DYNAMICS OF THE MAGNETIZATON OF A THIN-WALLED SUPERCONDUCTING CYLINDER IN A PARALLEL FIELD

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Abstract

The dynamics of the penetration of an applied parallel magnetic field into a hollow superconducting cylinder have been studied by measurement of the voltage fluctuations on a 4-kc/s resonant circuit that was magnetically coupled to the cylinder. On samples with appropriate wall thickness the field was found to penetrate as a random stream of individual flux quanta \( \phi_0 \) in a temperature region down to 0.7 °K below \( T_c \). The value of \( \phi_0 \) as determined from the fluctuations agrees with the theoretical value \( \hbar/2e \). Flux quantization has been demonstrated to occur at least up to \( 3 \cdot 10^6 \) enclosed quanta \( \phi_0 \). If the wall thickness \( d \) is smaller than the penetration depth \( \lambda \), a dynamic flux quantum is found that is equal to \( (d/\lambda) \phi_0 \). Such quanta are found to enter in groups of two or three.

1. Introduction

The dynamics of the magnetically induced superconducting-to-normal phase transition of a solid lead cylinder have been investigated earlier \(^1\) by using an experimental set-up as sketched in fig. 1. The cylinder was inserted in a coil \( L \) which was part of an LC circuit with a resonant frequency at about 18 kc/s. The circuit was connected to an a.c. amplifier with a voltage amplification of

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The output of the amplifier was fed into a loudspeaker and into the Y-axis of an XY-recorder. The X-axis of the recorder gave the magnetic-field strength of a copper-wound solenoid placed outside the helium bath and lined up parallel with the lead specimen. Over the length of the specimen the field was constant to within 0.01%.

When the field strength was increased from zero upwards, so as to make the specimen normal, no output of the amplifier was observed except around the critical field strength. Here a series of cracks, or noise, was observed. When the field was decreased, so as to make the specimen superconducting again, an output was observed only at the critical field.

The explanation of this noise occurring at the magnetically induced phase transition can be given in terms of a jerky or domain-like propagation of the superconducting and normal boundaries. The phenomenon is somewhat similar to the Barkhausen noise found during the magnetization of a ferromagnetic. However, whereas in a ferromagnetic the change of the magnetic induction may be as large as $10^4$ gauss, in a superconductor this change is equal to the critical field, being only about 500 Oe in lead at 4.2 K. For a given domain size the effect in a superconductor is thus one or two orders smaller than in a ferromagnetic.

The superconductor noise was observed both on polycrystalline and on monocrystalline lead cylinders, all having little demagnetization as their diameter was 0.9 mm and their length 40 mm.

**Historical note**

Noise has also been found to occur if a superconductor enters the intermediate state. London suggested that if the picture of the intermediate state as an intimate mixture of superconducting and normal regions were right — which was still an open question then — it should be possible to prove this with techniques similar to that used for demonstrating the domains in a ferromagnetic.

Shubnikov et al. made a set-up somewhat similar to the one described above (fig. 1) which could detect Barkhausen noise in nickel. The sensitivity, however, was too low for the detection of any effect in a superconductor. This was realized by Justi who, using a more powerful amplifier, was successful in demonstrating that on a tin sphere noise starts at a field strength where the intermediate state begins, i.e. at $H = \frac{1}{2} H_c$, and can be observed, on changing the applied field, up to the critical field $H_c$.

One may wonder why this dynamic technique for the investigation of the intermediate state has received far less attention than the stationary bismuth-probe technique with which intermediate state structures have been mapped.

The propagation of the normal front during the magnetically induced phase transition is controlled according to Faber and Pippard by eddy-current damping. In very pure metals the relaxation time associated with this damping may be of the order of minutes, as was shown recently by David with ultrasonic attenuation on aluminium.

