MEASUREMENT OF THE RESISTIVITY AND THICKNESS OF A HETERO TYPE EPITAXIALLY GROWN SILICON LAYER WITH THE SPREADING-RESISTANCE METHOD

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Abstract

The contact resistance to a thin epitaxial layer on an heterotype substrate is calculated. Two geometries are analysed: a square contact on a bar-like structure and a circular contact on a semi-infinite structure. In the latter case the contact resistance is related to the shallow component due to microcontacts of a spreading-resistance probe in the author's arrangement and to the zero-bias barrier resistance in other arrangements. The applicability of this theory as a measuring method is shown to exist in principle. It can be of some practical use only as soon as a four-point probe of spreading-resistance quality is available. Then the resistivity and the thickness of the layer will be obtained from two measurements with the same instrument on the same spot in a four- and three-point-probe arrangement, with an error which is fairly large due to the widely different resolving powers of the methods.

1. Introduction

In the last years the spreading-resistance technique has become an indispensable tool for evaluation of the resistivity distribution in semiconductors, in particular silicon. The distribution can be investigated either horizontally by probing the slice surface or in depth by probing a beveled surface which exposes the profile magnified by the bevel angle.

In a semi-infinite geometry arrangement the spreading resistance has been reported to be qual to

$$R = \frac{\rho}{4A} k(\rho),$$

where $\rho$ is the resistivity, $A$ the contact radius usually equal to a few microns, and $k(\rho)$ a correction factor of order unity. Mazur\(^1\)\(^2\) attributes the departure from proportionality between $R$ and $\rho$ to a series interface resistance

$$R_i = \frac{\rho}{4A} [k(\rho) - 1]$$

between the metal and silicon due to a Schottky-barrier diode at zero bias. Although Keenan et al.\(^3\) and Schumann\(^4\) also appear to do so, in the actual measurement procedure described by Gardner et al.\(^5\), $A/k(\rho)$ is considered as the effective contact radius which depends on $\rho$.

The present author\(^6\)\(^7\) has already reported on spreading-resistance measurements with a specially hardened-steel probe softer than silicon, where $R$ is pro-
portional to $\rho$ over four decades of n-type-Si resistivity. Converging electrical and microscopic evidence has been produced which proves that the resistance measured on a semi-infinite structure consists of two components of widely different resolving powers. The interface resistance $R_i$ which is due to $n$ microcontacts of average radius $a$ acts as a by no means negligible series resistance $\rho/4na$ to the macrocontact contribution $\rho/4A$. The microcontact contribution covers at most a few tenths of a micron. Further interesting details in particular on the mechanical aspects of this model will be published shortly 8).

This paper deals with some corrections which should be applied for finite geometry, in particular when the layer thickness $d \ll A$. It seems reasonable to apply this correction only to the component that is controlled by the deep spreading resistance, and not by the shallow microcontacts resistance or zero-bias barrier resistance. Schumann, Gardner and Gorey 9,10) have analysed a two-layer structure with parameters $q_1, d_1$ and $q_2, d_2$. The measurement arrangement assumed for the computation of the correction factors consisted of three probes with equal spacings $s$ in-line on top of the layer. They presented computed correction factors to be applied to the semi-infinite-geometry spreading resistance as a function of $d_1/A$ for $d_2 = \infty$ and various resistivity ratios $q_1/q_2$. That analysis also includes the case of a thin layer on an isolating substrate of which an n-type epitaxial layer on a p-type substrate can be considered to be an example. Mazur and Dickey 11) have considered a two-probe measuring arrangement, and Dickey 12) presented a correction formula which was found to agree with the results of Schumann and Gardner 9) for large thickness, but to differ by about 10% at $d_1 = A$. For both solutions a resistance proportional to the sheet resistance $R_s = q_1/d_1$ of the thin layer which acts as a series resistance to the spreading resistance is dominant. Because Gardner et al. 5) consider the spreading resistance given in eq. (1) as the semi-infinite-geometry spreading resistance, the correction can be applied directly to eq. (1). Mazur 1,2) applies the correction to $\rho/4A$ and adds $R_i(\rho)$ simply as a series resistance.

