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ON A NEW TYPE OF X-RAY GENERATOR*)

BY

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Abstract

The principle of the described X-ray generator is a variation upon the direct-current inductor, the grid-controlled X-ray tube itself acting as the interrupter. The anode of the X-ray tube is connected to a constant-voltage generator via a high-inductance coil. The capacitance is constituted by the sum of the capacitances of the tube, the leads and the coil. The main feature of the X-ray tube is the electron gun, consisting of a multi-grid construction of the Pierce type and a dispenser cathode with a concave emitting surface. The tube current is controlled by variation of the accelerating-grid voltage. By applying periodically a negative voltage pulse of rectangular wave form to the accelerating grid the capacitance is loaded at a low voltage and discharged at a high voltage alternately. The main data are: maximum voltage 250 kV, maximum tube current 300 mA, repetition frequency about 1100 c/s, discharge time about 50 μs. Also a spectrometer is described specially designed for measuring the high voltage and the spectral-energy distribution of the X-radiation produced.
ERRATA

Equation (1.2) on p. 1 should read

\[ V_2' = i_1' \left( \frac{L_1}{C_2} \right)^{1/2} \]

Equation (2.2) on p. 7 should read

\[ (i)_{t=0} = i_{a_0}; \left( L \frac{d i}{d t} \right)_{t=0} = V_{a_0} \]

Equation (3.6) on p. 17 should read

\[ \omega L = \left( \frac{L}{C} \right)^{1/2} = 15 \frac{V_{a_0}}{i_{a_0}} = 10^6 \Omega \]

Equation (3.13) on p. 24 should read

\[ \frac{d}{R_c} = 0.32 \]

Equation (5.3) on p. 41 should read

\[ \alpha = 0.001 \]

Equation (5.4) on p. 41 should read

\[ \Delta \lambda = 2d \alpha = 0.006 \text{ Å} \]
1. INTRODUCTION

The famous discovery by Röntgen 1) in 1895 of a new kind of radiation generated by bombarding matter with electrons was soon followed by an ever-increasing application of these X-rays in many domains of science and engineering. The extension of the applications was in turn accompanied by progressive improvements of the X-ray apparatus.

1.1. Survey of high-voltage-generator development

The high-speed electrons necessary for producing X-rays can be obtained in two essentially different ways.

The first method consists in accelerating the electrons in a static or quasi-static electric field. The final speed is determined by the difference in potential between the electrodes.

The alternative method consists in multiple acceleration of the particles along a distinct track by high-frequency electric fields until the desired speed is attained. Lawrence 2) was the first to realize this type of particle acceleration in the so-called cyclotron. The first attempt for electrons was made by Wideröe 3). After many years Kerst 4) succeeded in making a successful machine, the betatron. This was the beginning of intense activity in this field, stimulated by the urgent need for particles with great kinetic energy in nuclear physics.

In X-ray technique the indirect method is not likely to replace the direct method in the voltage region below a few million volts because of the greater complexity of the apparatus. In the multimillion-volts region the linear accelerator and the betatron are the most promising X-ray machines. In the following we shall restrict ourselves to the "classic" method.

The oldest high-tension source for X-ray tubes is the direct-current inductor or "open-core" transformer (fig 1.1). Denoting the primary peak current and

![Fig. 1.1. Direct-current inductor.](image)

the secondary peak tension by $i_1'$ and $V_2'$ respectively, we have the energy equation

$$\frac{1}{2}L_1i_1'^2 = \frac{1}{2}C_2V_2'^2.$$  \hspace{1cm} (1.1)

Hence,

$$V_2' = i_1' \frac{L_1}{C_2}. \hspace{1cm} (1.2)$$
Although the normal "closed-core" transformer was already known, the inductor type of high-voltage generator combined with the gas-discharge X-ray tube was maintained for many years. One of the reasons was that the inductor turned out to be specially adapted for the gas-filled tube because of the negligible voltage in the reverse direction. Up to then the only existing rectifier was of the mechanical type and was considered a cumbersome complication because of its size, noise, and ozon production. In spite of these disadvantages the mechanical rectifier later found a fairly wide application in high-power installations because of its reliability. Another reason for the prolonged use of the inductor was the fact that the voltage of the public power supply was mainly d.c. in those days.

The advent of the Coolidge tube produced a great change. Under certain conditions the high-vacuum X-ray tube could now be made self-rectifying. Moreover, the introduction of the vacuum-rectifying valve made the application of rectified a.c. voltage much more attractive. In addition the voltage of the public power supply was changing from d.c. to a.c. The low-frequency transformer soon became generally accepted.