In order to make a quantitative study of the dynamics feasible, including assessment of the domain size, the time constant of the elementary process giving rise to a noise output in the set-up of fig. 1 must be far smaller, e.g.
10 times, than the period of the resonant circuit, which is $5 \times 10^{-5}$ seconds. In the normal state of the solid lead cylinder, with a resistivity of $5 \times 10^{-9} \, \Omega \text{cm}$ at 4.2 °K, the time constant for eddy-current damping following a change in the applied field can be calculated \(^{(10)}\) to be about $5 \times 10^{-4}$ seconds, which is too large.

A much shorter relaxation time is obtained by using instead of a solid sample a thin-walled hollow cylinder, having the same outer dimensions, so that the flux coupled to the pick-up coil remains the same. For a wall thickness of 1000 Å the relaxation time in the normal state is less than $10^{-6}$ second, which is sufficiently small.

A hollow cylinder of this description was made by evaporating a lead layer of about 500 Å on to a quartz rod. It was tested in the apparatus shown in fig. 1. The result was entirely different from the behaviour of a solid cylinder, noise now being observed at a constant level over the entire field region, i.e. from virtually zero field up to the critical field. Apparently the magnetic field already starts to penetrate into the cylinder at very low fields, and the penetration continues until the cylinder is normal. The phase transition is thus smeared out over quite a large region of the field, which means that its effects can be studied in a steady state over a long period, e.g. up to minutes, depending on the actual value of the rate of field change and of the critical field. This hollow cylinder is therefore almost ideal for noise measurements and the present investigation is devoted entirely to such samples. In the case of the solid cylinder, which is quite different, the noise occurs as a transient only, lasting some tenths of a second.

Before presenting the detailed experimental results in sec. 4 of this paper, a theory \(^{(11)}\) on the mechanism for the field penetration into a hollow superconducting cylinder and a calculation of the accompanying fluctuations on the resonant circuit in fig. 1 will be given in the next section.

2. Theory

2.1. Mechanism of the field penetration

If a weak magnetic field $H$ is applied parallel to a hollow superconducting cylinder, a Meissner current ($M$ in fig. 2) is set up so as to shield the super-

Fig. 2. Cross-section of a hollow superconducting cylinder in a parallel field $H$. The directions of the Meissner current ($M$) and of the Abrikosov current ($A$) are given by the arrows.
conducting material, and consequently the interior, from the applied field. The magnetization energy per unit length of the cylinder due to this shielding current is equal to $r^2 H^2 / 8$, where $r$ is the radius of the cylinder. If the applied field is increased, the Meissner current will still shield the cylinder until the magnetization energy equals the condensation energy of the superconducting volume, which is equal to $2 r d H_{cb}^2 / 8$ per unit length, $H_{cb}$ being the bulk critical field and $d$ being the wall thickness; we assume $r \gg d$. By equating the two energy expressions, it follows that there is a critical field $H_1$ which is given by

$$H_1 = H_{cb} (2d/r)^{1/2}, \quad \text{with } r \gg d.$$  

(1)

Above $H_1$ the superconducting state is no longer stable unless the magnetic field is allowed to penetrate so as to prevent the magnetization energy from becoming larger than the condensation energy. Substitution of $d = 1000 \, \text{Å}$ and $r = 0.35 \, \text{mm}$ into eq. (1) gives $H_1 = 0.02 \, H_{cb}$, which allows for the almost immediate observation of noise, as mentioned in the introduction, upon increase of the field from zero. Equation (1) can also be obtained from a more general and stringent treatment given by Hsü Lung-Tao and Zharkov 12) and by Douglass 13).

The next step is to consider how the field penetrates into the cylinder. A very convenient way is to describe this penetration in terms of a “paramagnetic” or Abrikosov current (A in fig. 2) which has a direction opposite to the Meissner current 13). This current will start to flow as soon as the applied field exceeds $H_1$ and will balance any further increase of the Meissner current. The Meissner current will increase continuously with the applied field. The Abrikosov current, however, can increase only by discrete steps, as the flux enclosed by it must be quantized in units of the flux quantum $\phi_0 = h/2e = 2.07 \times 10^{-15} \, \text{V s}$ ($h = \text{Planck's constant, } e = \text{charge on the electron}$). These discrete steps by which the Abrikosov current changes, and which correspond to the flux quantum, give rise to the observed noise voltage on the resonant circuit.