It is the purpose of this paper to show that this is not always allowed and to present a theory which incorporates into the correction formula for hetero-type epitaxial structures also the interface resistance $R_i$, of whatsoever nature. In the next section this theory is presented as a corollary to a paper 13) on the four-point-probe method, in particular on the correction to be applied in the event that the junction resistance of an heterotype epitaxial structure is not negligible. In secs 3 and 4 experimental arrangements and some results are discussed which corroborate the theory. In these sections the relevance of the method as a technique for measuring the resistivity $\rho$ and the thickness $d$ of the layer is also treated. It is shown that in order to obtain $\rho$ and $d$ separately the measurements should be supplemented with other data which, because the sheet resistance $\rho/d$ occurs explicitly, can be four-point-probe measurements.
Of course, it is also possible to find \( \rho \) or \( d \) explicitly with the method described, if the other parameter, preferably the most uniform one, is supplied by a different measurement method. Because in general \( d \) is the more uniform one, it would be reasonable to use the infrared-multiple-interference or capacitance-voltage techniques for thickness measurement. It will be made clear that due to the widely different resolving powers of the two methods used and to the very nature of the equations a considerable error is introduced into the final result. The most important conclusion is that, provided the interface resistance is known as a function of the resistivity, also on isolated thin layers more or less reliable spreading-resistance measurements are possible.

It is worth stressing that applying the finite-geometry correction only to the macrocontact and adding the interface resistance simply in series is the right procedure with isotype epitaxial structures.

2. Theory of contact resistance to a thin layer

In a preceding paper \(^{13}\) on the four-point-probe method the current distribution was calculated in a silicon layer epitaxially grown on an opposite-type substrate. The analysis has been presented for a bar-shaped geometry and for a circularly symmetric structure of infinite extent around a probe.

As a corollary to this theory the current distribution below a square contact covering the full width \( B \) of the bar with a contact resistance \( R_c \) (in \( \Omega \text{ cm}^2 \)) on top of a layer of thickness \( d \), resistivity \( \rho \), and resistance per unit length...
$R_0 = \rho/dB$, shown in figs 1a, b and c, can be easily calculated. The contact material at potential $V_0$ represents an equipotential surface. The dependence of the potential $V$ on the coordinate $x$ is given by the solution to eq. (4) of ref. 13, repeated in the appendix as (A4). The appropriate boundary conditions $i = 0$ at $x = 0$ and $i = i_0$ at $x = B$ yield the solution

$$V(x) - V_c = i_0 R_0 \frac{A}{\sinh (B/A)} \cosh (x/A),$$

where

$$A^2 = R_0 d/\rho.$$  \tag{2}

Hence the resistance $R_m$ which would be measured between the contact and an imaginary probe near $x = B$, where the current lines are parallel to the surface, is equal to

$$R_m = R_0 A \coth \frac{B}{A}. \tag{4a}$$

This resistance $R_m$ is properly speaking a geometrically constrained spreading resistance under a square contact expressed in terms of $R_e = R_e B^2$ and $R_0 = R_e/B$. When $R_m$ is determined by the geometry rather than by $R_e$, implying $R_0 B \gg R_e/B^2$ or $B \gg A$, this resistance can be approximated to give

$$R_m = R_0 A = \left(\frac{\rho}{d} R_e\right)^{1/2} \frac{1}{B}. \tag{4b}$$

Essentially this expression is quoted by Yu 14), when referring to unpublished work of Shockley 15), by Wagner and Besocke 16), who discuss the impedance between equidistant parallel metal strips on a thin layer and by Murrmann and Widmann 17). In the other extreme $B \ll A$, which implies $\rho/d \ll R_e/B^2$, the resistance $R_m$ is found, as expected, to be caused by the contact resistance $R_e$ only:

$$R_m = \frac{R_e}{B^2}. \tag{4c}$$

As a further corollary to the theory outlined in the preceding paper 13), the current distribution below a circular contact of radius $A$ on top of a similar layer shown in fig. 1d, can be calculated *). The dependence of the potential $V$ on the coordinate $r$ is given by the general solution to eq. (15) of ref. 13, repeated in the appendix as (A7). However, the solution valid for the arrangement discussed there cannot be used here because the boundary conditions are different. The full solution is given by 19)

* Upon completion of this paper a brief paper by Niskov and Kubetskii 18) on the resistance of ohmic contacts between metals and semiconductor films where the same result is derived, came to the author's attention.
where $I_0(z)$ and $K_0(z)$ are the modified Bessel functions of the first and second kind, $a$ and $b$ are constants and $A$ is given by eq. (3). From the boundary condition $i = i_0$ at $r = A$ the resistance measured between the contact and a probe very near $r = A$ is found to be

$$R_m = \frac{I_0(z) + c K_0(z)}{I_1(z) - c K_1(z)} \frac{1}{2 \pi d},$$

where

$$c = \frac{b}{a} \quad \text{and} \quad z^2 = \frac{A^2}{L^2} = \frac{A^2 q}{R_c d} = \frac{R_s}{\pi R_t}.$$