The subsequent period that continues up to the present is chiefly characterized by improvements to the apparatus. The addition to the high-voltage transformer of rectifying valves and condensers resulted in the well-known circuits bearing the names of Grätz, Villard, Wittka, Greinacher, Cockcroft and Bouwers. At the same time the dimensions were reduced as a result of better insulation and lower losses. In a class by itself is the moving-belt generator of Van de Graaff, one of the most successful types in the group of electrostatic generators.

Although nowadays the low-frequency transformer is the most widely used high-voltage source, several attempts have been made to find other solutions based on the possibilities offered by modern high-frequency techniques. This research was stimulated by a problem encountered in the construction of X-ray apparatus, for which there are two essentially different alternatives, i.e. (1) the X-ray tube is connected to the generator by means of high-voltage cables, or (2) X-ray tube and high-voltage transformer are enclosed together in one envelope.

The first method has the advantage of a light-weight tube which is easy to manipulate and requires only a small stand. This is, for example, a very important point in medical diagnostic practice. This advantage is lost, however, at higher voltages owing to the fact that the high-voltage cables become heavy and less flexible, with a practical limit of 300 kV d.c. symmetrical voltage. It was therefore obviously desirable, especially for high voltages, to look for ways and means of reducing the weight and size of the apparatus. For low-power apparatus solution (2) is used because it may lead to a cheaper construction and the
absence of the vulnerable H.T. cables can be important for transportable and
dismountable equipment.

One of the first successful solutions was the combination of a resonance
transformer and an X-ray tube in a high-pressure gas tank, designed by
Charlton and Westendorp. They succeeded in making an industrial one-
million-volt unit of reasonable size and advantageous features. As a compromise
between excessive losses on the one hand and too many secondary turns on the
other, they chose 180 c/s for the resonance frequency. The absence of an iron
core not only meant a considerable reduction in weight but furthermore per-
mitted the location of the X-ray tube in the axis of the transformer.

The application of a pulse transformer is another interesting development.
Originally the radiography of rapidly moving metal parts was performed
exclusively by discharging a high-voltage condenser through an X-ray tube.
These units are rather bulky and restricted in control of pulse width and repe-
tition rate. Slack et al. describe an X-ray unit for high-speed radiography
using a pulse transformer. The tube head, which contains the X-ray tube and
the transformer, is connected to the power supply by a 20-kV cable. The power
supply comprises a voltage-doubling choke circuit to charge a condenser. The
discharge is controlled by a hydrogen thyratron. With this apparatus it is
possible to produce 10-μs pulses at a repetition rate from 50 to 150 pulses/s.
The tube current during the discharge is 60 A, the peak voltage 150 kV.

Also founded on the achievements in the field of high frequencies is the
combination of an X-ray tube and a cavity resonator, realized by Mills. He
obtained a mean current of 70 μA at a peak voltage of 1·1 MV by exciting
the resonant cavity with a magnetron operating at a wavelength of 25 cm. The
pulse width is 5 μs at a repetition rate of 200 pulses/s.

It should be mentioned here that these pulsating X-ray machines introduce
a serious tube problem, owing to the fact that very high tube currents are
required. The use of a tungsten filament entails a considerable filament power
and a limited life of the filament. If a gas-discharge tube is used the cathode is
damaged within a short time and it is difficult to obtain an acceptable focal
spot. In all cases a relatively quick anode breakdown can be expected. A
comparative review of different types of tube has been given by Clayton.

A major improvement was made by Dyke et al., who developed an X-ray
tube with a cold cathode, giving a tube current of 1400 A and a current density
up to 10⁸ A/cm² at 100 kV. The pulse width is 0·03 μs. Since there is no heater
supply and the insulation strength is much higher than normal due to the short
pulse length, an important gain could be obtained in size and weight of the
equipment.

The following pages contain a treatise on yet another kind of X-ray generator,
based on a suggestion of Douma. The principle of the idea is given in the
next section. Chapter 2 contains an analysis of the circuitry, and chapter 3 deals
with the problems connected with design and construction of the X-ray tube. In chapter 4 a review is given of the electrical generating and control equipment. Chapter 5 concludes with a description of a spectrometer of special design for measuring peak voltage and spectral-energy distribution. In addition the results of dose measurements are given and some final remarks are made.

1.2. Outline of new X-ray generator

The principle of the design is a variation upon the direct-current inductor, the grid-controlled X-ray tube itself acting as the interrupter.