2.2. The mean-square noise voltage on the resonant circuit

In the previous section the penetration of the magnetic field has been described in terms of changes of the Abrikosov current. In the present section it will be easier sometimes to speak in terms of flux quanta entering or leaving the cylinder.

The mean-square noise voltage will be calculated on the basis of the following assumptions:

(1) all steps of the Abrikosov current correspond to only one flux quantum,
(2) there is no correlation between two successive steps of the Abrikosov current.

It is further assumed that the wall thickness $d$ of the cylinder is larger than the penetration depth $\lambda$, i.e. $d/\lambda > 1$. The case $d/\lambda < 1$ will be dealt with in sec. 4.2.
The effect of a single flux quantum $\phi_0$ jumping into the hollow cylinder is such as to give rise to a damped oscillation of the resonant circuit. If the time needed for the flux in the cylinder to rise by one quantum is small compared with the period of the resonant circuit, then the voltage on the capacitor $C$ of fig. 1 is, in M.K.S. units, given by

$$V(t) = n\phi_0\omega_0 \exp\left(-\frac{Rt}{2L}\right) \sin \omega_0 t,$$

with $4Q^2 = 4L/R^2C \gg 1$, (2)

where $n$ = number of turns on the pick-up coil,

$\omega_0 = (LC)^{-1/2}$,

$L$ = inductance of the pick-up coil,

$R$ = damping resistance of the pick-up coil,

$Q$ = quality factor.

In the introduction $10^{-6}$ second was estimated to be a lower limit for the relaxation time of the cylinder. Experiments by Kwiram and Deaver $^{14}$) on hollow tin cylinders with wall thicknesses of 500 and 5000 Å have shown that the time for entering or leaving of a flux quantum is smaller than $10^{-5}$ second. The period of the actual resonant circuit was $2.5 \times 10^{-4}$ seconds (sec. 3.1).

In the stationary state that is due to a stream of $N$ flux quanta per second, the voltage on the capacitor can be calculated from the effect of each individual quantum by summation over a time which is large compared with $t_l = 2L/R$ (see eq. (2)). If the last quantum to be taken into account jumped at $t = 0$, then the effect of the $m$th quantum before this last one is given by

$$V_m(t) = n\phi_0\omega_0 \exp\left[-\frac{(R/2L)(t + mt)}{2L}\right] \sin (\omega_0 t + mt),$$

(3)

where $\tau = 1/N$ is the mean time between two successive quanta. We are interested in the mean-square voltage after the stationary state has been reached. This voltage is given by the time average of the square of the summation over $m$ of eq. (3) and can be written as

$$\langle \delta V^2 \rangle = \left\langle \left[ \sum_{m=0}^{\infty} V_m(t) \right]^2 \right\rangle = \frac{1}{2} \sum_{m=0}^{\infty} n^2\phi_0^2\omega_0^2 \exp\left[-\frac{(R/2L)(t + mt)}{2L}\right].$$

(4)

The factor $\frac{1}{2}$ on the right-hand side in eq. (4) is due to the mean-square value of the sine functions of eq. (3). On account of assumption (2) the stream of flux quanta is considered to be incoherent. This means that the cross-terms in $\sum_{m=0}^{\infty} V_m(t)^2$ do not contribute to its time-average value. Putting $t = 0$, as the last quantum to be considered arrived at $t = 0$, and carrying out the summation, we get from eq. (4):

$$\langle \delta V^2 \rangle = \frac{1}{2} n^2\phi_0^2\omega_0^2 \frac{L}{R \tau}, \text{ with } \tau \ll \frac{1}{2} \tau_1.$$ (5)

