suitable for direct computation. It will easily be verified, however, that after some rearrangement we arrive at the following precept.

First we prescribe
the cathode area $O$,
the anode current $I_{a0}$ at $V_{g1} = 0$,
the cut-off potential $V_{c0}$ at which $I_a = 0$,
the screen-grid potential $V_{g2}$,
the value of $S$ according to (18), and
the value of $B$ according to (19).

From these data we first compute

$$D_2 = -\frac{V_{c0}}{V_{g2}} \tag{20}$$

and

$$C_1 = \frac{1}{\frac{SD_2}{2} - 2} \tag{21}$$

With this value of $C_1$ we then obtain

$$l_1 = \left(\frac{A\hat{h}}{I_{a0}}\right) \left(-\frac{V_{c0}}{C_1 + D_2}\right)^{3/2} \tag{22}$$

$A$ being $2.33 \times 10^{-6}$ amp. volt$^{-3/2}$,

and then

$$l_2 = \frac{3l_1}{4D_2} (C_1 - 1) \tag{23}$$

$$d = \frac{2\pi D_2}{B} l_2 = \frac{3\pi l_1}{2B} (C_1 - 1) \tag{24}$$

$$e = \frac{d}{2\pi} e^{-B} = \frac{3l_1}{4B} (C_1 - 1) e^{-B} \tag{25}$$

Thereby the design of the valve has been completely specified.

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\textbf{REFERENCES}

5) Reference 3, equation (16).
7) Jahnke-Emde, Tables of functions, 1938, p. 32.