The anode of the X-ray tube is connected to a constant-voltage generator $V_{ao}$ via a high inductance coil $L$ (fig. 1.2). The total capacitance constituted by the tube, the leads and the coil is denoted by $C$. When the tube current is cut off with a suitable grid voltage the X-ray tube and the coil are virtually connected in parallel. The voltage across the coil and thus across the X-ray tube rises rapidly to a high value. When the voltage is near its peak value the tube is made conductive again by restoring the normal grid voltage. In consequence $C$ discharges through the tube. In this way pulsed X-rays are obtained with a high tube current during the pulse. The repetition rate is determined by the time required for the current through the coil to increase again.

![Fig. 1.2. Basic diagram of X-ray generator.](image)

The characteristics of the X-ray tube are those of a pentode. The tube is given a pulsed grid drive of rectangular wave form and negative sign. In the stationary condition a constant current $i_a = i = i_{ao}$ passes through the coil and the tube. This means that in the magnetic field of the coil a certain amount of energy has been accumulated, given by $E = \frac{1}{2} L i_{ao}^2$. At a certain moment the tube current is cut off. In consequence the capacitor $C$ is charged by the current through the coil and the energy of the magnetic field is converted into the electrical energy of the capacitor. Then the tube is made conductive again and the capacitor discharges through the tube. X-rays are now generated until the anode voltage has dropped to a very low value. After that the current passing through the tube and the coil again increases until the peak value $i_{ao}$ is reached. At that moment the tube current is cut off again, and so on.

If the conversion of magnetic energy into electrical energy is complete, the peak voltage of the tube is found from the relation
Hence,\[ \frac{1}{2} L i_a^2 = \frac{1}{2} CV^2. \] (1.3)

Hence,\[ V = i_a \sqrt{\frac{L}{C}}. \] (1.4)

From this formula it can be seen that the higher the peak current the smaller the inductance may be for a given peak voltage. As compared with a high-voltage transformer the inductance coil has the advantage of smaller size and weight. If it is possible to realize an X-ray tube that fulfils all requirements, then the combination of X-ray tube and inductance coil may lead to a compact lightweight tube head. The investigations dealt with in the next pages mainly concern the practicability in principle of the method described.
2. CIRCUIT CONSIDERATIONS *)

2.1. Determination of optimum conditions

First of all we wish to obtain an impression of the role played by the various electric quantities in order to be able to determine the conditions at which the highest X-ray yield may be expected. For that purpose we introduce some simplifications. In the circuit in fig. 1.2, L and C are considered to be free of any losses, while the \((i_a, V_a)\) characteristics of the X-ray tubes have an idealized shape with an infinitely high internal resistance (fig. 2.1). The grid voltage is given in fig. 2.2. The pulses are considered to be truly rectangular. The maximum grid voltage \(V_g = V_{g0}\) corresponds to the maximum anode current \(i_a = i_{a0}\). At all values of \(V_g\) below a distinct value \(V_g = V_{g\text{min}}\) the tube current is cut off for all anode voltages.

![Fig. 2.1. \((i_a, V_a)\) characteristics of X-ray tube.](image)

Supposing at \(t = 0\) the anode current has just reached the value \(i_{a0}\), the anode voltage still being zero, and taking the pulse width \(t_1\) as a variable, we may determine the value of \(T\) in such a way that after each (identical) cycle all quantities again have the same value as at \(t = 0\).

![Fig. 2.2. Grid voltage.](image)

![Fig. 2.3. Circuit diagram for \(0 \leq t < t_1\).](image)

*) The contents of this chapter have been provided mainly by Mr T. Douma, former member of the staff of Philips Research Laboratories, as described in his internal report nr. 2467.
At $t = 0$ the tube current is cut off. The circuit is given by fig. 2.3, where $i$ is the current through the coil. From that moment the following equation applies:

$$L \frac{di}{dt} + \int_0^t \frac{i}{C} \, dt = V_{a0}, \quad (2.1)$$

with initial conditions

$$(i)_{t=0} = i_{a0}; \quad \left( L \frac{di}{dt} \right)_{t=0} = V_a. \quad (2.2)$$

The solutions are

$$i = \frac{V_{a0}}{\omega L} \sin \omega t + i_{a0} \cos \omega t \quad (2.3)$$

and

$$V_a = V_{a0} (1 - \cos \omega t) + i_{a0} \omega L \sin \omega t \quad (2.4)$$

with

$$\omega^2 = \frac{1}{LC}. \quad (2.5)$$

From eq. (2.4) it can be concluded that $V_a$ remains positive at least as long as $\omega t \leq \pi$. For the case of very high anode voltages it is justified to suppose $i_{a0} \omega L \gg V_{a0}$. The terms containing $V_{a0}$ may be neglected then for $\omega t < \pi$, and we get

$$i = i_{a0} \cos \omega t \quad (2.3a)$$

and

$$V_a = i_{a0} \omega L \sin \omega t. \quad (2.4a)$$

Fig. 2.4. Circuit diagram for $t_1 \leq t < t_2$.