The number of flux quanta entering the cylinder per second, $N = 1/\tau$, is related to the rate of field change. For $H$ fields above $H_1$ the increase of the
field inside the cylinder equals the increase of the applied field, so that

$$N\phi_0 = \phi_0/\tau = \mu_0\dot{H}A,$$

(6)

where $\mu_0 = 4\pi.10^{-7}$ henry/m, $\dot{H} =$ rate of field change and $A = \pi r^2$ is the cross-sectional area of the cylinder. Substitution of eq. (6) into eq. (5) yields, with $Q = (1/R)(L/C)^{1/2}$, the final expression for the mean-square noise voltage:

$$\langle \delta V^2 \rangle = \frac{1}{2} \phi_0 n \mu_0 \dot{H} A n Q \omega_0 = \frac{1}{2} \phi_0 V_{dc} n Q \omega_0.$$

(7)

Here $V_{dc} = n \mu_0 \dot{H} A$ is the d.c. voltage on the capacitor that is due to the quantized flux inleak. The value of $\langle \delta V^2 \rangle$ is large if $\dot{H}, A, n, Q$ and $\omega_0$ are large. Not all these quantities, however, can be varied independently of each other. The value of $\omega_0$ is limited by the restriction that the period of the resonant circuit, $2\pi/\omega_0$, be large compared with the relaxation time of the cylinder.

Two numerical assumptions, viz. $4Q^2 \gg 1$ (eq. (2)) and $\tau \ll \frac{1}{2} \tau_1$ (eq. (5)) have been made in order to simplify the final expression eq. (7). They are justified by the actual values $Q \approx 22$ (sec. 3.1), $\tau = 0.68.10^{-5}$ second (sec. 4.1) and $\frac{1}{2} \tau_1 = 0.87.10^{-3}$ second (sec. 3.1).

In sec. 4.1 the measured mean-square noise voltage is compared with the value calculated from eq. (7). The agreement found there is a justification afterwards for the assumptions (1) and (2) and thereby supports the picture of the mechanism of the field penetration.

The method by which eq. (7) has been derived was used earlier by Fürth 15) for the mean-square voltage on a resonant circuit in the anode lead of a vacuum diode. There the fluctuations (shot noise) are caused by the fact that the electrons arrive singly and independently of each other on the anode, each electron causing a damped oscillation of the circuit. This method has the advantage that no special theorems on fluctuation phenomena are involved. A much quicker way for arriving at eq. (7), however, is by using Campbell's theorem 16). This states that the mean-square fluctuations caused by a random stream of single events, at a rate of $N$ per second, are obtained from the effect of the single event (eq. (2)) as follows:

$$\langle \delta V^2 \rangle = N \int_0^\infty V^2(t)dt = N \int_0^\infty n^2 \phi_0^2 \omega_0^2 \exp(-Rt/L) \sin^2 \omega_0 t dt = \frac{1}{2} n^2 \phi_0^2 \omega_0^2 \frac{L}{R} N.$$

Given $N = 1/\tau$, this result is identical with eq. (5).

3. Experimental

3.1. Set-up

The set-up shown in fig. 1 was modified in two ways. First, its resonant frequency was changed from 18 kc/s (see sec. 1) down to 4 kc/s, corresponding
to a period of 2.5 \times 10^{-4} \text{ seconds}, by enlarging the capacitance \( C \) to about 10 nF. Secondly, the loudspeaker was replaced by a mean-square-voltage meter containing two thermocouple valves Philips type TH1 feeding a Philips type GM 6020 d.c. microvoltmeter (fig. 3). The output of the thermocouple valves is directly proportional to the mean-square value of the input, so that background noise can be subtracted linearly from the reading. The large time constant of the thermocouples, being 10 seconds, ensures proper averaging of the fluctuating signal.

The microvoltmeter was calibrated at the level of the measured noise voltages by feeding into the amplifier a sine wave of known amplitude from an RC generator, Philips type GM 2317, set at the resonant frequency of the circuit (fig. 3). The amplifier was a Philips type EL 6400 modified by having its two microphone stages in series for increased amplification.

