THE INFLUENCE OF NON-UNIFORM BASE WIDTH ON THE NOISE OF TRANSISTORS

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Summary
It is shown that the discrepancy between the experimentally observed values of the noise resistance $R_n$ and the values predicted by the noise theory of transistors is due to the presence of one or more regions with a non-uniform base width. In this case the effective base resistance, which mainly determines the noise resistance, is larger than that obtained experimentally from admittance measurements.

Résumé
On montre que la différence entre les valeurs expérimentales et les valeurs théoriques de la résistance de bruit $R_n$ d'un transistor est due à la présence d'une ou de plusieurs régions où l'épaisseur de la base n'est pas uniforme. Dans ce cas, la résistance de base effective, qui est le principal facteur déterminant la résistance de bruit, est supérieure à la valeur déduite de mesures d'admittances.

Zusammenfassung
Es wird gezeigt, daß der Unterschied zwischen den Meßwerten des Rauschwiderstandes $R_n$ und den durch die Theorie des Transistorrauschens vorausgesagten Werten durch ein oder mehrere Gebiete mit einer ungleichmäßigen Dicke der Basisschicht verursacht wird. In diesem Fall ist der effektive Basiswiderstand, der hauptsächlich den Rauschwiderstand bestimmt, größer als der Experimentalwert, den man aus Admittanzmessungen bekommt.

1. Introduction

The noise of transistors is mainly caused by statistical fluctuations in (a) the diffusion process by which the minority carriers move from emitter to collector through the base region, (b) the generation-recombination process by which electron-hole pairs are created thermally and disappear by recombination.

The fluctuation currents due to these processes, represented by current sources, can be calculated theoretically, as has been done by Van der Ziel 1) and Becking 2). By suitably inserting these noise-current sources in the equivalent circuit of the transistor, a noisy four-terminal network is obtained which may be studied as described by Becking, Groendijk and Knol 3). Proceeding in this way we obtain expressions for the noise quantities in terms of the elements of the equivalent circuit of the transistor, and these elements can be determined experimentally from measurements of the elements of the admittance matrix.

The results of these calculations and experiments will be reported in the near future and are summarized in the next section, but the main purpose of this paper is to describe an effect which we observed during the experiments. It
appeared that the values of the noise resistance $R_n$ shown by some of the transistors were too high, being a factor 1.5 to 2 above the theoretical value. The same transistors showed a discrepancy in the frequency dependence of the input admittance, and it appeared to be possible to explain both discrepancies by a non-uniformity in the base width of the transistor.

In this treatment the low frequency, so-called $1/f$ noise, will be neglected, since all observed noise quantities were independent of the frequency in the lower part of the observed frequency region.

2. Noise quantities of transistors

2.1. Introduction

If the applied a.c. signals are sufficiently low, a transistor can be considered as a linear noisy four-pole, which may be replaced \(^3\) by a linear noiseless four-pole preceded by a noise voltage generator $E$ and a noise-current generator $J$; fig. 1.

![Fig. 1. Equivalence of a transistor with internal noise sources and a noise-free transistor preceded by two external noise generators.](image)

The noise generators $E$ and $J$ represent the Fourier components at frequency $f$ of a fluctuating voltage $e(t)$ and a fluctuating current $i(t)$ respectively; so this representation is only valid for a narrow frequency band $df$. The spectral densities $S_E$ and $S_J$ of these Fourier components $E$ and $J$ are given by the relations

\[
EE^* = S_E(f) df \quad \text{and} \quad JJ^* = S_J(f) df,
\]

and the cross-correlation density $S_{EJ}$ is given by

\[
EJ^* = S_{EJ}(f) df,
\]

where the asterisk denotes the complex conjugate and the bar denotes averaging over a period long compared to $1/df$.