Let the tube become conductive at $t = t_1 < \pi/\omega$. The corresponding circuit diagram, given by fig. 2.4, is mathematically expressed by the equations

$$L \frac{di}{dt} + V_{a1} + \int_{t_1}^t \frac{i_c}{C} \, dt = V_{a0} \quad (2.6)$$

and

$$i_c + i_{a0} = i,$$
with initial conditions:

\[ (i)_{t=t_1} = i_1; \quad \left( L \frac{di}{dt} \right)_{t=t_1} = V_{a_0} - V_{a_1}, \quad (2.7) \]

where \( i_c \) is the current to the capacitor and \( i_1 \) is the current through the coil given by eq. (2.3) at \( t = t_1 \).

The solutions are

\[ i = \frac{V_{a_0}}{\omega L} \sin \omega t + i_{a_0} \cos \omega t + i_{a_0} \{1 - \cos \omega(t - t_1)\} \quad (2.8) \]

and

\[ V_a = V_{a_0}(1 - \cos \omega t) + i_{a_0} \omega L \sin \omega t - i_{a_0} \omega L \sin \omega(t - t_1), \quad (2.9) \]

or, written otherwise,

\[ i = \frac{V_{a_0}}{\omega L} \sin \omega t + 2i_{a_0} \sin \frac{\omega t_1}{2} \cos (\omega t + \varphi) + i_{a_0} \quad (2.8') \]

and

\[ V_a = V_{a_0}(1 - \cos \omega t) + 2i_{a_0} \omega L \sin \frac{\omega t_1}{2} \sin (\omega t + \varphi), \quad (2.9') \]

with

\[ \varphi = \frac{\pi}{2} - \frac{\omega t_1}{2}. \quad (2.10) \]

The expressions (2.8), (2.9), (2.8') and (2.9') are valid from \( t = t_1 \) until \( t = t_2 \), during which time \( V_a \) decreases from its maximum value to zero. With \( i_{a_0} \omega L \gg V_{a_0} \), eqs (2.8') and (2.9') become

\[ i = 2i_{a_0} \sin \frac{\omega t_1}{2} \cos (\omega t + \varphi) + i_{a_0} \quad (2.8a') \]

and

\[ V_a = 2i_{a_0} \omega L \sin \frac{\omega t_1}{2} \sin (\omega t + \varphi), \quad (2.9a') \]

with

\[ \varphi = \frac{\pi}{2} - \frac{\omega t_1}{2}. \quad (2.10) \]

At \( t = t_2 \), \( V_a \) has decreased to zero. From eq. (2.9a') this is only possible when \( \sin (\omega t + \varphi) = 0 \). Substitution in eq. (2.8a') gives

\[ i_2 = i_{a_0} - 2i_{a_0} \sin \frac{\omega t_1}{2}. \quad (2.11) \]

Now this leads to two possibilities:

\[ (A). \quad t_1 < \frac{\pi}{3} \quad (i_2 > 0), \]
(B) \[ t_1 > \frac{\pi}{3} \ (i_2 < 0). \]

(A) At the moment \( t = t_2 \) the anode voltage has decreased to zero and the current through the coil is positive \((i_2 > 0)\). From that moment the circuit diagram is given by fig. 2.5 and the corresponding equation by

\[
L \frac{di}{dt} = V_{a0} .
\]  

(2.12)

The solution is

\[
i = \frac{V_{a0}}{L} (t - t_2) + i_2
\]

(2.13)

and \( i \) increases until it has attained the maximum value \( i_{a0} \). This happens after an interval \( T \), given by

\[
T = t_2 + \frac{L(i_{a0} - i_2)}{V_{a0}},
\]  

(2.14)

\( t_2 \) being found from eq. (2.9) by determining \( t \) at \( V_a = 0 \). Substitution of the derived value in eq. (2.8) gives \( i_2 \). With \( i_{a0} \omega L \gg V_{a0} \) the values are

\[
\omega t_2 = \frac{\pi}{2} + \frac{\omega t_1}{2}
\]

(2.15)

and

\[
i_2 = i_{a0} \left( 1 - 2 \sin \frac{\omega t_1}{2} \right).
\]

(2.16)