In order to determine the resonant frequency \( \omega_0/2\pi \) and the quality factor \( Q \), the thermocouple output was measured as a function of frequency around \( \omega_0/2\pi \), with the output voltage of the RC generator kept constant. By plotting the thermocouple output against frequency, a resonance curve was obtained from which \( \omega_0/2\pi \approx 4 \text{ kc/s} \) was found and, after due correction for the loading resistor of 1 M\( \Omega \), \( Q \approx 22 \). Precise values will be given in sec. 4. They vary by a few per cent, which is probably due to a variation of the stray capacitance as the resonant circuit with the coil in liquid helium had very long leads, extending over several metres. From these values for \( \omega_0 \) and \( Q \) we find \( Q/\omega_0 = L/R = \tau_1/2 = 0.87 \times 10^{-3} \) second. The impedance of the circuit at resonance follows from \( Z_p = Q(\omega_0 C)^{-1} \) and is about 88 k\( \Omega \). A typical value for the mean-square noise voltage \( \langle \delta V^2 \rangle \) is 30 \( \mu \text{V}^2 \). The noise power on the resonant circuit \( \langle \delta V^2 \rangle/Z_p \) is then equal to 3.10^{-16} \text{ watts} which is very small. The background noise was about 20 \( \mu \text{V}^2 \). Note: \( \mu \text{V}^2 \) stands for \( 10^{-12} \text{ V}^2 \).

### 3.2. Specimens

The specimens were prepared by vacuum deposition of tin on to a rotating quartz cylinder. They were protected against mechanical damage, likely to occur on insertion into the pick-up coil, by dip-coating in a plastic. Except for the first
experiment, tin cylinders were used and not lead cylinders, as lead films deteriorate rapidly. Another practical advantage of tin over lead is its critical temperature of 3·7 °K against 7·2 °K of lead. This allows the tin cylinder to be studied over a range of temperatures both below and above its critical temperature by using the conventional method of pumping on the helium.

3.3. Experimental procedure

The mean-square noise voltage is proportional to the rate of field change $\dot{H}$ (eq. (7)). In the experiments the field was changed linearly with time by means of an automatic device so that $\dot{H}$ was constant. This was checked by putting the field on the $Y$-axis of the recorder while the $X$-axis varied linearly with time. A straight line was obtained. From its slope the value of $\dot{H}$ was evaluated.

At a given temperature of the helium bath the field was increased from zero until the specimen was normal. A recording of the noise output against the magnetic field was taken and the mean-square noise voltage was read on the microvoltmeter. From the recording the critical field was determined. The procedure was repeated at different temperatures. This was done in order to determine to what extent the variation with temperature of the penetration depth with respect to the wall thickness, $d/\lambda$, would influence the mean-square noise voltage.

4. Results and discussion

4.1. $d/\lambda > 1$

We will first discuss a specimen with $d/\lambda > 1$ at all temperatures except near $T_c$ where $\lambda \to \infty$. (fig. 4). As anticipated, $\langle \delta V^2 \rangle$ is temperature-independent (the region near $T_c$ will be discussed in sec. 4.2). Its value $(28 \pm 2) \mu V^2$ agrees within experimental error with $\langle \delta V^2 \rangle_{\phi_0} = (2.6 \pm 0.7) \mu V^2$ which is calculated from eq. (7) upon substitution of the following numerical data: $\phi_0 = 2.07.10^{-15} V s$; $n = 1.3.10^4$; $\mu_0 = 4\pi.10^{-7} H m^{-1}$; $\dot{H} = (7.15 \pm 0.07).10^2 A m^{-1}s^{-1}$ (9.0 Oe s$^{-1}$); $A = (3.42 \pm 0.10).10^{-7} m^2$; $Q = 19.6 \pm 0.3$;

