PARAMETRIC-AMPLIFIER ELECTRON GUN

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Summary

Two guns of high perveance \((3 \times 10^{-6} \text{AV}^{-3/2})\) and low beam voltage \((6 \text{ V})\) are described. One gun is of the Brillouin type, the other of the immersed-flow type. The guns are intended for application in transverse-velocity parametric beam amplifiers. Experimental Adler tubes were built and tested at a signal frequency of 550 Mc/s. The minimum noise figure obtained with the Brillouin gun is 1.31 \((90^\circ \text{K})\) at a maximum gain of 30 dB, and the saturation gain is 50 dB. The corresponding quantities obtained with the immersed-flow gun are 1.4 \((116^\circ \text{K})\), 22 dB and 30 dB, respectively. This indicates that the gain obtained with a Brillouin gun is higher, whilst its noise figure is not inferior to that of the immersed-flow gun. Measurements of the internal rotation frequency of the electrons in the beam together with measurements of the saturation gain are in agreement with the theory that a high space-charge density is desirable in order to prevent beam defocussing in an inhomogeneous pumping field.

Résumé

On décrit deux canons à électrons d'une forte pervéance \((3 \times 10^{-6} \text{AV}^{-3/2})\) et à faible tension anodique \((6 \text{ V})\), l'un du type Brillouin, l'autre du type à immersion. Ils sont destinés aux amplificateurs paramétriques de faisceaux d'électrons à composant transversal. Pour les besoins de l'expérience on a construit des tubes du type Adler qu'on a essayé à une fréquence de signal de 550 MHz. Avec le canon Brillouin on obtient un facteur de bruit minimum de 1,31 \((90^\circ \text{K})\) pour une amplification maximale de 30 dB, et une amplification de saturation de 50 dB. Les valeurs correspondantes pour le type à immersion étaient de 1,4 \((116^\circ \text{K})\), 22 et 30 dB. Ce qui montre qu'avec le type Brillouin, l'amplification est plus forte, le facteur de bruit n'étant pas moins défavorable qu'avec le type à immersion. Les mesures de la fréquence de la rotation propre des électrons du faisceau, ainsi que celles de l'amplification de saturation, confirment la théorie selon laquelle une forte densité de charge d'espace permet d'éviter les variations de mise au point du faisceau électronique en cas de non-homogénéité du champ de pompage.

Zusammenfassung

1. Introduction

In Adler tubes and in similar devices using an axial focussing magnetic field and in which amplification is accomplished by means of inhomogeneous electric or magnetic fields a well-focussed beam of high space-charge density is desirable. A large bandwidth and a low noise figure require a high beam perveance and a constant electron velocity over the beam cross-section.

Two types of electron gun in which the above specifications are met will be described. They have been tested in Adler tubes at a signal frequency of 550 Mc/s. One gun is of the Brillouin type using a magnetically shielded cathode, the other is of the immersed-flow type. Otherwise the guns are identical.

The noise of the Brillouin-type gun is higher; in certain cases this is not a serious disadvantage, e.g. in Adler tubes or similar amplifiers where the beam is denoised before the amplification takes place. However, for the same value of magnetic field the Brillouin gun allows a higher space-charge density in the beam and a more constant axial electron velocity over the cross-section. A high beam perveance is attained by means of a low-perveance gun, either Brillouin or immersed flow, followed by a decelerating lens system. This is made possible by operating the gun at a considerably higher anode voltage than the beam voltage. The divergent effect which a simple decelerating lens would have is avoided by the use of a more complex system.

2. Gun design

The low-perveance gun is a diode gun with a beam-shaping electrode (fig. 1).

![Fig. 1. Electron gun with small-diameter cathode.](image-url)
To avoid partition noise and secondary electrons, which cause bad focussing and premature gun saturation, the best solution is to use a cathode diameter equal to the diameter of the anode aperture, or even smaller, since then the anode is not hit by electrons of the beam. Mechanical simplicity pleads strongly in favour of the application of a larger-diameter cathode and a small aperture in the anode $a_1$ (fig. 2), as then no beam-shaping electrode is necessary. Experiments with both types of cathode show that for Adler tubes the disadvantages of the latter type of construction are not serious.