It is reasonable to assume that $c = 0$, because for any non-zero value of $c$, a value of $A/L$ would exist where $R_m$ changes sign, which is not acceptable. Moreover, for $A \ll L$ the two modified Bessel functions of the first kind $I_0 \to 1$ and $I_1 \to 0$ and $R_m$ would increase as $\log z$. Therefore, it must be concluded that $c = 0$ and that $R_m$ is given by

$$R_m = \frac{q}{2 \pi d} F(z),$$

where $F(z) = I_0(z)/z I_1(z)$ is plotted in fig. 2. The resistance $R_m$ is properly speaking a geometrically constrained spreading resistance under a circular contact expressed in terms of $R_c = R_t \pi A^2$ and $R_s = q/d$. When $R_m$ is determined by the geometry rather than by $R_c$, implying $q/d \gg R_c/A^2$ or $A \gg L$, the function $z F(z)$ equals unity and

![Fig. 2. The function $F(z)$ with $z^2 = A^2 q/R_c d$.](image)
From the expansion of $F(z)$ near $z = 0$ it is found that for $A \ll \lambda$, which means that the contact resistance is dominant,

$$R_m = \frac{\varrho}{2 \pi d} \frac{1}{z} = \frac{1}{2 \pi \lambda} \left( \frac{\varrho R_e}{d} \right)^{1/2}.$$  \hfill (7b)

It was shown in a previous paper \(^6\) that the contact between a probe of specially hardened steel and silicon is spreading-resistance-controlled and multi-tipped. Later \(^7\) it was shown that this necessarily implies that the resistance $R_s$ should be considered as a series combination of a shallow contribution $R_i$ due to microcontacts and a deep contribution $R_a$ due to the macro-contact. The shallow contribution can be obtained by subtracting $R_a$ from the measured $R_s$:

$$R_i = \frac{\varrho}{4 \pi a} = R_s - \frac{\varrho}{4 \pi A},$$  \hfill (8)

where $n$ is the effective number of microcontacts of average contact radius $a$. However, $n a$ can for this purpose be considered as a formal parameter, between 1 and 10 $\mu$m for a new and an old probe, respectively. On introduction of the shallow contribution $R_i \pi A^2$ as $R_e$ into eq. (7a) the desired equation is found, namely

$$R_m = \frac{\varrho}{2 \pi d} \sqrt{\frac{4 \pi a}{\pi d}}.$$  \hfill (9)

It is worthwhile at the end of this section to comment on a fundamental limitation of the presented theory which is implied by the model chosen. It has been assumed that for a square or a circular contact a vertical field $E_y$ exists immediately below the contact over the resistance $R_e / B^2$ or $R_e / \pi A^2$ and that the remaining part of the layer is dominated by a horizontal field $E_x$ or $E_y$, respectively. This means that the theory is applicable only on condition that $d \ll B$ or $A$.

Equations (4b) and (7b) have limited validity and what is really the best solution depends on the parameter which produces the inequality. With $R_e = 0$ the appropriate expressions for a circular contact have been calculated by Schumann, Gardner and Gorey \(^9,10\) and for a rectangular contact the current density has been calculated by Overmeyer \(^20\). For $d \gg A$ the resistance should then be the spreading resistance $\varrho/4A$. Theoretically, if $R_e$ has a finite value, a diameter can always be chosen which is such that the theory is valid. This holds for contact studies, but for microcontact-dominated spreading-resistance measurements $n a$ increases with $A$ and increasing the diameter is
no remedy. It is worth noting by the way that in many contact studies with contact resistance $R$ the parameter $R \pi A^2/\rho$ is about equal to one of the dimensions used: the contact radius $A$ or the thickness $d$ of the layer. This indicates that what is actually measured is controlled by geometrical constraints and not by the physics of the contacting process.

The experimental arrangement for measuring $R_m$ can be the same as used for a normal three-point-probe spreading-resistance measurement. When in addition for instance the sheet resistance $\rho/d$ is measured in the usual way with the four-point-probe method, $\rho$ and $d$ can be obtained separately from this equation. The implications of this method will be discussed in more detail in the next section.