![Fig. 4. Mean-square voltage fluctuations on the resonant circuit during magnetization of a hollow tin cylinder with $d/\lambda_0 = 3.32$, measured at various temperatures.](image-url)
\[ \omega_0 = 2\pi (4.02 \pm 0.04) \times 10^3 \text{s}^{-1}. \] According to eq. (6) the mean time between two successive quanta is \( \tau = \phi_0/\mu_0 \dot{H}A = 0.68 \times 10^{-5} \text{s} \), with the above data. At a given field strength the total number of quanta in the cylinder is given by \( \mu_0 \dot{H}A/\phi_0 \). Substitution of \( H = 226 \text{ Oe} \), being the critical field at 3.05 °K, yields a number of \( 3.7 \times 10^6 \) enclosed quanta.

The agreement between theory and experiment means that the essential assumptions regarding the flux quanta entering singly and independently of each other are justified here. Or, reversing the argument, the present experiment yields a method for determining the value of \( \phi_0 \) from fluctuations, in the same way as the value of the electron charge has been obtained by measurement of the current fluctuations (shot noise) of a vacuum diode \(^{17}\).

The value of \( \phi_0 \) has been measured for the first time by Deaver and Fairbank \(^{16}\) and by Doll and Nåbauer \(^{19}\). These authors used a static method in which the frozen-in flux of 1, 2, 3 or 4 \( \phi_0 \) in a hollow cylinder of about 10 \( \mu \text{m} \) diameter was measured. The present method is dynamic and has shown quantization to occur at least up to \( 3.7 \times 10^6 \phi_0 \). The random entrance of about ten individually observed flux quanta into a solid type-II superconducting cylinder with a diameter of 2 to 3 \( \mu \text{m} \) has recently been reported by Boato et al. \(^{20}\).

The process of entry of single flux quanta is confined to a limited temperature region. Below 3.05 °K the flux suddenly started to enter the specimen of fig. 4 as large bundles that could be detected separately and contained up to \( 10^4 \phi_0 \). It is probable that with decreasing temperature the entry of flux into the cylinder becomes increasingly difficult (pinning).

4.2. \( d/\lambda < 1 \)

A specimen for which \( d/\lambda < 1 \) over the entire temperature range will now be discussed (fig. 5). The value of \( \langle \delta V^2 \rangle \) varies with temperature and is less than its theoretical value \( \langle \delta V^2 \rangle_{\phi_0} = (80 \pm 5) \mu \text{V}^2 \), which is calculated from eq. (7)

\[ \frac{d}{\lambda_0} = 0.88 \]

![Fig. 5. Mean-square voltage fluctuations for a cylinder with \( d/\lambda_0 = 0.88 \).](image)
upon substitution of $\phi_0 = 2.07 \times 10^{-15}$ V s; $n = 1.3 \times 10^4$; $\mu_0 = 4\pi \times 10^{-7}$ H m$^{-1}$; 
$H = (1.36 \pm 0.03) \times 10^3$ A m$^{-1}$ s$^{-1}$ (17.1 Oe s$^{-1}$); $A = (4.59 \pm 0.12) \times 10^{-7}$ m$^2$; 
$Q = 22.3 \pm 0.6$; $\omega_0 = 2\pi (4.06 \pm 0.04) \times 10^3$ s$^{-1}$: Below 3.34 °K the flux enters in large jumps, as was also observed in the specimen of fig. 4 below 3.05 °K.

We tried to find whether the observed reduction of noise could be explained by the reduction of the flux quantum that occurs in very thin-walled cylinders $^{21}$. If $d < \lambda$, the closed integral path leading to the fluxoid quantization $^{22}$ cannot be chosen in a current-free region so that the flux $\phi < \phi_0 = h/2e$ and is given by $^{21}$)

$$\phi = \phi_0 \frac{1}{1 + (2d/r)(\lambda_L/d)^2},$$  

(8)

where $\lambda_L$ is the London penetration depth. Working out eq. (8) numerically for the worst case, i.e. taking $\lambda_L = \lambda$ and the case where $(\lambda/d)^2$ is largest — the point closest to the origin in fig. 6 — yields $\phi = 0.98 \phi_0$. Experimentally at this point $\langle \delta V^2 \rangle = 0.05 \langle \delta V^2 \rangle_{\phi_0}$ (fig. 6) or $\phi = 0.05 \phi_0$, the mean-square noise voltage being proportional to the flux associated with a single step of the Abrikosov current. It will be clear that eq. (8) does not account for the experimental results.

