MECHANICAL EXCITATION OF HELICON WAVES

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Abstract

It is shown that helicon resonances can be excited by sliding along each other two metal plates which are oriented perpendicularly to an applied static magnetic field, provided that they are electrically connected along their circumferences. As expected, the resonance frequencies and Q-values turn out to be the same as those derived by Chambers and Jones for the case of excitation with an external a.c. magnetic field. The relation of the phenomenon with the interaction between helicons and ultrasound is commented on and the efficiency of this method of excitation is evaluated and found to be considerable. The mechanical impedance involved displays resonant behaviour and can become quite large. Mass multiplication by a factor of $10^4$ is e.g. quite feasible.

1. Introduction

One well-studied aspect of the interplay between helicon propagation in solids and mechanical movement of the lattice is the interaction between helicon waves and ultrasound. This interaction, of course, primarily occurs when the phase velocities of the two kinds of waves are equal. For propagation parallel to the applied static magnetic field $H_0$ the condition for this is

$$\omega^{1/2} = (qnH_0)^{1/2} v_s,$$

where $\omega$ is the angular frequency, $q$ the elementary charge, $n$ the concentration of charge carriers, which we assume all to be of the same sign, and $v_s$ the velocity of sound. With reasonable magnetic fields (1) yields for metals $\omega$-values which are situated in the gigacycle range. On the other hand we see from (1) that an interaction between helicons and mechanical movement at much lower frequencies might be expected if a phase velocity much less than $v_s$ could be assigned to the latter.

In this paper a method of mechanically exciting helicon waves is studied which may be regarded as an example of this. In fact we shall consider (fig. 1) two parallel metal plates of equal thickness $\frac{1}{2}d$, oriented perpendicularly to an applied magnetic field which are slid along each other with angular frequency $\omega$. Obviously the relevant wavenumbers $k$ involved in such a movement are entirely determined by $d$ and therefore the relevant “phase velocities” $\omega/k$ are controlled by $\omega$. We shall show that by adjusting $\omega/k$ to the phase velocity of helicon waves the latter can be excited in the structure with considerable efficiency. The resonances to be found will have the same resonance frequencies and $Q$-values.
as in the case of excitation with an external a.c. magnetic field, as derived and demonstrated by Chambers and Jones 1).

It should already be emphasized at this stage that the electrical currents associated with the helicon waves, which flow in the plane of the plates, must be allowed to leave and to enter the plates at their circumferences. The circumferences must therefore be electrically connected. If the connection is adequate one can assume the plates to extend infinitely in two directions, thus ignoring the connection, which permits the use of one-dimensional geometry. It has been shown elsewhere 2, 3) that at least rigid structures for which this procedure is correct can be made.

In sec. 2 we shall first consider the excitation of helicon waves by a circularly polarized travelling displacement wave of arbitrary $\omega$ and $k$ in an infinite medium. From the results the excitation of helicons by a plane-polarized standing displacement wave is easily evaluated. In sec. 3 it is shown that sliding two metal plates of finite thickness along each other can be considered as a superposition of such waves which satisfy the boundary conditions for the excited electromagnetic fields. In sec. 4 we calculate the actual helicon response to this superposition and in sec. 5 we consider the (large and resonant) mechanical impedance involved. The validity of the assumption, made in sec. 3, that no significant deformation of the plates occurs is also investigated there.

2. Excitation of helicon waves by displacement waves of arbitrary $\omega$ and $k$ in an infinite medium

We consider a classical electron gas, the electrons having a concentration $n$ and an effective mass $m$, which is embedded in a positive background, with which the electrons interact with a relaxation time $\tau$. We assume this background to move with a velocity $v$, which is a circularly polarized wave, travelling in the $z$-direction, with real but otherwise arbitrary $\omega$ and $k$:  

![Fig. 1. Configuration considered in this paper.](image-url)
Here $\omega$ and $k$ should be considered to be positive and hence the upper sign pertains to right-handed, and the lower sign to left-handed circular polarization.