(B) At the moment \( t = t_2 \) the anode voltage has decreased to zero and the current through the coil is negative \((i_2 < 0)\). In that case the circuit diagram is given by fig. 2.3. The equation is again given by (2.1) with initial conditions:

\[
(i)_{t=t_2} = i_2; \quad \left( L \frac{di}{dt} \right)_{t=t_2} = V_{a0}.
\]

(2.17)

The solutions are

\[
i = \frac{V_{a0}}{\omega L} \sin \omega (t - t_2) + i_2 \cos \omega (t - t_2)
\]

(2.18)

and

\[
V_a = V_{a0} \{1 - \cos \omega (t - t_2)\} + i_2 \omega L \sin \omega (t - t_2).
\]

(2.19)
Neglecting the terms containing \( V_{ao} \) this situation remains until \( V_a \) switches from a negative to a positive sign, i.e. after a half period. Suppose \( V_{a3} = 0 \) at \( t = t_3 \), then \( i \) has become positive with a value \( i_3 = -i_2 \). From \( t = t_3 \) eq. (2.12) is valid. The solution is

\[
i = \frac{V_{ao}}{L} (t - t_3) + i_3; \tag{2.20}
\]

\( i \) increases to \( i_{ao} \). This happens after an interval \( T \), given by

\[
T = t_3 + \frac{L(i_{ao} - i_3)}{V_{ao}}. \tag{2.21}
\]

The exact value of \( t_3 \) is found by solution from eq. (2.19) at \( V_a = 0 \). Substitution of this value in (2.18) gives \( i_3 \).

Now it will be clear that nothing can be gained by admission of a period during which the anode voltage becomes negative. For that reason we restrict ourselves to the situation where \( \omega t_1 \leq \pi/3 \). If the terms containing \( V_{ao} \) are omitted where it is allowed, the important relations become from \( t = 0 \) to \( t = t_1 \):

\[
i = i_{ao} \cos \omega t, \tag{2.3a}
\]

\[
V_a = i_{ao} \omega L \sin \omega t, \tag{2.4a}
\]

\( i_a = 0 \);

from \( t = t_1 \) to \( t = t_2 \):

\[
i = i_{ao} \cos \omega t + i_{ao} \{1 - \cos \omega (t - t_1)\}, \tag{2.8a}
\]

\[
V_a = i_{ao} \omega L \sin \omega t - i_{ao} \omega L \sin \omega (t - t_1), \tag{2.9a}
\]

\( i_a = i_{ao} \);

from \( t = t_2 \) to \( t = T \):

\[
i = \frac{V_{ao}}{L} (t - t_2) + i_2, \tag{2.13}
\]

\( V_a = 0 \),

\( i_a = i \);

with

\[
\omega t_2 = \frac{\pi}{2} + \frac{\omega t_1}{2}, \tag{2.15}
\]

\[
i_2 = i_{ao} \left(1 - 2 \sin \frac{\omega t_1}{2}\right), \tag{2.16}
\]

\[
T \omega = \frac{\pi}{2} + \frac{\omega t_1}{2} + \frac{2i_{ao} \omega L \sin \frac{1}{2} \omega t_1}{V_{ao}}; \tag{2.14a}
\]
\( \omega T \) increases from \( \pi/2 \) at \( t_1 = 0 \) to \( 2\pi/3 + i_{ao}\omega L/V_{ao} \) at \( \omega t_1 = \pi/3 \).

At \( \omega t_1 = \pi/3 \), \( T \) is largest because the conversion of the magnetic energy \( \frac{1}{2} L i_{ao}^2 \) into anode energy is then complete. After that the current through the coil is raised by the battery from zero to \( i_{ao} \) in \( i_{ao}L/V_{ao} \) s.

For the energy supplied by the battery during one period we find, after some calculation,

\[
W_b = V_{ao} \int_0^T i \, dt = V_{ao}i_{ao} (t_2 - t_1) + \frac{1}{2} L(i_{ao}^2 - i_2^2).
\]

(2.22)

With \( i_{ao}\omega L \gg V_{ao} \) the energy per second becomes

\[
W_s = V_{ao}i_{ao} \left( 1 - \sin \frac{\omega t_1}{2} \right).
\]

(2.23)

From this equation the anode dissipation for \( \omega t_1 = \pi/3 \) is found to be

\[
W_s = \frac{1}{2} V_{ao}i_{ao}.
\]

(2.24)

In figs 2.6, 2.7, 2.8 and 2.9 the graphs are given of \( V_{ao}, i_a \) and \( i \) as a function of \( \omega t \) for \( \omega t_1 = \pi/6 \), \( \omega t_1 = \pi/3 - \epsilon \), \( \omega t_1 = \pi/3 + \epsilon \) and \( \omega t_1 = \pi/2 \), respectively, with \( i_{ao}\omega L/V_{ao} = 15 \). The symbol \( \epsilon \) is used to indicate a very small deviation.