From the spectral densities and the cross-correlation density the following noise quantities have been defined \(^3,^4\):

- noise resistance \( R_n = \frac{EE^*}{4kTdf} \)
- noise conductance \( G_r = \frac{JJ^*}{4kTdf} \)
correlation numbers
\[ \zeta = \frac{EJ^* + E^*J}{4kTd} \]
and
\[ \kappa = \frac{EJ^* - E^*J}{j4kTd} \]

The minimum noise figure \( F_{\text{min}} \) (that is, the noise figure at a properly chosen value of the internal impedance of the signal source) is then given by

\[ F_{\text{min}} - 1 = \zeta + \sqrt{4R_nG_r - \kappa^2}. \]

In order to calculate the noise quantities of a transistor one can proceed in the following way. Starting from a properly chosen equivalent circuit the different noise sources are inserted. The noise sources can be distinguished in two kinds:
(a) the noise sources, representing the fluctuations in the generation-recombination process of the charge carriers and in the diffusion process of the minority carriers, diffusing from the emitter to the collector,
(b) the thermal noise sources, associated with the dissipative elements of the equivalent circuit.

Now the transition to the noiseless fourpole preceded by the two noise generators \( E \) and \( J \) is made and these noise generators are calculated. A calculation of this kind is given in the Appendix. The last step is then to calculate the four noise quantities \( R_n, G_r, \zeta \) and \( \kappa \), yielding expressions in terms of the elements of the equivalent circuit and of d.c. currents flowing in it. In order to compare the theoretical results with the experiments, the elements of the equivalent circuit must be determined, which is generally accomplished by measuring the fourpole admittances (sometimes additional measurements are required).

2.2. Simplified equivalent circuit

If we are interested only in the noise quantities of common practical transistors, at frequencies below the cut-off frequency of the current-amplification factor, considerable simplifications can be made.

Consider fig. 2, where the elements have the following meaning:
\( R_b = \) base resistance, \( C_e = \) emitter capacitance, \( r_e = \) emitter differential resistance = \( kT/qI_e \), \( a' = \) current-amplification factor in common-emitter configuration: \( a' = a/(1-a) \).

The circuit contains two formal noise-current generators \( i_{e,n} \) and \( i_{c,n} \), of which the spectral densities are given by

\[ S_{i_e} = i_{e,n} i_{e,n}^*/df = -2qI_e, \]
\[ S_{i_c} = i_{c,n} i_{c,n}^*/df = -2qI_c. \]
and the cross-correlation density by

\[ S_{te,te} = \frac{i_{e,n} i_{e,n}^*}{df} = 0, \]

where \( I_e \) and \( I_c \) are the emitter and collector d.c. currents respectively, having their positive directions towards the transistor, so that \( S_{te} \) is a negative quantity.

![Simplified equivalent circuit of a transistor in common-emitter configuration.](image)

\[ i_b = Y_{ie} V_{be} + Y_{re} V_{ce} \]
\[ i_c = Y_{fe} V_{be} + Y_{re} V_{ce} \]

Fig. 2. Simplified equivalent circuit of a transistor in common-emitter configuration.

Obviously the circuit of fig. 2 is incomplete in several respects. For instance for this circuit \( Y_{oe} = Y_{re} = 0 \), in contrast with the actual state of affairs in a transistor. Also the current generators give an incomplete description of the actual noise sources. However, the values of \( Y_{te} \) and \( Y_{fe} \) are identical with those of a complete equivalent circuit. Furthermore it can be shown that, in the considered frequency range, the noise quantities \( R_n, G_r, \xi \) and \( \kappa \), calculated with the aid of the simplified circuit of fig. 2, are very closely equal to those obtained from a complete circuit with complete noise sources, provided one adds a term \(-1\) to the calculated value of \( \xi \). An extensive justification of this statement will be published in the near future.

The input admittance in common-emitter configuration with short-circuited output terminals,

\[ Y_{te} = G_{te} + j\omega C_{te} = \left( \frac{i_b}{V_{be}} \right)_{V_{ce} = 0}, \]

can be calculated from fig. 2, yielding

\[ G_{te} = \frac{(1 - a_0)/r_e + (f/f_b)^2/R_b}{1 + (f/f_b)^2} \quad \text{and} \quad C_{te} = \frac{C_1}{1 + (f/f_b)^2}, \quad (1) \]

where

\[ f_b = (2\pi R_b C_1)^{-1} \quad \text{and} \quad C_1 = MC_e. \]