We defer the question whether such movement is possible and how it can be brought about until later.

The linearized equation of motion for the electrons is

$$m \frac{du}{dt} = -qE - q(u \times B_0) \frac{mu}{\tau} + \frac{mv}{\tau},$$

where $u$ is the velocity of the electrons, $E$ the self-consistent electric field and $B_0$ the applied magnetic induction, assumed to be directed along the $z$-axis.

Furthermore, we have Maxwell's equations

$$\text{curl } E = -\frac{\partial B}{\partial t},$$

and

$$\text{curl } H = J + \frac{\partial D}{\partial t},$$

where $B$ is the excited magnetic induction, $H$ the excited magnetic field and $J$ the current density

$$J = qn(v - u),$$

and where we neglect the dielectric displacement $D$. The solution of (3), (4), (5) and (6) for $J$, $E$, $H$, $B$ and $u$ in terms of $v$ is easily obtained. In particular we find

$$E_x = \pm jE_y = \frac{\omega \mu_0 qn(\omega \mp \omega_c) v_x}{\{1 + j(\omega \mp \omega_c) \tau\} k^2 + j\omega \mu_0 \sigma},$$

where $\omega_c = qB_0/m$ is the cyclotron frequency, $\mu_0$ the magnetic permeability and $\sigma$ the normal d.c. conductivity. We remark parenthetically that the use of real $\omega$ and $k$ implies that in what follows $E$ and $J$ are everywhere and always perpendicular to each other, as can be seen from (4) and (5). This does not mean that there is no dissipation, since work is still done by the mechanical forces which drive the plates, $E$ and collisions being the agents for transferring part of this work to the electron gas.

Addition of the results for both circular polarizations given in (7) yields the response to a plane-polarized travelling displacement wave:

$$v_x' = 2v_0 \exp\{j(\omega t - kz)\}$$

and

$$v_y' = 0.$$
We obtain from (7), assuming as usual $\omega \ll \omega_c$,

$$E_x' = \frac{-j\omega_c^2 \tau^2 k^2 \omega \mu_0 q n v_x'}{(k^2 + j\omega \mu_0 \sigma)^2 + \omega_c^2 \tau^2 k^4}$$

(9)

and

$$E_y' = \frac{j\omega_c \tau (k^2 + j\omega \mu_0 \sigma) \omega \mu_0 q n v_x'}{(k^2 + j\omega \mu_0 \sigma)^2 + \omega_c^2 \tau^2 k^4}$$

(10)

3. Considerations on finite structures

We must now study finite structures. Adhering to the one-dimensional approach adopted above we consider, to begin with, a metal plate of thickness $d$, oriented perpendicular to the $z$-direction (i.e. perpendicular to $B_0$). We assume the plate to be situated between $z = 0$ and $z = d$ and to extend infinitely in the $x$- and $y$-directions. We subject this plate to plane-polarized movement with angular frequency $\omega$ in the $x$-direction, the movement being uniform in the $xy$-plane, but not necessarily so in the $z$-direction.

In order to be able to apply the results of the previous section we have to expand this movement in Fourier components with various wavenumbers. This must be done in such a way that for each Fourier component the boundary conditions for the electromagnetic fields associated with it are satisfied. We therefore must formulate these first.

Consider a travelling plane-polarized displacement wave (8). The electromagnetic fields associated with it in particular satisfy Maxwell's equation (4), i.e. we have apart from the electric field (9) and (10) a magnetic field satisfying the relations

$$-E_y' |H_x'| = E_x'|H_y' = \omega \mu_0 / k.$$  

(11)

It has been mentioned in the introduction that we are chiefly interested in situations where $\omega / k \ll v_s$ and we may therefore take $\omega / k$ to be much smaller than the velocity of light $(\varepsilon_0 \mu_0)^{-1/2}$, where $\varepsilon_0$ is the dielectric constant of vacuum. We thus have from (11):

$$|E_y'|/|H_x'| = |E_x'|/|H_y'| \ll (\mu_0/\varepsilon_0)^{1/2}.$$  

(12)