3. Discussion on measurement procedures

When the layer to be evaluated with the spreading-resistance technique is thin with respect to the probe-contact radius $A$, at first sight it would seem reasonable to separate the interface resistance $R_i A^2$, whether this be due to microcontacts or to a zero-bias barrier resistance, from the corrected macro-contact spreading resistance $\rho/4A$, to calculate the correction formula to this component and add the shallow component simply in series. It is the gist of the theory outlined above to demonstrate that this is not correct for heterotype structures. On the other hand, for the evaluation of epitaxially grown heterotype layers there is an urgent need for a measurement technique which will obviate the difficulties encountered up to now in obtaining the resistivity and thickness separately. It would be still more desirable to measure the resistivity distribution in the top layer with the spreading-resistance resolving power. In principle, the method described in the preceding section would permit this. However, the errors allowed with experiments which illustrate a theory and with those which prove the feasibility of a measurement method may differ by one or two orders of magnitude. It will be shown that in this case feasibility is intimately related to the uniformity of the layer and to the quality of the measuring instrument, provided the theory is accurately applicable.

Ideally, for obtaining resistivity $\rho$ and thickness $d$, a single instrument must be used. In this case it seems most appropriate to use a good four-point probe, by which is meant a four-point probe with which spreading-resistance measurements can also be made. This imposes some restrictions as to play, probe material, sliding, velocity of impact, etc. on the probes. Indeed, as far as the author is aware, no commercially available four-point-probe head satisfies these requirements. Therefore the three- and four-point-probe measurements required cannot sufficiently reliably be done with the same instrument for evaluation purposes.

As discussed elsewhere, in working with three- or four-point-probe arrangements the probe distances $s_i$ and contact diameters $2A_i$ should be taken into account. When current is fed through the probes (1) and (3), and the poten-
tial of one of them, the common probe (1), is measured with respect to another probe (2), it can easily be found that for this three-point-probe arrangement

$$ R_{213} = \frac{V_{12}}{i_{13}} = R_m + \left( \frac{\varrho}{2\pi d} \right) \ln \left( \frac{(s_{12} - A_1) (s_{13} - A_3)}{A_1 (s_{23} - A_3)} \right). $$

(10a)

where $R_m$ is the geometrically constrained spreading resistance under probe (1) derived in the preceding section, eq. (7). Here $\langle \rangle$ stresses the averaged nature of the resistivity in contrast to the local character in eq. (9). The sheet resistance should be measured separately with a four-point-probe head, and, since in all three expressions, $R_{2314}$, $R_{2413}$ and $R_{3412}$, the averaged values over all spacings occur, it is most appropriate to measure, as usual, $R_{2314}$, where

$$ R_{2314} = \frac{V_{23}}{i_{23}} = \left( \frac{\varrho}{2\pi d} \right) \ln \left( \frac{(s_{24} - A_4) (s_{13} - A_1)}{(s_{34} - A_4) (s_{12} - A_1)} \right). $$

(11a)

In case both measurements are done with one four-point-probe head, in a first-order approximation the contact radii can be neglected with respect to the probe spacings and supposing the three probe spacings to be equal, the simplified expressions

$$ R_{213} = R_m + \frac{\varrho}{2\pi d} \ln \frac{2s}{A} = R_m + \frac{\varrho}{2\pi d} \left( \ln \frac{s}{A} + 0.693 \right) $$

(10b)

and

$$ R_{2314} = \frac{\varrho}{2\pi d} 1.386 $$

(11b)

are obtained.

In order to obtain $R_m$ the layer should to a certain extent be uniform because in eq. (10b) $R_{2314}$, which is proportional to the average sheet resistance, is subtracted from $R_{213}$ and $R_m$ refers to the local conditions under the probe contact only. The values of $R_{2413}$ and $R_{213}$ can be read generally with an error equal to about 1% and the resultant error in $R_m$ depends on their relative magnitudes. Because the logarithmic factor $f$ in the second term of eq. (10a) equals about 6 in the actual arrangement, whereas $F(z)$ often equals about unity, the error in $R_m$ will increase to about 10%. In the spreading-resistance tracks and calibration-curves published earlier it has clearly been shown that for a certain probe, $a n$ is a constant in a measurement series. Hence in an ideal, uniform sample the thickness of the layer follows from eqs (7a), (10a) and (11b) combined as

$$ F\left( \sqrt{\frac{4 \cdot a \cdot n}{\pi d}} \right) = 1.386 \frac{R_{213}}{R_{2314}} - f. $$

(12a)

