ON THE THEORY OF SECONDARY EMISSION
OF METALS

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Summary

The angular distribution of the secondary emission of nickel is measured with small intervals of energy. The results allow us to make a hypothesis about the mechanism of the emission, about the energy distribution, and about the scattering of secondary electrons inside the metal. An explanation is proposed for the cosine distribution of the emitted secondary electrons.

Résumé

La distribution angulaire de l'émission secondaire du nickel est mesurée pour de faibles intervalles d'énergie. Les résultats nous permettent de bâtir une hypothèse sur le mécanisme de l'émission, sur la distribution énergétique et sur la dispersion des électrons secondaires à l'intérieur du métal. On propose une explication de la distribution cosinusoidale des électrons secondaires émis.

Zusammenfassung


1. Introduction

When electrons having an energy of 25 eV or more impinge on a metal target, secondary electrons are liberated. Theories that try to explain this phenomenon have to deal (a) with the generation of secondary electrons inside the material, and (b) with the mechanism of the emission of the secondary electrons generated.

Our knowledge of these subjects is far from complete, because it is difficult to obtain experimental confirmation. At present, experimental data are available about:

(1) the ratio \( \delta \) of the number \( N_s \) of emitted secondary electrons (integrated over all energies and over all directions) to the number \( N_p \) of bombarding primary electrons, as a function of the energy of the latter, \( \delta = \frac{N_s}{N_p} = f_1(\varepsilon V_p) \);

(2) the energy distribution of the secondary electrons (for each energy integrated over all directions), \( dN_s = f_2(\varepsilon V_s) \, d(\varepsilon V_s) \);

(3) the angular distribution of the secondary electrons (for each direction integrated over a certain energy range), \( dN_s = f_3(\theta) \, d\Omega \).
The measurements of the relations mentioned under (1) and (2) are given in literature for many materials.

In former papers \(^1\)\(^2\), we described measurements as mentioned under (3). We found that the angular distribution of the true secondary electrons (low energies) leaving a smooth metal surface is nearly a cosine distribution. This also was the case with the electrons with moderate energies. Another result we found was that for both groups this distribution is practically independent of the angle of incidence of the primary electrons (fig. 1).

![Fig. 1. The effect of the angle of incidence upon the angular distribution of secondary electrons (from ref. 1), fig. 12e.](image)

On their way to the surface, the secondary electrons liberated inside the metal will be affected by collisions, which can be elastic or inelastic. The latter result in a loss of energy or a total absorption. At the surface, the electrons can be refracted or rejected by the surface barrier.

All these effects will influence the experimental curves considerably. In the existing theories \(^3\), some of these effects were taken into account and others were neglected, and it is difficult to prove from the experiments so far available, which supposition is correct.

The experimental data mentioned above under (1), (2) and (3) are not very suitable to test these theories. More information may be expected, at least about the way emission takes place, from measurements of the angular distribution of the true secondary electrons in much smaller energy intervals than were used so far. We shall describe these experiments in the following section.

2. Method of measurement

In a special test tube with a retarding field between two concentric
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spheres \(^1\), we measured the current of secondary electrons to a collector that could be placed under different angles to the surface of the target. In this way we performed some measurements of the angular distribution of secondary electrons of different energies.

However, the curves published in the paper cited only show some mean effect of all secondaries within a rather large energy interval (width \(>\) 10 eV). The sensitivity of the vibrating-condenser electrometer used was about \(5 \times 10^{-3}\) volts. This voltage was developed across a resistance of \(10^9 \Omega\), so that the minimum current that could be measured was \(5 \times 10^{-12}\) amperes. The ratio between the areas of half the sphere and that of the collector was 1 to \(6 \times 10^{-4}\). So the minimum secondary-emission current that could be measured was, integrated over all angles, \(8 \times 10^{-9}\) amperes. As the primary current was limited by space charge to \(10^{-6}\) amperes, the angular distribution could only be measured if the ratio \(I_{\text{total}}/I_p\) was not less than \(10^{-2}\).

In order to reduce the energy intervals, we had to improve the sensitivity, which was done by dynamically scanning the integrated retarding-voltage curve. This method gives directly the slope of the curve, i.e. the number of electrons as a function of their energy, \(dN_s = f_s(eV_s)\ \text{d}(eV_s)\).

To eliminate the influence of inter-electrode capacities, a rather low frequency of 70 c/s was used. The scanning voltage ranged from 0.01 to 0.3 volt r.m.s., and the sensitivity was increased to \(10^{-14}\) amperes. In this way it became possible to measure in a narrow energy interval the angular distribution of the secondary electrons with energy levels higher than 0.5 eV. As a target, a single-crystal nickel disc was used, which was bombarded normal to the \((111)\) surface.

At these low secondary energies, the compensation of the contact potentials is of great importance. As the primary electrons have high energies in our measurements, the only important contact potentials are those between target and inner sphere \((V_{ti})\), between collector and outer sphere \((V_{co})\), and between target and collector \((V_{tc})\). The contact potential between cathode and target \((V_{kt})\) is not important so far as the influence upon the secondary electrons is concerned. For the determination of the contact potential \(V_{tc}\) between target and collector, however, \(V_{kt}\) also had to be found (see (c) below).