It is advantageous to choose the aperture diameter smaller than the final beam diameter thereby reducing transverse velocities of the electrons in the beam $^{10,11})$. Expressed in terms of electron temperature $T$, the ratio of velocities is $T_1/T_2 = A_2/A_1$ where $A_1$ and $A_2$ are the corresponding beam areas.

To obtain Brillouin-type focussing the anode $a_1$ is made of iron and forms part of an iron cylinder shielding the cathode from the magnetic field. The aperture of anode $a_1$ is tapered to avoid reflections of electrons.

For the immersed-flow-type gun the anode construction is identical but molybdenum is used instead of iron.

A 6-volts beam with a current of 40 $\mu$A and a beam diameter of 0·4 mm was aimed at. The anode aperture $a_1$ was chosen 0·25 mm, yielding a cathode current density of 0·1 A/cm$^2$. The lens system following the gun is formed with the anodes $a_2$, $a_3$, $a_4$, $a_5$ and $a_6$. Exact calculation of this system is not possible. Therefore the electron paths are studied by means of an electrolytic tank combined with an automatic plotter $^{12})$. One can object that space-charge forces
and forces exerted by the magnetic field are not accounted for. However, once the beam is launched properly those forces are approximately balanced. Provided no strong lenses occur one may assume that this is also approximately the case in the gun itself. The latter reasoning was suggested to us by Dr Groendijk.

For an experimental gun it is desirable to have a few more electrodes than is strictly necessary to obtain a parallel beam.

From the electron paths (fig. 3), it can be concluded that a system with a

![Fig. 3. Electron paths of three-anode gun.](image)

![Fig. 4. Electron paths of five-anode gun.](image)
total of three electrodes ($a_1$, $a_2$ and $a_3$) of simple geometry does not offer a good solution: Neither do four electrodes, but five do (fig. 4). The anode $a_2$ is given a high positive voltage. A negative voltage will do also (fig. 5) but, as the paths are then strongly curved, aberration results.

Actual guns built with altogether five electrodes do launch well-focussed beams of sufficiently large current densities. Noise measurements show, however, that the addition of another electrode is necessary to attain a low noise figure. We suppose that in this case the axial electron velocity is made more constant over the beam cross-section.

The final electrode $a_6$ acts as a collimator. The voltages at $a_2$, $a_3$, $a_4$ and $a_5$ should be chosen such as to minimize interception current at $a_6$. To avoid secondary electrons, which disturb the beam, this anode is constructed of two thin metal sheets with a small aperture separated by a thick metal plate with a larger aperture (figs 1 and 2).

3. Saturation gain

Let us assume that the beam can be represented adequately by a cylinder of circular cross-section and of constant space-charge density, characterized by $f_p$, the plasma frequency. The frequencies at which the internal electron rotations in the beam occur are then given by $^{11)}$:

$$f_+ = \frac{1}{2}f_e + \frac{1}{2}f_e \left[ 1 - 2(f_p/f_e)^2 \right]^{\frac{1}{2}},$$  \hspace{1cm} (1)

$$f_- = \frac{1}{2}f_e - \frac{1}{2}f_e \left[ 1 - 2(f_p/f_e)^2 \right]^{\frac{1}{2}}.$$  \hspace{1cm} (2)
Here $f_c$ is the cyclotron frequency,

$$f_c = \frac{eB}{2\pi m}, \text{ and } f_p = \frac{1}{2\pi} \left( \frac{eI_0}{m\varepsilon_0 u_0 A} \right)^{\frac{1}{2}},$$

where $e/m$ is the ratio of electron charge and mass, $\varepsilon_0$ the dielectric constant of vacuum, $A$ the beam cross-section area, $I_0$ the beam current, and $u_0$ the axial beam velocity. The rotation of the beam's centre of mass occurs at $f_c$; the transverse motion of a particular electron is composed of three rotations viz. at $f_0, f_+ \text{ and } f_-^{(13)}$.