Assuming that the error in $z$ is about equal to the error in $R_m$, the error in the required result $d$ will amount to about 20%.
It is evident that, although the sheet resistance measured with the four-point-probe method is a very obvious parameter to introduce into eq. (10a), similar results can be obtained when either \( q \) or \( d \), preferably the more uniform one, is measured with a different method. When \( d \) is the more uniform parameter, measured by the infrared-multiple-interference or capacitance–voltage techniques, eqs (7a), (10a) and (11a) can be combined to

\[
R_{213} = \frac{1}{2\pi d} [q F(z(d)) + \langle q \rangle f].
\]  

(12b)

Since the dominant term in the measurement result, \( R_{213} \), in eqs (12a) and (12b) is proportional to an integrated value of the resistivity, which is necessarily smoother than the actual resistivity profile \( q(x) \), \( R_m \) will show a much more pronounced structure than either \( R_{213} \), \( R_{2314} \), or \( \langle q \rangle f \).

It cannot be denied that these two aspects: that is, the increased error and the smoothed-out value \( \langle q/d \rangle \) included in \( R_{2314} \), severely limit the applicability of this theory. The instrument should be of high quality and the slice should be fairly uniform, though no more so than the four-point-probe method in any case requires.

4. Experimental results and discussion

A real epitaxial layer is, of course, not uniform, but, as reported for instance by Eversteyn et al. \(^{21}\), epitaxial layers can be grown which are uniform in thickness to within 1% over a slice. This has been measured with the infrared-multiple-interference and capacitance–voltage techniques on \( n \)-on-\( n^+ \) layers and similar results have been found on heterotype structures with the ball-sleeve and stain technique. It is well known that the resistivity is a more difficult parameter to control, but in good epitaxy the sheet resistance \( R_{2314} \) measured with the four-point-probe method is found to be uniform over a slice to within a few per cent, at most. The second source of variations in the results of four- and three-point-probe measurements is due to non-reproducible mechanical properties of the instrument, in particular the probe spacings and contact diameters. Those two causes of errors should be clearly distinguished by suitable experiments. Because the feasibility of the method described depends strongly on the instrumental performance particularly of the four-point probe, this will be discussed in some detail now.

As discussed elsewhere \(^{13}\), a first criterion for the four-point-probe performance is the value and reproducibility of the ratio \( R_{2314}/R_{2413} \) on substrate material. If the three probe spacings are equal and the four contact diameters are equal or negligible with respect to the probe spacings, the ratio should be equal to 1·262. For four different types of four-point-probe heads the ratio varied between 1·22 and 1·29 and the reproducibility between 0·3 and 1·5%. This discrepancy could be correlated to the different values of \( s \) and \( \Delta \) and...
the error could be attributed to the non-reproducible positioning, due in particular to sliding of the probes.

With a Dumas four-point-probe head, a series of 2-Ω cm n-type, 3 μm thick layers epitaxially grown on p-type substrates was measured in the direction of the gas flow at nine positions each. The ratio $R_{2314}/R_{2413}$ shows a distribution over the slices which is not much different from the distribution over each individual slice and is equal to $1.275 \pm 0.005$. This was the substrate value for that particular probe head at that time. It is worth noting that these measurements are made at millivolt level, as advocated elsewhere\(^{13}\).

As an example of the theory outlined in the preceding section typical results of two slices are plotted in figs 3a and b. The sheet resistance $R_{2314}$ measured with the four-point-probe method proves to be constant to within about 2% in the direction of the gas flow. Perpendicular to that direction the slices are much less uniform, which certainly affects the accuracy.

![Diagram](image)

Fig. 3. The four-point-probe-measured sheet resistance $R_{2314}$ (dots) yields with the local spreading-resistance data the value of $R_m$ (crosses) and hence of $z$ (circles), for slices RiR 17-3 (a) and 17-9 (b).
The three-point-probe measurement of $R_{213}$ is done with the automated spreading-resistance equipment described earlier. The distance $s_{12}$ between the common and current probes amounted to 4.9 mm whereas the voltage probe was fixed to the slice at a minimum distance from the current probe $s_{23} = 15$ mm. Taking into account the error involved in the value of $A$, the distances $s_{13}$ and $s_{23}$ can be considered to be equal and constant during the measurement. For slice 17-3 the spreading resistance $R_{213}$ measured with the arrangement with the three-point probe on top and plotted with 100-μm steps is presented in fig. 4a.