The reduction of noise and its temperature dependence can be explained, however, in the following way. Suppose that the flux associated with each step of the Abrikosov current is reduced due to the fact that in the case of $d < \lambda$ only a fraction $d/\lambda$ of the current can be accommodated in the superconducting layer. We assume that the flux coupled with an elementary step of the Abrikosov current is reduced by the same fraction:

$$\phi = (d/\lambda)\phi_0, \quad \text{with } d/\lambda \leq 1.$$  

(9)

Equation (9) will be a reasonable approximation only if the current density is uniform, i.e. for the case $d \ll \lambda$. Equation (9) does not imply a violation of the flux quantization as it gives only the dynamic part of the flux quantum. The Meissner current can shield only the same fraction $(d/\lambda)$ of the applied field, so that a fraction $(1 - d/\lambda)$ of the flux quantum will leak into the cylinder continuously. The remainder $(d/\lambda)$ is provided for by the steps of the Abrikosov current and this part only gives rise to noise. The number of steps of the Abrikosov current per second $(N)$ is still given by eq. (6). The expression for the mean-square noise voltage now becomes, after substituting in eq. (5) $\phi_0$ by $(d/\lambda)\phi_0$ (eq. (9)) and $1/\tau$ by $\mu_0 H A/\phi_0$ (eq. (6)):

$$\langle \delta V^2 \rangle = (d/\lambda)^2 \langle \delta V^2 \rangle_{\phi_0}, \quad \text{with } d/\lambda \leq 1;$$  

(10)

$\langle \delta V^2 \rangle_{\phi_0}$ stands for the m.s. noise voltage in the case $d/\lambda > 1$ (eq. (7)).
In order to compare eq. (10) with the measurements, \((d/\lambda)^2\) must be known as a function of temperature. This dependence is found from the Ginzburg-Landau expression for the critical field \(H_{CF}\) of thin films. For small values of \(d/\lambda\), i.e. \(d/\lambda\) up to the order of unity, this expression is given by

\[
(d/\lambda)^2 = 24 \left( \frac{H_{CB}(T)}{H_{CF}(T)} \right)^2, \tag{11}
\]

where \(H_{CB}\) is the bulk critical field.

The field \(H_{CF}(T)\), above which noise was observed no longer, was obtained from recordings of the noise versus the applied field (sec. 3.3); \(H_{CB}(T)\) was calculated from \(H_{CB}(T) = H_{CB0}(1 - t^2)\) with \(H_{CB0} = 308\) Oe. It appeared that the dependence of \((d/\lambda)^2\) on temperature, calculated in this way, could be represented by

\[
(d/\lambda)^2 = (d/\lambda_0)^2(1 - t^4), \tag{12}
\]

where \(\lambda_0\) is the penetration depth at 0 °K; \(\lambda_0\) increases with decreasing film thickness. From eq. (12) the value of \(d/\lambda_0\) for each cylinder was obtained (see table I).