(a) Contact potential between target and inner sphere \((V_{ti})\)

If there is a contact potential between target and inner sphere, the space in this sphere is not field-free and the secondary electrons leaving the target at a large angle of emergence will be deflected from the pure-radial orbit. The correct value of \(V_{ti}\) can be determined from the curves giving the angular distribution of very slow secondary electrons (0.5 eV, e.g.), measured with different values of potential applied between target
and inner sphere (fig. 2). When this contact potential is properly compensated, the tangent to the angular-distribution curve through the origin of the curve must be flush with the surface of the target.

![Fig. 2. The influence of a small difference between the potentials of the target ($V_t$) and the inner sphere ($V_{ip}$) upon the observed angular distribution of very slow secondary electrons (energy about 0.5 eV).](image)

(b) **Contact potential between collector and outer sphere ($V_{co}$)**

At the setting for low secondary energies (0.5 eV, e.g.), the alternating current to the collector is plotted against a variable d.c. voltage put in series with the loading resistor of the collector (see fig. 3).

When the collector is at a lower d.c. potential than the outer sphere, the retarding voltage or the energy level of the secondary electrons is given by the potential difference between target and collector. When the potential of the collector is the same as that of the outer sphere or slightly higher, the retarding voltage is given by the potential difference between target and outer sphere. In the first case the energy of the secondary electrons is higher than in the second case, and the alternating current to the collector will be correspondingly higher.

At greater potential differences between collector and outer sphere, the alternating current to the collector will decrease because of the action of the electrostatic lens between collector and outer sphere.

![Fig. 3. The influence of a small d.c. voltage, applied between collector and outer sphere, upon the alternating current to the collector. The knee $A$ in the curve indicates the point where the potentials of collector and outer sphere are equal. From this point the contact potential $V_{co}$ between collector and outer sphere can be determined.](image)
(c) Contact potential between target and collector ($V_{tc}$).

This contact potential can be determined in two steps only. First the contact potential between cathode and collector ($V_{kc}$) is found from the energy-distribution curve (fig. 4). The energy of the elastically reflected electrons is equal to the primary energy $eV_p$, and $V_{kc}$ is given by the shift of the peak of the reflected electrons in the energy-distribution curve with respect to the energy $eV_p$.

![Figure 4](image)

Fig. 4. The contact potential $V_{kc}$ between cathode and collector can be found from the displacement of the peak of the elastically reflected electrons in the observed energy-distribution curve with respect to the energy $eV_p$.

Secondly the contact potential between cathode and target ($V_{kt}$) is found by plotting log $I = f(V)$, in which $V$ is a voltage applied between target and cathode and $I$ the current to the target measured at low cathode temperature. The knee in the curve indicates the correct value of $V_{kt}$ (fig. 5). We then have

$$V_{tc} = V_{kc} + V_{kt}.$$  

When measuring at low secondary energies, we must have a very clean target surface. In our tube this was the case only during a few hours after flashing the target at 1000 °C.

![Figure 5](image)

Fig. 5. From the diode characteristic taken from the target, the contact potential between target and cathode can be determined.
3. Discussion of the measurements

The measurements made with these precautions confirm more in detail our former observations\(^1\). All secondary electrons on any given energy level between 1.5 and 20 eV are emitted with a cosine-like distribution (fig. 6). Neither with a poly-crystal target nor with a single-crystal target could we find a fine structure in this distribution. Again we found a small deviation of the cosine law, such that for lower energies the curves are slightly flattened.

Fig. 6. The angular distribution of secondary electrons of different energy levels, with normal incidence of the primary beam.

Theories are found in literature (e.g., Baroody\(^8\)), showing that the cosine distribution is caused by the loss of velocity of the electrons passing the surface barrier. Electrons reaching the surface are not able to escape if their velocity component perpendicular to the surface is not sufficient. This will be the case for many electrons in the low-velocity range. At greater angles of incidence less electrons within this energy range will be able to pass the surface barrier than at smaller angles, resulting in a cosine-like angular distribution for the emitted electron current. However, we also found a cosine distribution for electrons with energies large compared to the energy loss in the surface barrier (energies up to 100 eV).

As we found that electrons with such different energies all have approximately the same angular distribution outside, the occurrence of this angular distribution cannot be based upon the influence of the constant energy loss in passing the surface barrier.

The following calculation, which takes into account the diffraction at the surface, shows that the influence of the surface barrier upon the angular distribution of the emitted secondary electrons can only be
neglected if the secondary electrons that reach the barrier from the inside already have a cosine distribution 4).