So the ratios $E_y'/H_x'$ and $E_x'/H_y'$ are much smaller in the metal than in vacuum, where they are of course equal to the right-hand member of (12). Standard reflection theory shows that this implies that the electromagnetic fields are totally reflected at $z = 0$ and $z = d$ with a node in $H$ and an antinode in $E$. We
therefore conclude that in order to avoid surface excitation, i.e. the occurrence of additional fields which do not satisfy (9) and (10), the movement to be imposed on the plate must be a superposition of standing displacement waves.

Now the relation between $E$ and $v$, (9) and (10), contains only even powers of $k$ which means that the response to a standing plane-polarized displacement wave

$$v_x'' = 4v_0 \exp(j\omega t) \cos(kz),$$
$$v_y'' = 0$$

is simply (9) and (10) with $v_x'$ replaced by $v_x''$, i.e. when the plate is subjected to a superposition of standing displacement waves, the $z$ dependence of electric field and displacement is the same. From this it follows that the Fourier components of the movement of the plate must have antinodes at the boundaries $z = 0$ and $z = d$. This condition obviously leaves us with a Fourier expansion

$$v_x''' = \sum_{N=0}^{\infty} A_N \cos(k_N z) \exp(j\omega t),$$

with uniquely determined wavenumbers

$$k_N d = N\pi, \quad (N = 0, 1, 2, \ldots).$$

Clearly the simplest way of obtaining this type of movement is to divide the plate in a number of thinner plates, which together have a total thickness $d$ and which are moved relative to each other in the $x$-direction. Since the excited electrical currents have only $x$- and $y$-components, the plates need in practice only be electrically connected along their circumference in order to simulate the one-dimensional picture we are using.

In the following we restrict ourselves to two plates of thickness $\frac{d}{2}$. In this case the relative movement of the system is antisymmetrical with respect to $z = \frac{d}{2}$ (fig. 1) and therefore the electric field, having the same $z$ dependence as the movement is also antisymmetrical. Equation (4) shows that the magnetic field now is symmetrical, and it can therefore be detected with the usual open-circuited pick-up coil.

Assuming that the excited waves do not cause significant distortion, which we shall prove in sec. 5, (13) in this case takes the form

$$v_x''' = \sum_{N=1}^{\infty} (-1)^{(N+1)/2} \frac{4v}{N\pi} \cos(k_N z) \exp(j\omega t),$$

(15)

where $v$ is the velocity amplitude with which each of the two plates moves uniformly.
4. Helicon response to sliding two metal plates of thickness $\frac{1}{2}d$ along each other

From (9), (10) and (15) the electric-field response can immediately be written down:

$$E_x''' = \sum_{N=1, odd}^{\infty} (-1)^{(N+1)/2} \frac{4v}{N\pi} \frac{-jk_N^2 \omega^2 \tau^2 \omega \mu_0 q_n}{(k_N^2 + j\omega \mu_0 \sigma)^2 + k_N^4 \omega^2 \tau^2} \exp(j\omega t) \cos(k_N^2),$$

Equation (16) can conveniently be written as

$$E_y''' = \sum_{N=1, odd}^{\infty} (-1)^{(N+1)/2} \frac{4v}{N\pi} \frac{j(k_N^2 + j\omega \mu_0 \sigma)\omega \mu_0 q_n}{(k_N^2 + j\omega \mu_0 \sigma)^2 + k_N^4 \omega^2 \tau^2} \exp(j\omega t) \cos(k_N^2).$$

We thus see that the electric-field response consists of a series of resonances with the same $Q$-values and resonance frequencies as those found by Chambers and Jones for excitation with an external a.c. magnetic field. This, of course, is not surprising since the $Q$-values and resonance frequencies which characterize the response of a linear system to an a.c. excitation are solely determined by the system and not by the way of excitation. The present analysis may therefore be regarded as an alternative derivation of the $Q$-values and resonance frequencies obtained by Chambers and Jones, which explicitly demonstrates the influence of the two circular polarizations on $\omega_N$ and $Q$ (in their paper this influence comes in implicitly where they transform with Fourier methods from the response to a step function to the response to a.c. excitation).