From the foregoing it can be concluded that \( \omega t_1 = \pi/3 - \epsilon \) gives the best operating conditions, because in that case the total amount of magnetic energy is converted into electric energy, the anode never becoming negative. In addition \( T \) has to be fixed in such a way that the current through the coil after the discharge has again reached the value \( i_{ao} \).

Fig. 2.6. \( V_{ao}, i_a \) and \( i \) as a function of \( \omega t \) with \( \omega t_1 = \pi/6 \).
Fig. 2.7. $V_a$, $i_a$ and $i$ as a function of $\omega t$ with $\omega t_1 = \pi/3 - \varepsilon$.

Fig. 2.8. $V_a$, $i_a$ and $i$ as a function of $\omega t$ with $\omega t_1 = \pi/3 + \varepsilon$. 
The quality of the excited X-rays depends on the shape of the anode voltage during the discharge. The highest output would be obtained if the anode voltage remained constant. In our case the anode voltage has a convex shape given by fig. 2.10, as may be derived from eq. (2.9a') by writing

\[ V_a = 2i_{a0}\omega L \sin \frac{\omega t_1}{2} \cos \left( \omega (t - t_1) + \frac{\omega t_1}{2} \right). \]  

(2.25)

It is easily seen that with \( \omega t_1 = \pi/3 \) the anode voltage is symmetrical. About 80 per cent of the energy is dissipated between \( V_{a\text{ max}} \) and \( \frac{1}{2} V_{a\text{ max}} \).
2.2. Deviations from ideal conditions

We may now determine roughly the influence of various approximations used in the calculations of the preceding section.

First of all there will be no ideal pentode characteristic, but a more realistic curve, as given in fig. 2.11. This results in a non-linear increase of the anode current. Instead of eq. (2.12) the following equation applies:

\[ \frac{di}{dt} + i \frac{r}{L} = V_{ao}, \]  

(2.26)

with solution

\[ i = \frac{V_{ao}}{r} \left( 1 - \exp \left[ \frac{r}{L} (t_2 - t) \right] \right) + i_2 \exp \left[ \frac{r}{L} (t_2 - t) \right], \]  

(2.27)

where

\[ r = \frac{V_{cr}}{i_{ao}}, \]  

(2.28)

and where \( V_{cr} \) is the lowest anode voltage for which \( i_a = i_{ao} \). As long as \((r/L)T \ll 1\) the deviation from eq. (2.13) is very small.

The anode dissipation has to be revised as well. Using eq. (2.13) the anode dissipation from \( t_2 \) to \( T \) becomes

\[ \int_{t_2}^{T} r i^2 dt = \frac{3}{2} \frac{V_{cr}}{V_{ao}} \left( i_{ao}^2 - i_2^2 \right). \]  

(2.29)

With \( \omega t_1 = \pi/3 \) this expression alters into

\[ \frac{3}{2} \frac{V_{cr}}{V_{ao}} \frac{1}{2} L i_{ao}^2. \]  

(2.30)

Consequently a small value of \( V_{cr} \) is necessary to keep additional anode losses within bounds.

A third deviation is due to the finite resistance of the inductor. If we take into
account this resistance $R$ we find instead of eq. (2.3):

$$i = i_a \exp\left(-\frac{R}{2L} t\right) \cdot \left(\frac{R}{2\omega L}\sin \omega t + \cos \omega t\right),$$

(2.31)

with

$$\omega^2 = \frac{1}{LC} - \left(\frac{R}{2L}\right)^2$$

(2.32)

In the interval $\omega t < \pi/2$, eq. (2.31) will not differ much from eq. (2.3a) as long as $\omega L/R \gg 1$. For a first-order approximation it is therefore permitted to consider the coil loss-free as far as the value of the current is concerned. We now suppose that the same is true during the intervals $(t_1, t_2)$ and $(t_2, T)$.