When the beam passes through a transverse quadrupole pump field, it can be seen from fig. 6 what happens to the motion of the centre of mass of the beam and from fig. 7 what happens to the internal motions (see appendix).

In these diagrams the reduced pump amplitude $q$ is plotted against the normalized pump frequency $\nu = f_q' / f_c$, $f_q'$ being the pump field frequency as seen by the electron. The differential equations describing the system possess exponentially growing solutions if the point $(\nu, q)$ lies in the shaded area of the diagram. If $f_q'$ is chosen equal to $2f_c$, i.e. if $\nu = 2$, then it can be seen from fig. 6 that exponential growth of the cyclotron motion occurs for arbitrarily small values of the pump field. At the same time it follows from fig. 7 that for $\nu = 2$ the pump field should exceed a minimum value $q_+$ before exponential growth of the internal motions is possible; the value $q_+$ is larger if $f_+$ and $f_c$ are more

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**Fig. 6.** Stability diagram beam's centre of mass motion.

**Fig. 7.** Stability diagram of internal electron motions.
apart. Consequently the saturation gain, i.e. the maximum gain before a serious beam defocussing occurs, is larger.

If we choose \( v = 2f_+/f_0 \) instead of 2 then amplification of the internal electron motion at \( f_+ \) occurs for arbitrarily small pump fields (fig. 7). The motion of the centre of mass (fig. 6) is not amplified in this case.

The occurrence of an instability due to a small pump field at \( v = 2f_+/f_0 \) suggests a method of beam testing, since if \( f_+ \) is known \( f_p \) can be calculated from eq. (1), and from \( f_p \) in its turn we can find \( \rho_b \), the beam radius.

When the rotations at \( f_+ \) are amplified to such an extent that beam interception occurs, a dip in the collector current can be observed. So \( f_+ \) can be determined in principle by tuning the pump oscillator to \( 2f_+ \). In practice resonances in the quadrupole and related circuitry can cause serious errors.

In ref. 13 a method is described where the magnetic field is increased in order to increase \( f_0 \) to \( f_0' \) and \( f_+ \) to \( f_+ = f_0' \); the dip in the collector current is here also the criterion. As a side effect of this method a change of electron paths in the gun can occur while the spread of the beam is reduced; the method is therefore liable to yield incorrect values of \( f_p \).

A combination of the two methods is free of these disadvantages. The pump frequency is adjusted until the defocussing dip of the collector current, when the magnetic field is changed, occurs at the original field value. The frequency response of the quadrupole circuitry should be broad enough. Consequently it is not possible to measure \( f_- \) without constructing a special tube.

4. Discussion of results

The guns have been tested in Adler tubes. The measurements can be subdivided roughly into three parts: (a) the potential profile in the gun, (b) the noise behaviour, (c) the saturation gain and the rotation frequencies:

(a) Table I gives the anode voltages, the current \( i_e \) intercepted by the collimator anode \( a_5 \) and the collector current \( i_c \).

The adjustments under I, II and III refer to an immersed-flow gun, and those under columns IV, V and VI refer to a Brillouin gun. The profiles of I and IV represent the case of maximum collector current with adjustments similar to those of fig. 4, obtained with the electrolytic tank. The profiles of II and V give the maximum collector current with similar adjustments as in fig. 5. The adjustments needed for minimum noise figure are collected under III and VI.

As the maximum collector current of I is larger than that of III whereas the collector current of IV is not larger than that of VI, we can conclude that the tank measurements are reliable for immersed-flow guns, but are not so in the case of Brillouin-flow guns where strong magnetic lenses occur. For an ideal Brillouin gun the adjustment for maximum beam current is the same as that for a uniform axial electron velocity. This may be the reason that the adjust-
TABLE I

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<tr>
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<th>immersed flow</th>
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<th>Brillouin flow</th>
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<tr>
<td></td>
<td>I</td>
<td>II</td>
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<td>IV</td>
<td>V</td>
<td>VI</td>
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<td>15</td>
<td>12-5</td>
<td>14·5</td>
<td>18·5</td>
<td>14</td>
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<td>-3</td>
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<td>14</td>
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<tr>
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<td>13</td>
<td>12</td>
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<tr>
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<td>7-5</td>
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<td>7·5</td>
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<td>31</td>
<td>32</td>
<td>39</td>
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I and IV: Adjustment similar to fig. 4.
II and V: Adjustment similar to fig. 5.
III and VI: Adjustment for minimum noise figure.