Using (20) it is easily seen from a comparison of corresponding terms in (16) and (17) that in the limit of large $\omega_c \tau$ we have $E_x = jE_y$ for $\omega = \omega_N$ as far as the $N$th term is concerned. This shows that in resonance we are really exciting a circularly polarized electromagnetic wave, i.e. the current due to the plane-
polarized motion of the lattice has become negligible compared with the
current due to the (circularly polarized) motion of the electron gas. Note that
the wave has the circular polarization of the "propagating" helicon mode,
which one expects.

The signal picked up with an open-circuited coil in the \( y \)-direction can now
be evaluated too. This signal is proportional to

\[
\frac{\partial H_y'''}{\partial t} dt = \int_0^d \frac{\partial H_y'''}{\partial t} dz = \frac{-1}{\mu_0} \int_0^d \frac{\partial E_x'''}{\partial t} dz = \\
= \sum_{N=1}^{\infty} (N-1)^{1/2} \frac{4v}{N\pi} \frac{\omega_c^2 \tau m/q\mu_0}{1 - jQ(\omega_N/\omega - \omega/\omega_N)} \exp (j\omega t). \tag{21}
\]

It is interesting to compare this result with the corresponding response in the
case of excitation with an a.c. magnetic field \( H \) in the \( x \)-direction. Chambers
and Jones obtained for this response

\[
\frac{\partial H_y'''}{\partial t} dt = \sum_{N=1}^{\infty} \frac{4\omega \omega_c \tau}{\pi^2 N^2} \frac{\omega_c^2 \tau m/q\mu_0}{1 - jQ(\omega_N/\omega - \omega/\omega_N)} \exp (j\omega t). \tag{22}
\]

From (20) and (14) we easily see that in resonance (\( \omega = \omega_n \)), corresponding
terms of (21) and (22) have the same magnitude for

\[
H = \frac{qnd\nu}{N\pi} \frac{\omega_c \tau}{(1 + \omega_c^2 \tau^2)^{1/2}}. \tag{23}
\]

For \( \nu = 10^{-3} \text{ m s}^{-1}, \ d = 10^{-3} \text{ m}, \ n = 10^{29} \text{ m}^{-3}, \ N = 1 \) and \( \omega_c \tau \gg 1 \) we
thus find \( H \approx 60 \text{ Oe} \). It should be noted in this context that \( H = 1 \text{ Oe} \) in our
equipment amply suffices to excite readily detectable resonances.

5. Mechanical impedance presented to the driving machine

For the external force per unit of volume required to move the plates we can
obviously write

\[
K = j\omega q\nu'' - qn(\nu''' \times B_0) - qnE'' + \frac{mn}{\tau} (\nu''' - u'''),
\]

where \( q \) is the density of the material.
In accordance with the preceding sections we shall restrict ourselves here to the case in which suitable constraints ensure that \( v_y = 0 \). Then \( K \) has only a component \( K_x \) and, substituting (6), we obtain

\[
K_x = j\omega \varphi v_x''' - qnE_x''' + \frac{m}{\varphi}. 
\]  

(24)

In (24) we can write on account of Maxwell theory

\[
J_x''' = \frac{-j}{\omega \mu_0} \frac{\partial^2 E_x'''}{\partial z^2}. 
\]  

(25)

Substitution of (18) and (25) in (24) yields, with the aid of (19) and (20), for the right-hand plate \((\frac{1}{2}d < z < d)\):

\[
K_x = \left[ j\omega \varphi v + \sum_{N=1}^{\infty} \frac{(-1)^{(N+1)/2}}{N\pi} \frac{2\nu}{\omega \epsilon^2 \tau n} \times \right. \\
\left. \times \left\{ \frac{1 - j(\omega N/2Q\omega)}{1 - jQ(\omega N - \omega/\omega_N)} \right\} \cos (k_N z) \right] \exp (j\omega t). 
\]  

(26)

For the left-hand plate we have, of course, the same expression but of opposite sign.