In the same way the transfer admittance,

\[ Y_{fe} = G_{fe} + j\omega C_{fe} = \left( \frac{i_c}{V_{be}} \right)_{V_{ce} = 0}, \]
can be obtained from fig. 2, which yields

\[ G_{fe} = \frac{a_0/r_e - 2f_C C_1 (f/f_C)^2}{1 + (f/f_C)^2} \quad \text{and} \quad C_{fe} = \frac{-1}{2\pi} \frac{a_0/r_C f_C + 2C_1}{1 + (f/f_C)^2}. \quad (2) \]

In these calculations the following approximation has been used for the frequency dependence of the current-amplication factor \( a \):

\[ a = a_0 \exp \left( -j\phi p \right) \quad (3) \]

where \( p = f/f_C, f_C = \text{cut-off frequency for } a \), that is, the frequency for which \( |a|^2 = \frac{1}{2} a_0^2 \) holds, \( \phi = \text{factor depending on the drift field in the base region, being equal to } 0.221 \text{ radian at zero drift field.} \)

This approximation holds for transistors with a uniform drift field, as has been shown by Te Winkel 5), who also introduced the factor \( M \), which is equal to 1.5 if there is no drift field in the base region.

It is clear from eqs (1) and (2) that measurement of the h.f. value of \( C_{te} \) will deliver the value of the emitter capacitance \( C_e \), and a measurement of the h.f. value of \( G_{te} \) can be used to determine the base resistance \( R_b \).

The four noise quantities \( R_n, G_r, \zeta \) and \( \kappa \) can now be calculated, using eq. (3), which yields for not too low values of the d.c. emitter current:

\[ R_n = \frac{1}{2a_0 r_e} \left[ r_e^2 + 2R_b r_e + R_b^2 (1-a_0^2) + R_b^2 p^2 \right] \approx R_b + \frac{1}{2} r_e + \frac{R_b^2}{2r_e} p^2, \quad (4) \]

\[ G_r = \frac{1}{2r_e} \left[ \frac{I_b}{I_e} - \left( \frac{1}{a_0^2} - 1 \right) \frac{I_e}{a_0} + \frac{1}{a_0^2} p^2 \right] \approx \frac{1}{2a_0' r_e} + \frac{1}{2r_e} p^2, \quad (5) \]

\[ \zeta = \frac{1}{r_e} \left[ R_b \frac{I_b}{I_e} - r_e + R_b \left( 1-a_0^2 \right) \frac{I_e}{a_0} + \frac{R_b}{a_0} p^2 \right] - 1 \approx \frac{1}{a_0} \left( 1 + \frac{R_b}{r_e} \right) + \frac{R_b}{r_e} p^2, \quad (6) \]

\[ \kappa = \frac{1}{r_e} \frac{\omega C_e I_e}{|a Y_e|} \approx -0.82 p, \quad (7) \]

where \( a_0 \) and \( a_0' \) denote the low-frequency values of \( a \) and \( a' \), respectively, and \( I_b \) is the d.c. base current.

To derive the right-hand part of the expressions (4) to (7) the following approximations have been made:

\[ a_0 = -\left( \frac{dI_e}{dI_e/dv_{eb}=0} \right) = -\left( \frac{I_e}{I_e/dv_{eb}=0} \right) \sim -1. \]

Recently Hibberd has derived these approximate expressions in a somewhat different way 6).
3. Experimental results and discussion

Experiments to check the theoretical results of sec. 2 have been carried out on alloy transistors of the type OC44 and OC45 in the frequency range 0·1 to 12 Mc/s.

From the results of our measurements it appeared that some transistors showed too high values of $R_n$ at low frequencies, being a factor 1·5 to 2 above the theoretical value. This means that the noise figure of these transistors exceeded the theoretical value by approximately a factor 1·5. The transistors having this discrepancy in $R_n$ also showed a discrepancy in the behaviour of the input admittance as a function of frequency.