The adjustment of minimum noise figure coincides with that of maximum current, as can be seen from VI in comparison with IV and V. Maximum collector current and minimum noise figure of the immersed gun do not coincide (I and III). The difference between I and III is probably due to lens effects in the gun, which in the latter case cause rotations in the beam when it expands and thus provide a more uniform axial electron velocity. At the same time aberrations are introduced, and therefore the current of III is lower. As was expected the adjustments of II and V yield results inferior to those of I and IV, owing to aberrations. In both cases the collector current and the ratio of transmitted and intercepted current ($i_6$) was lower.

(b) For the Brillouin gun the minimum noise figure obtained is $F = 1·31$ (90 °K) (over a band of 40 Mc/s) at the maximum gain of 30 dB. The results obtained with the immersed-flow gun are substantially the same; only the maximum gain should be 22 dB instead of 30 dB, and $F = 1·4$. This marked difference can be attributed to second-order pumping of synchronous waves 14) or to a less perfectly focussed beam. The noise figures are slightly higher than those quoted by Adler. This is due to the fact that our cathode has a higher working temperature, being $\approx 1400$ °K instead of $\approx 1100$ °K in Adler’s tubes. Moreover in the Brillouin gun amplification of the noise waves occurs 6). Indeed it was found that the equivalent temperature of the noise removed from the beam in the Brillouin gun is $\approx 5000$ °K. This temperature measured with the immersed-flow gun is $\approx 3000$ °K. One should expect this to be $\approx 1400$
The source of this increase may be rotations of the beam caused by the lenses of the gun or fluctuations of the work function over the cathode surface.

(c) The saturation gain and the rotation frequency $f_\pm$ were measured with both types of gun. It was found that the saturation gain (fig. 8) is much higher for the Brillouin gun at large values of beam current. They are approximately the same, however, at low values of beam current.

Figures 9 and 10 show the theoretical curves of $f_\pm$ and $f_p$ calculated on the assumption of a beam radius $r_b = 0.4$ mm and a beam voltage of 6 V. These figures relate to the immersed and the Brillouin gun respectively.

The measured values of $f_\pm$ and their corresponding values of $f_p$ are indicated by round dots. It can be seen that for low values of current the measured values follow essentially the theoretical curves, but at higher beam currents a deviation occurs. This effect occurs at lower currents in the immersed flow gun. In fig. 11 are indicated the beam radii derived from the values of $f_p$ calculated from $f_\pm$ (eq. (1)). It can be seen that defocussing occurs at higher values of beam current; for comparison the assumed value of $r_b = 0.4$ mm, being the aperture diameter of anode $a_0$, is also drawn: The value of $f_\pm$ is the same for the two guns at
Fig. 9. $f_+^+$ and $f_p^+$ of immersed-type gun.

Fig. 10. $f_+^+$ and $f_p^+$ of Brillouin-type gun.
lower beam currents and the saturation gain is also essentially the same. This is in agreement with the theory that beam defocussing is prevented by space charge in the beam. It is amazing how accurately the measured values of \( f_0 \) agree with the theoretical values, despite the rather crude assumption of a constant space-charge density in the beam. There is no agreement with the theory \(^{14}\) which ascribes the limitation of gain to current interception in the quadrupole region by second-order pumping of synchronous waves, not even for low values of current: Here one would expect agreement in the case of immersed flow since the equations are certainly correct in the case of negligible space charge. It is highly probable, however, that the assumptions of laminar flow and no electron rotations about the axis do not apply to our gun. As a matter of interest the values of \( f_0 \) measured by the method of ref. 13 are indicated in figs 9 and 10 by square dots. For the immersed-type gun good agreement exists, but agreement in the case of the Brillouin gun is bad. Hence the method is less suited for systems which include strong magnetic lenses. If there are no such lenses the method offers the advantages of simplicity and swiftness.