One period of $i$ consists of three parts, i.e.

$$i = i_a \cos \omega t \quad \text{for} \quad 0 \leq t < \frac{\pi}{3},$$

$$i = i_a \left[1 + \cos \left(\omega t + \frac{\pi}{3}\right)\right] \quad \text{for} \quad \frac{\pi}{3} \leq t < \frac{2}{3},$$

(2.33)

$$i = \frac{V_a}{\omega L} \left(\omega t - \frac{2\pi}{3}\right) \quad \text{for} \quad \frac{2}{3} \leq t < \frac{2}{3} + \frac{i_a L}{V_a}.$$ 

After some calculation we find for the effective value of the current

$$\left(\frac{1}{T} \int_0^T i^2 dt\right)^{1/2} > \frac{i_a}{\sqrt{3}}.$$ 

(2.34)

Consequently, the losses in the coil will be $\frac{1}{3} i_a^2 R$ at the least.
3. THE GRID-CONTROLLED X-RAY TUBE

3.1. Introduction

Before dealing with the problems connected with the design and construction of the X-ray tube we have to establish the conditions to be satisfied by the tube. The main requirements, some of which have already been mentioned in chapter 2, can be formulated as follows.

(a) The tube must be capable of giving a high anode current. The relation (1.4), \( V = i (L/C)^{1/2} \) shows that a high peak voltage in the first place depends on a high value of the anode current.

(b) It must be possible to cut off the anode current by a suitable grid voltage for any anode voltage up to the peak value.

(c) The tube must have pentode properties. Moreover, the slope of the \((i_a, V_a)\) curve should be as steep as possible in order to keep losses low during the time that the current through the coil is increasing from zero to the maximum value \(i_{a_0}\).

(d) For the time that the tube conducts as an X-ray tube, i.e. during the discharge of the condenser through the tube, the electron beam must give a well-defined focal spot on the anode. We are not interested in any focal spot during the loading period.

(e) The tube must be capable to withstand an anode voltage of approximately 250 kV.

In addition to those mentioned above there are a number of other requirements concerning insulation, anode dissipation etc., but since these are not at variance with those underlying ordinary X-ray tubes, they will not be discussed here.

The requirements (a), (b) and (c) are connected with the switching properties of the tube, while requirements (d) and (e) make the tube an X-ray tube. It is possible, of course, to divide the required properties over two different tubes, a switching tube and a normal X-ray tube, but it will be clear that such a proposition is not at all attractive and furthermore by no means simplifies the problem.

In order to ascertain the practicability of the proposed method we now have to fix the values of the various quantities by making our choice out of rather numerous possibilities. First of all we decide on a total power

\[ P = 3.10^3 \text{ W} \]  

and an anode discharge current

\[ i_{a_0} = 0.3 \text{ A.} \]  

The mean value of the anode current is 0.15 A. It follows from (2.24) that
With the assumption
\[
\frac{i_{ao} \omega L}{V_{ao}} = 15
\]
(3.4)
the absolute peak voltage becomes 300 kV, and by substitution in \((2.4a)\) the practical limit given by the best tuning of the circuitry becomes
\[
V_p = 300 \sin \frac{\pi}{3} = 259.8 \text{kV}.
\]
(3.5)
Further we have
\[
L = \left( \frac{L}{C} \right)^{1/2} = 15 \frac{V_{ao}}{i_{ao}} = 10^6 \Omega
\]
(3.6)
or
\[
L = 10^{12} \text{C}.
\]
(3.7)

If, for example, we estimate the total capacitance at \(50.10^{-12} \text{F}\), the self-inductance of the coil becomes 50 H. Substitution of these values in the equations of chapter 2 gives a repetition frequency of 1165 c/s. The anode-pulse rise time becomes 52 \(\mu\text{s}\), the discharge time is again 52 \(\mu\text{s}\), and therefore the total anode pulse lasts 104 \(\mu\text{s}\).

3.2. The cathode

In most X-ray tubes a tungsten filament is used as an electron source. One has to accept the rather poor efficiency of electron emission, because tungsten has some other properties making it in X-ray tubes far superior to an oxide-coated cathode.

As will be discussed later, the thermionic emitter in our tube has to satisfy three conditions in addition to those known from normal X-ray-tube practice:

(a) a high specific emission of electrons,
(b) a homogeneously emitting surface,
(c) no deformations.

Although a tungsten filament fulfills these requirements to a certain extent only, this filament was mainly used for lack of a better during the initial experiments. Thoriated tungsten gives a slight improvement in thermal yield but this advantage does not counterbalance the laborious pumping and degassing process and the increased chance of deformation.