We shall first use this equation to verify our assumption that no significant distortion of the plates is caused by the presence of the helicon waves. In order to do this we have to consider the non-uniform forces exerted on the plates, i.e. the sum in (26). If we write for the \( N \)th term of this sum \( K_{N_d} = K_{N_d}^0 \exp (j\omega t) \cos (k_N z) \) the equation of motion of the lattice as far as non-uniform velocities \( v_{dN} = v_{dN}^0 \exp (j\omega t) \cos (k_N z) \) due to this term are concerned, is

\[
\frac{\partial K_{N_d}}{\partial t} + c \frac{\partial^2 v_{N_d}}{\partial z^2} = q \frac{\partial^2 v_{N_d}}{\partial t^2}, 
\]  

(27)

where \( c \) is the stiffness of the material and where the subscript \( d \) stands for distortion.

We have assumed from the outset that \( \omega /k \ll v_x = (c/\varphi)^{1/2} \) and therefore we may write (27) as

\[
|K_{N_d}| = \frac{ck_{N_d}^2}{\omega} |v_{N_d}|. 
\]  

(28)

Now it is clear that in (26) the expression between braces cannot exceed an absolute value of about unity, which is attained for \( \omega = \omega_N \). For this case (26) yields, when combined with (28),
where $4v/N\pi$ is the velocity amplitude of the $N$th term corresponding with the uniform movement of the plates (see (15)). The ratio (29) can be kept well below a small fraction of unity, for example with $B_0 = 10^4$ gauss, $\tau = 10^{-10}$ s, $c = 10^{11}$ Nm$^{-2}$ and $m = 10^{-30}$ kg it is about $10^{-4}$. Thus distortion is of no importance under a wide variety of realistic conditions.

We can also obtain the mechanical impedance per unit area of plate $Z$ from (26) by integrating from $\frac{1}{2}d$ to $d$ and dividing by $v \exp(j\omega t)$:

$$Z = j\omega q \frac{d}{2} + \sum_{N=1 \text{ odd}}^{\infty} \frac{2d\omega_c^2 \tau m}{N^2\pi^2} \frac{1 - j(\omega_N/2Q\omega)}{1 - jQ(\omega_N/\omega - \omega/\omega_N)}.$$  

(30)

Simple computations show that this impedance can greatly exceed the first term which represents normal mass. For instance when $\omega = \frac{1}{2}\omega_1$ and for $Q^2 \gg 1$ the series represents an inductive impedance (equivalent to mass) per unit area of about $(\omega_1 \rho d/8)(\omega_c m/\omega_1 M)$ in series with a resistive impedance which is a factor of $\approx 3Q/5$ smaller. Here $M$ is the atomic mass of the material. With $\omega_c = 10^{11}$ cs$^{-1}$, $M/m = 10^4$ and $\omega_1 = 10^3$ cs$^{-1}$ this corresponds to a mass multiplication of about a factor $5.10^3$. For $\omega = \omega_1$ we have, of course, essentially a very large resistance and for $\omega = 2\omega_1$ the impedance is largely capacitive.

We conclude this paper by noting that for $\omega \to 0$ the series (30) becomes

$$\sum_{N=1 \text{ odd}}^{\infty} \frac{d\omega_c^2 \tau m}{N^2\pi^2 Q^2} = \frac{1}{2} \frac{d m}{\tau} \frac{\omega_c^2 \tau^2}{1 + \omega_c^2 \tau^2}.$$  

(31)

which indeed is the easily derivable expression for the resistance presented by an electron gas which is not allowed to build up Hall voltages, to a lattice which moves with very low frequency (see sec. 1).

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