![Figure 3. Input capacitance as a function of the frequency.](image1)

![Figure 4. Input conductance as a function of the frequency.](image2)
As an example figs 3 and 4 show the experimentally obtained curves of $C_{ie}$ and $G_{ie}$ versus frequency of such a transistor together with theoretical curves with suitably chosen parameters. The other fourpole admittances showed similar deviations, but we will restrict ourselves in this paper to the input admittance.

A possible qualitative explanation of the observed behaviour of $C_{ie}$ and $G_{ie}$ is to assume that the transistor under consideration cannot be represented by the simple equivalent circuit of fig. 2, but by a number of these circuits, connected in parallel, each circuit having different values of the corresponding elements.

![Fig. 5. Parallel connection of two circuits of fig. 2.](image)

The most simple form of this is the parallel arrangement of two circuits of fig. 2, as shown in fig. 5, where the noise sources have been omitted. Figures 6 and 7 give the frequency dependence of the total input conductance $G_{ie}$ and the total input capacitance $C_{ie}$ of the configuration of fig. 5, assuming that $C_{e1} > C_{e2}, R_{b1} > R_{b2}$ and $r_{e1} < r_{e2}$.
$C_{e1} > C_{e2}$, $R_{b1} > R_{b2}$ and $r_{e1} < r_{e2}$. By increasing the number of parallel circuits the frequency dependence of $G_{ie}$ and $C_{ie}$ will tend towards the dashed lines in figs 6 and 7 and these are in qualitative agreement with the experimental curves of figs 3 and 4.

Fig. 7. Input capacitance of the circuit of fig. 5. It is assumed that $C_{e1} > C_{e2}$, $R_{b1} > R_{b2}$ and $r_{e1} < r_{e2}$.

An equivalent circuit of the described form is necessary to represent a transistor in which one or more regions exist with a non-uniform base width. In fig. 8 three possible geometries with a non-uniform base width are given, of which we will consider fig. 8a in more detail. The main part of the emitter current of this transistor will flow through part 1, which has the smaller base width, so $r_{e1} < r_{e2}$ and $C_{e1} > C_{e2}$, due to the smaller base width $R_{b1} > R_{b2}$; assuming

that $a_{01} \approx a_{02}$ but $1 - a_{01} < 1 - a_{02}$ it is clear that the frequency dependence of $G_{ie}$ and $C_{ie}$ of this transistor is expected to follow the solid lines in figs 6 and 7. The structures of fig. 8b and c can be represented by a large number of parallel circuits, exhibiting the frequency dependence of $G_{ie}$ and $C_{ie}$ given by the dashed lines.

Fig. 8. Possible geometries having a non-uniform base-width.
In order to see whether there is any reason to assume the existence of regions having a non-uniform base width, a cross-section has been made of a transistor showing the discrepancies mentioned in the beginning of this section. This cross-section showed the structure of fig. 8c, so that this assumption seems to be quite reasonable indeed. We will now consider the influence of this effect on the noise quantities of the transistor.

4. Influence of non-uniform base width on the noise quantities

To simplify the treatment the parallel connection of only two transistors will be considered (fig. 9). Each transistor is represented by its admittance matrix $|Y|$ and its equivalent noise generators, and we have to calculate the resulting noise generators $E$ and $J$ of the parallel connection (fig. 10). It is shown in the Appendix that the resulting noise generators $E$ and $J$ are given by

$$E = \frac{1}{Y_{fe,1} + Y_{fe,2}} (Y_{fe,1} E_1 + Y_{fe,2} E_2),$$

$$J = J_1 + J_2 \frac{Y_{fe,1} Y_{fe,2} - Y_{te,2} Y_{fe,1}}{Y_{fe,1} + Y_{fe,2}} (E_1 - E_2).$$
Substituting into these equations
\[ \frac{Y_{fe,1}}{Y_{fe,1} + Y_{fe,2}} = B \quad \text{and} \quad \frac{Y_{te,1} Y_{fe,2} - Y_{te,2} Y_{fe,1}}{Y_{fe,1} + Y_{fe,2}} = A, \]
we obtain
\[ E = BE_1 + (1 - B)E_2, \quad (10) \]
\[ J = J_1 + J_2 - A(E_1 - E_2). \quad (11) \]