Finally we conclude from the above considerations and figures that the Brillouin focussed beam is not inferior as regards the minimum noise figure that can be attained, and that it is decidedly superior as far as defocussing effects are concerned.

**Fig. 11. Beam diameter.**

**Acknowledgement**

We would like to acknowledge the stimulating discussions we have had with Dr H. Groendijk and the valuable assistance of Messrs H. Bouman and L. Kranenbarg.

_Eindhoven, June 1961_
Appendix

The quadrupole field can be given as

\[ V = Qr^2 \cos (2\theta - \beta q z) \cos \omega_q t, \tag{3} \]

where \( Q \) is the field amplitude, \( r, \theta \) and \( z \) are cylindrical coordinates, \( \beta q \) is the phase constant of the quadrupole pump wave travelling in the positive \( z \) direction, the pump frequency being \( \omega_q = 2\pi q_0 \).

The \( z \) coordinate of an electron travelling at velocity \( u_0 \) and starting at \( t = t_0 \) from \( z = 0 \) is given by \( z = u_0(t - t_0) \).

We rewrite the field as

\[ V = \frac{1}{2} Qr^2 \cos (2\theta - \omega q' t + \beta q u_0 t_0) + \frac{1}{2} Qr^2 \cos (2\theta - \omega q'' t + \beta q u_0 t_0). \tag{4} \]

The frequencies as seen by the electron are written as

\[ \omega_q' = \omega_q + \beta q u_0 \quad \text{and} \quad \omega_q'' = -\omega_q + \beta q u_0. \]

The frequency of rotation of the electron being \( \omega_r \), the effect of the transverse pump field can be calculated using only that part of the field which varies at the rate \( \omega_q' \approx 2\omega_r \), and neglecting the field varying at the rate of \( \omega_q'' \). The potential caused by the space charge \( \rho_0 \) of the electron beam is \(-e\rho_0 r^2/4\epsilon_0\).

The electron trajectories in the field caused by these two potentials are given by the equations

\[ \frac{d^2 r}{dt^2} - j\omega_0 \frac{dr}{dt} = \frac{e}{m} Qs \exp \left[ j(\omega_q' t - u_0 \beta q t_0) \right] + \frac{1}{2} \omega_p^2 r \tag{5} \]

and

\[ \frac{d^2 s}{dt^2} + j\omega_0 \frac{ds}{dt} = \frac{e}{m} Qr \exp \left[ -j(\omega_q' t - u_0 \beta q t_0) \right] + \frac{1}{2} \omega_p^2 s. \tag{6} \]

Here \( r = \frac{1}{2}(x + jy) \), \( s = \frac{1}{2}(x - jy) \), \( x, y \) and \( z \) being the Cartesian coordinates, while \( \omega_p = 2\pi p_0 \) and \( \omega_0 = 2\pi f_0 \).

Two of the solutions are unstable if the reduced pump field, given by \( q = eQ/m\omega_0^2 \), is in the shaded area of fig. 7. On the abscissa is plotted \( \nu = f_p/f_0 \), the reduced pump frequency.

The boundaries of the shaded area are given by the intersection of the two parabolas

\[ q = \frac{1}{4}(\nu - 1)^2 - \frac{1}{4}(1 - 2\mu^2) \]

and

\[ q = -\frac{1}{4}(\nu - 1)^2 + \frac{1}{4}(1 - 2\mu^2), \]

where \( \mu = f_p/f_0 \).

Apart from the motion of each individual electron the motion of the centre
of mass of the beam is important, as this motion represents the signal. The
equations are the same as eqs (5) and (6) with \( \omega_q = 0 \).

The instabilities of this motion, which cause the signal amplification, are
given in the shaded area of fig. 6.

REFERENCES

   1960.