If the electron from the inside of the metal reaches the surface with a velocity \( v_i \) under an angle \( p \) to the normal, and after emission has a velocity \( v_o \) and an angle \( q \) (fig. 7 *)

\[
\sin \frac{q}{p} = \frac{v_i}{v_o} = n = \text{constant for one given energy.}
\]

![Fig. 7. The diffraction of one electron passing the surface barrier.](image)

Differentiation gives:

\[
n \cos p \, dp = \cos q \, dq.
\]

Let a current \( d\sigma \) in a solid angle \( d\Omega \) inside the metal be located in a solid angle \( d\Omega \) outside the metal. The ratio of the current \( J_o \) per unit solid angle outside the material to the current \( J_i \) per unit solid angle inside follows from

\[
\frac{J_o}{J_i} = \frac{d\sigma/d\Omega}{d\sigma/d\Phi} = \frac{d\Phi}{d\Omega} = \frac{2\pi \sin p \, dp}{2\pi \sin q \, dq} = \frac{1}{n^2 \cos p}.
\]

Now if \( p = 0 \) and consequently \( q = 0 \), we have

\[
\frac{J_o(0)}{J_i(0)} = \frac{1}{n^2},
\]

so that

\[
\frac{J_o}{J_i} = \frac{J_o(0) \cos q}{J_i(0) \cos p}.
\]

*) \( v_o < v_i \), because an electron is losing an amount of energy equal to \( \psi \), the "gross work function", by passing the surface barrier coming from the inside of the metal.
Our measurements have shown that the angular distribution of the emitted electrons follows a cosine law, i.e. \( J_\theta = J_\theta(0) \cos \varphi \). From the former equation we may conclude that the current inside will then also follow that law and thus will be independent of the energy of the electrons.

Some time ago \(^4\) we discussed, assuming absorption and a large mean free path \( k \) (no elastic scattering), the possibility that the secondary electrons reaching the surface from the inside will have a cosine-like distribution outside. Some other properties of secondary emission could also be understood with these assumptions. To explain by this theory our recent experimental results, however, one would have to accept that secondary electrons of very different energies have the same absorption coefficient, which is not likely so. Moreover, the existence of the supposed large value of \( k \), based on Katz’s results \(^5\), can be doubted in view of later experiments of Berger \(^6\).

Another argument for the assumption of a small value of \( k \) could be found in the fact that the angular distribution of the secondary electrons is not really affected by the angle of incidence of the primary electrons; see ref. \(^6\) and fig. 1. If the secondaries are scattered many times on their way out, their original direction gets lost. On the other hand, we should not forget that the supposition of many authors, that the path of the primary electrons in the metal would be a straight line, does not agree with reality, and it is more probable that we have to expect something like fig. 8. The original distribution of the liberated secondary electrons at the end of the path of the primary electrons will in practice be more or less isotropic instead of parallel to the surface, as Fröhlich \(^7\) suggested for free electrons.

Assuming now many elastic and inelastic collisions before the secondaries reach the surface, we can explain our experimental results on the

\[ \text{Fig. 8. Scattering of the primary beam in the metal.} \]
angular distribution if we make the following suppositions for the secondary electrons of one energy level:

1. the mean free path \( k \) is small in comparison with the penetration depth of the primary electrons (many collisions);
2. the scattering of the secondary electrons takes place homogeneously in the metal;
3. the mean chance for one electron to move in a certain direction after being scattered several times is the same for all directions.

Then we see in fig. 9 that electrons that just are able to reach the surface in a direction perpendicular to it, originate from a layer of thickness \( k \). Electrons reaching the surface under an angle \( \rho \) originate from a layer of a thickness \( k \cos \rho \). It is clear that the secondary electrons reaching the surface will have a cosine distribution inside the metal if the three assumptions mentioned above are satisfied. As follows from the given calculation, they will then have a cosine distribution outside as well. As the energy level does not play a role in our argumentation, our conclusion must hold for all energies.

Assuming a small mean free path for elastic collisions \( k \), the existence of secondary electrons inside the metal could be compared with an electron gas at a very high temperature (10000 to 30000 °K). The energy of the emerging electrons will then be given by a Maxwell distribution and the angular distribution by a cosine law. Seen in this way, secondary emission is roughly similar to thermionic emission at very high temperatures.

In fig. 6 we notice a small deviation from the cosine law for electrons with low energies. This may be explained by the fact that the fraction of electrons reflected at the surface is larger for low-energy than for high-energy electrons.

We can make the following simple calculation:

\[
\frac{J_o(0)}{J_i(0)} = \frac{1}{n^2} = \frac{v_o^2}{v_i^2} = \frac{\varepsilon V_s}{\varepsilon V_i} = \frac{\varepsilon V_s}{\varepsilon V_s + \varepsilon\psi},
\]
where $\varepsilon \psi$ is the gross work function. It will be possible, starting from the observed energy distribution of the secondary electrons outside, to calculate the energy distribution of the same secondary electrons inside the metal just before their emission (fig. 10).

In conclusion, with finer measurements of the angular distribution, a contribution can be made towards better understanding of some of the processes underlying the phenomenon of secondary emission.

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REFERENCES

6) W. Berger, Naturwissenschaften 41, 59, 1954.