From the local values of $R_{213}$ and $R_{2314}$, eqs (10a) and (11a) provide the local value of $R_m$. These and the ensuing values of $z$ are also plotted in figs 3a and b. The distribution of the result is due partly to lack of precision and partly to real structure in the epitaxial layer. If it were due only to the former cause, the error would have the right order of magnitude, assessed in the preceding section. In determining $d$ from $z$ the error in $a n$ should also be taken into account. The equivalent (not effective!) spreading-resistance radius $q/4R$ is accurately known to within 1%, but the value of $A$ is estimated rather subjectively under the microscope from the not perfectly circular print of the probe. Under the actual circumstances to which figs 3a and 4a have reference, $q/4R = 4.05$ μm, $A = 15.5$ μm, and hence with eq. (8), $a n = 5.3$ μm. With $z = 1.8$ the thickness of the epitaxial layer follows as $d = 2.4$ μm, which is within the limits of experimental error compared to the thickness $d = 3$ μm obtained with the usual ball-sleeve and stain technique. A similar and more constant value is found for 17-9, shown in fig. 3b.

Another typical result which illustrates the power of the technique in the detection of non-uniformities is shown in fig. 4b. The effect of the edges of the slice is shown in fig. 4c. The measurement track is always run through the middle of the former four-point-probe positions in order to furnish a maximum of relevant data.

5. Summary and conclusions

The resistance of a contact to a thin heterotype layer can always be separated into two contributions: the first is $R_c$ directly below the contact due to contact resistance $R_c$ and the second arises from the material between the two contacts proportional to the sheet resistance $q/d$. This effect has been analysed for a bar-shaped geometry with a square contact and for an infinite geometry with a circular contact. It turns out that the parameter $n a/d$ can be found if the parameter $q/d$, as valid under the probe, is known. The averaged value of $q/d$ can be obtained from a four-point-probe measurement, and if it is assumed to be equal to the local value and if $n a$ is known, $q$ and $d$ are found separately. This is particularly important for thin heterotype epitaxial layers. The procedure, conditionally phrased above, can be followed only with a four-point
Fig. 4. The spreading resistance plotted on the automated instrument for slice RiR 17-3 (a), slice 17-6, which shows an interesting detail (b), and slice 17-5 which shows the effect of the edges (c), because the left-hand side is less than a probe-print diameter from the edge. To show the reproducibility of the technique all plots are, at least partially, repeated at 100 μm distance running parallel in the opposite direction, as explained elsewhere⁵,⁷.)
probe with probes of spreading-resistance quality: *n* or *p* should be known and reproducible. In the author’s experience no four-point-probe head commercially available satisfies the requirements for reliable spreading-resistance measurements.

The theory presented has been tested, as far as the sample uniformity and the instruments allow, by measuring the sheet resistance at a number of positions and plotting a spreading-resistance track at the appropriate positions. Identifying the averaged sheet resistance measured with the four-point-probe method with the local value valid under the contact introduces an error amounting to about 20% which rules this out as a feasible measuring method.

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**Appendix**

From the equivalent circuit in fig. 1c the voltage drop in the x-direction can be seen to be given by

\[
\frac{d}{dx} V - V_0 = -R_0 i
\]

(A1)

and the current increase in the x-direction by

\[
\frac{di}{dx} = B J_y,
\]

(A2)

where the layer resistance per unit length is \( R_0 = \rho/B = R_s/B \). The junction characteristic is supposed to be uniform and equal to

\[
V_0 - V = R_c J_y.
\]

(A3)

Equations (A1), (A2) and (A3) can be combined to yield the differential equation

\[
\frac{d^2 V}{dx^2} - \frac{1}{A^2} (V - V_0) = 0,
\]

(A4)

where \( A^2 = R_c/B \). With the appropriate boundary conditions, eq. (2) can be found from this equation.

For circular geometry of a contact of radius \( A \) at potential \( V_0 \) similar equations can be written as

\[
\frac{dV}{dr} = -\frac{1}{2\pi r} \frac{\rho}{d} i
\]

(A5)
and the current increase in the radial direction as

\[ \frac{dI}{dr} = 2 \pi r J_x. \]  

With the same junction characteristic eq. (A3), eqs (A5) and (A6) can be combined to yield the differential equation

\[ \frac{d^2V}{dr^2} + \frac{1}{r} \frac{dV}{dr} - \frac{1}{A^2} (V - V_0) = 0. \]  

Equation (5) is the general solution to this equation.

REFERENCES

2) R. G. Mazur, to be published, Westinghouse Scientific Paper, 69-1F4-Pl.
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