In fig. 6 the data of fig. 5 are replotted. The measured values of \(\langle \delta V^2 \rangle\) divided by \(\langle V^2 \rangle_{\Phi_0}\) are plotted against the values of \((d/\lambda)^2\) as obtained from eq. (11). A straight line is found, in accordance with eq. (10). The slope of the line, however, is 2-5 or 3 times greater than calculated. This means that in this cylinder with \(d/\lambda_0 = 0.88\) the dynamic flux quanta do not enter singly, but in bundles containing on the average 2.5 to 3 quanta.
Figure 7 shows the results of a similar plot for a cylinder having a thicker wall \((d/\lambda_0 = 1.90)\). Here \(\langle \delta V^2 \rangle \phi_0 = (29.4 \pm 1.9) \mu V^2\), calculated from eq. (7) by substitution of \(\phi_0 = 2.07 \cdot 10^{-15} V s; \ n = 1.3 \cdot 10^4; \ \mu_0 = 4\pi \cdot 10^{-7} H m^{-1}\); \(H = (7.10 \pm 0.08) \cdot 10^2 A m^{-1} s^{-1}\); \(A = (3.31 \pm 0.10) \cdot 10^{-7} m^2\); \(Q = 22.3 \pm 0.6; \ \omega_0 = 2\pi (4.06 \pm 0.04) \cdot 10^3 s^{-1}\). The linear relationship of eq. (10) appears to hold reasonably well even up to \((d/\lambda)^2 = 1\). From the slope of the line it follows that the average number of dynamic quanta in a bundle is 2.5. Above \((d/\lambda)^2 = 1\) the m.s. noise voltage reaches a constant value, as should be expected. The initial drop of \(\langle \delta V^2 \rangle\) immediately above \((d/\lambda)^2 = 1\) is caused by the fact that the bundle size decreases from 2.5 dynamic quanta in the linear region to 2 \(\phi_0\) in the constant region. It is remarkable that the flux enters here consistently as double quanta 2 \(\phi_0\).

As is shown by figs 6 and 7, eq. (10) gives a quantitatively fair approximation for the dynamic part of the flux quantum in films with \((d/\lambda_0) < 2\). For thicker films with \((d/\lambda_0) > 2\), however, eq. (10) was found to give only a qualitative description of the experimental results. In table I \(d, \lambda_0, \xi_0\) (the coherence length at 0 °K) and \(\kappa_0\) are given for 4 specimens. Using the experimental results of Douglass and Blumberg\(^{23}\), \(d\) was worked out from the value of \(d/\lambda_0\); \(\xi_0\) was calculated from \(1/\xi_0 = 1/\xi_{B0} + 1/d\), \(\xi_{B0}\) being the bulk value at 0 °K. Although for the thicker films, with \(d/\lambda_0 = 3.32\) and 4.70, \(\langle \delta V^2 \rangle\) was also found
to vary linearly with \((d/A)^2\) near \(T_c\), in agreement with eq. (10), \(\langle \delta V^2 \rangle\) became constant not at \((d/A)^2 = 1\) but at 2-5 and 5-0, respectively. It seems that for these type-I films \((\kappa < 0.71)\) the appropriate scaling factor in eq. (10) should be \((d/\xi)^2 = (\kappa d/A)^2\) instead of \((d/A)^2\).

On the cylinder with the thickest wall \((d/\lambda_0 = 4.70)\) the noise was found to lie about 25\% below the level for single-flux entry. This is due to partial shielding of the interior by the Meissner current. The resulting difference between the applied field and the field inside the cylinder was annihilated with some very large flux jumps into the cylinder, at the moment the critical field was reached.

5. Conclusions

(1) An experiment has been described in which the value of the flux quantum \(\phi_0 = h/2e\) has been determined from measurements of voltage fluctuations.
(2) Flux quantization involving up to \(3.7 \times 10^6\) enclosed quanta has been demonstrated experimentally.
(3) During the magnetization of cylinders with a wall thickness \(d\) smaller than the penetration depth \(\lambda\), a dynamic flux quantum equal to \((d/\lambda)\phi_0\) has been found. These quanta enter the cylinder in groups containing 2 or 3 of them.
(4) The noise measurements have been confined to only one frequency that was low compared to \(\tau_0^{-1}\), where \(\tau_0\) is the time needed by a flux quantum to enter the cylinder. By extending the measurements to other frequencies so as to cover the whole power spectrum of the fluctuations, it should be possible to determine the value of \(\tau_0\). With such measurements the transit time of flux bundles that cross a type-II superconducting foil in the mixed state has been determined 24).

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