Using the definitions of sec. 2, bearing in mind that the noise generators \( E_1 \) and \( J_1 \) are uncorrelated with \( E_2 \) and \( J_2 \), and confining ourselves to the low-frequency region, as we are mainly interested in the l.f. behaviour, the calculations result in:
\[ R_{n0} = B_0^2 R_{n0,1} + (1 - B_0^2) R_{n0,2}, \quad (12) \]
\[ G_{r0} = G_{r0,1} + G_{r0,2} + A_0^2 (R_{n0,1} + R_{n0,2}) - A_0 (\zeta_{01} - \zeta_{02}), \quad (13) \]
\[ \zeta_0 = B_0 \zeta_{01} + (1 - B_0) \zeta_{02} - 2A_0 B_0 R_{n0,1} + 2A_0 (1 - B_0) R_{n0,2}, \quad (14) \]
where the subscript 0 denotes the l.f. value.

From the l.f. values of the fourpole admittances it can be derived (cf. eqs (1) and (2)) that
\[ B_0 = \frac{a_{01} \epsilon_2}{a_{01} \epsilon_2 + a_{02} \epsilon_1}, \quad (15) \]
\[ A_0 = \frac{a_{02} - a_{01}}{a_{01} \epsilon_2 + a_{02} \epsilon_1}. \quad (16) \]

In order to analyse this result we will assume that fourpole 1 represents the part with the smaller base width of a transistor having the structure of fig. 8a and of which the frequency dependence of \( G_{te} \) and \( C_{te} \) is given by the solid lines of figs 6 and 7. In this case \( \epsilon_1 < \epsilon_2 \), and as \( a_{01} \) and \( a_{02} \) will not differ very markedly, the value of \( B_0 \) will tend towards 1, indicating that the contribution of the region with the higher base resistance to the total noise resistance \( R_n \) and the correlation number \( \zeta \) will be relatively the larger; cf. eqs (12) and (14).

This conclusion is very important, since \( R_n \) and \( \zeta \) are usually calculated from eqs (12) and (14), after inserting in it the experimentally obtained value of \( R_b \). This value can be obtained either from measuring the h.f. input conductance \( G_{te} \) or from some other admittance measurement, but in each case something like the parallel connection of \( R_{b1} \) and \( R_{b2} \) in fig. 5 is measured, resulting in a value for \( R_b \) which is mainly determined by the lower base resistance. It will be clear that when this value is used to calculate \( R_n \) and \( \zeta \) from eqs (12) and (14) too low values are obtained.
To illustrate this we assume that 10% of the d.c. emitter current \( I_e = 1 \text{ mA} \) will flow through a part of the transistor of fig. 8a having a lower base resistance ("part 2"). Assuming the following properties:

| Part 1 | \( a_{01} = 0.99 \) | \( R_{b1} = 150 \Omega \) | \( r_{e1} = 28 \Omega \) |
| Part 2 | \( a_{02} = 0.90 \) | \( R_{b2} = 50 \Omega \) | \( r_{e2} = 250 \Omega \) |

the noise quantities of each part are (cf. eqs (4) to (7)):

| Part 1 | \( R_{n0,1} = 164 \Omega \) | \( G_{r0,1} = 0.179.10^{-3} \Omega^{-1} \) | \( \xi_{01} = 0.064 \) | \( \kappa_{01} \approx 0 \) |
| Part 2 | \( R_{n0,2} = 175 \Omega \) | \( G_{r0,2} = 0.2.10^{-3} \Omega^{-1} \) | \( \xi_{02} = 0.12 \) | \( \kappa_{02} \approx 0 \) |

From this it follows that \( A_0 = -3.3.10^{-4} \) and \( B_0 = 0.91 \).

The noise quantities of the complete transistor now become

\[
\begin{align*}
R_{n0} &= 136 + 14 = 137.4 \Omega, \\
G_{r0} &= (0.179 + 0.2 + 0.03 + 0.037).10^{-3} = 0.446.10^{-3} \Omega^{-1}, \\
\xi_0 &= 0.0582 + 0.0108 + 0.0095 - 0.0104 = 0.157.
\end{align*}
\]

If the base resistance of the complete transistor had been obtained from the measured h.f. value of the input conductance (cf. sec. 2), then for \( R_b \) we would have obtained

\[
1/R_b = 1/R_{b1} + 1/R_{b2} \text{ giving } R_b = 37.5 \Omega.
\]

Further \( a_0 = 0.98 \) and \( r_e = 25 \Omega \). In that case the noise quantities would have been:

\[
R_{n0} = 50 \Omega, \quad G_{r0} = 0.4.10^{-3} \Omega^{-1} \text{ and } \xi_0 = 0.05.
\]

Thus it follows from these calculations that the influence of the non-uniform base width is greatest on \( R_{n0} \) and \( \xi_0 \) and smallest on \( G_{r0} \). As it is rather difficult to determine these low values of \( \xi_0 \) with a good accuracy the discrepancy in this quantity will not easily be observed experimentally.

5. Conclusions

If a transistor contains a region with a smaller base width between emitter and collector junctions, a greater part of the current will flow through this region, and consequently the contribution of this region to the noise resistance and the correlation number of the transistor is relatively larger.

Generally the base resistance of a transistor is determined from admittance measurements and then the region with the smallest base resistance is the most important one. This means that if this value of the base resistance is used to calculate the noise resistance and the correlation number, the calculated values will be lower than the values observed experimentally.

In order to check the given explanation, transistors were selected having the right behaviour of \( C_{ie} \) and \( G_{ie} \) as a function of frequency. With these transistors good agreement was obtained between the theoretical and experimental values of \( R_{n0} \).
The author is indebted to Mr Memelink for his suggestion about the non-uniform base width and to Mr Brand for carrying out the measurements carefully.

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Appendix

The mesh and nodal equations of the circuit of fig. 9 are

\[ i_t = i_{b1} - J_1 + i_{b2} - J_2, \]
\[ v_t = \nu_{be1} - E_1 = \nu_{be2} - E_2, \]
\[ i_0 = i_{c1} + i_{c2}, \]
\[ v_0 = \nu_{ce1} = \nu_{ce2}. \]

From the admittance matrices it follows that

\[ i_{b1} = Y_{fe1} \nu_{be1} + Y_{re1} \nu_{ce1}, \]
\[ i_{c1} = Y_{fe1} \nu_{be1} + Y_{oe1} \nu_{ce1}, \]
\[ i_{b2} = Y_{fe2} \nu_{be2} + Y_{re2} \nu_{ce2}, \]
\[ i_{c2} = Y_{fe2} \nu_{be2} + Y_{oe2} \nu_{ce2}. \]

If the input signals are chosen as follows:

\[ i_t = -J \text{ and } v_t = -E, \]

then no input signal will exist at the input terminals of the fourpoles themselves, and as these fourpoles are noiseless no output signals will exist either. By making use of this property the equivalent noise generators \( E \) and \( J \) can be calculated after substituting

\[ v_0 = 0 \quad i_0 = 0 \quad v_t = -E \quad i_t = -J \]

in the equations. The exact proof of this statement has been given by Becking [7]. With this substitution the following set of equations is obtained:

\[ -J = i_{b1} + i_{b2} - J_1 - J_2, \quad i_{b1} = Y_{fe1} \nu_{be1}, \]
\[ -E = \nu_{be1} - E_1 = \nu_{be2} - E_2, \quad i_{c1} = Y_{fe1} \nu_{be1}, \]
\[ i_{c1} + i_{c2} = 0, \quad i_{b2} = Y_{fe2} \nu_{be2}, \]
\[ \nu_{ce1} = \nu_{ce2} = 0, \quad i_{c2} = Y_{fe2} \nu_{be2}. \]

From these equations \( E \) and \( J \) are easily solved, yielding:

\[ E = \frac{1}{Y_{fe1} + Y_{fe2}} (Y_{fe1} E_1 + Y_{fe2} E_2), \]
\[ J = J_1 + J_2 - \frac{Y_{fe1} Y_{fe2} - Y_{fe2} Y_{fe1}}{Y_{fe1} + Y_{fe2}} (E_1 - E_2). \]

REFERENCES