In order to approximate to a homogeneously emitting surface as far as possible, the filament was wound in the shape of a flat spiral helix. Nevertheless it soon became evident that eventual results would be dependent on a more suitable cathode.
Numerous investigators have carried out extensive research to improve the oxide-coated cathode. This research was stimulated by the urgent need for a better cathode in connection with new developments in the high-frequency field. Among the remarkable improvements achieved, the dispensor cathode holds a special place owing to the fact that it combines the favourable properties of both tungsten filament and oxide-coated cathode and lacks most of their unfavourable properties. In our case the L cathode meant the release from insuperable difficulties.

However, it was seriously doubted whether the L cathode could be used in X-ray tubes on account of the continuous evaporation of barium, which was expected to have an unfavourable influence upon the high-voltage properties. To clear up this problem a number of identical high-voltage rectifying valves were made, some provided with a tungsten filament and others with an L cathode. Without going into detail as to the construction of these rectifying valves we shall only remark that experiments with them showed a relatively small difference in maximum attainable reversed voltage, i.e. 180 kV for the valves with a tungsten filament against 150 kV for the valves with an L cathode. As the conditions in a rectifying valve are much more unfavourable than in an X-ray tube for uni-directional high voltage, we expected no difficulties from the evaporation of barium. The results obtained confirmed these expectations.

The L cathode used throughout the experiments had a cylindrical shape (fig. 3.1). Normally the emitting surface is flat and circular. It is possible, however, to work the tungsten block so as to obtain a particular profile. As will be seen in the following sections, an electron gun of the Pierce type is used in the X-ray tube. The cathode of this gun needs a concave emitting surface. This is easily produced in our case by turning the tungsten mass to the prescribed radius of curvature $R_c$ (fig. 3.2). An additional advantage of turning is that a sharp edge can be obtained which is essential for good performance of the gun. The cathode cylinder has a diameter of 10 mm and a height of 6 mm. The operating temperature is about 1150 °C which corresponds to a specific emission of about 25 A/cm². The heating power required is about 40 W.
3.3. The electron gun

To design an electron gun for 0.3 A without any restrictions is a fairly simple task. In our situation, however, a number of conditions must be satisfied. Besides those mentioned in sec. 3.1(a)-(d), in case an auxiliary accelerating electrode is used, its potential must be as low as possible, as it will always draw some current with the result that the gun temperature increases. A further limiting condition is that no method of magnetic focussing to obtain a well-defined focal spot on the anode can be applied because it would complicate in an intolerable manner the construction of the tube, i.e. a purely electrostatic solution has to be found.

3.3.1. The aligned-grid gun

The most obvious solution is to make use of a multi-grid construction. A drawback as compared with radio-tube design is the fact that focussing considerations rule out a concentric location of the grids. We started by making a number of experimental tubes comprising a flat circular cathode, two plane grids parallel to the cathode and an anode (fig. 3.3). The function of the first grid is to cut off, by applying a negative voltage to it, the anode current at the desired moment. The function of the second grid, that carries a constant positive voltage, is firstly to increase the electric-field strength in the space directly in front of the cathode in order to counteract the space-charge limitations of the
anode current during the "charging" period of the coil, when the anode voltage is low and secondly to reduce this field strength during the period that the anode current has to be cut off completely and the anode voltage rises to very high values, which otherwise might outweigh the influence of the negative voltage applied to the first grid. Each grid consisted of equidistant tungsten wires. The grids are aligned in such a way that the wires of the second grid are situated in the "electron shadow" of the wires of the first grid, to minimize the current to, and the heat dissipation in the second grid.

The problem is to determine all dimensions in such a manner that optimum gun performance is obtained. Only the distance between gun and anode is known in advance. The minimum of this distance is determined by the appearance of field emission, which phenomenon depends on various factors such as surface condition, gas pressure, etc. As 20 kV/mm is a normal figure in high-vacuum tubes, that would mean 25 mm for 250 kV in our tube. In connection with the very short interval during which the anode is at top potential we fixed the distance between gun and anode at 20 mm.

The strong space-charge effect complicates an exact calculation of the optimum values of the pitch of the grids, their mutual distances and the diameter of their wires. By using as a starting point the mathematical relations for the space-charge-free case one may, however, obtain useful information on these questions. This was done by Groendijk *), resulting in the directive that the distance from \( g_1 \) to \( g_2 \) should be equal to the pitch of \( g_1 \). For aligned grids \( g_1 \) and \( g_2 \) should have the same pitch.

We made a number of tubes based on these directives. In our case the number of variations was restricted by the following considerations.

(a) In order to prevent sparking between first and second grid, the distance between them must not be less than 1 mm. This fixes the pitch of both grids at the same time.

(b) Even with the best gun construction \( i_{g2} \) will be rather high during the beginning of the loading period when the anode current and therefore also the anode voltage are still low. As a result the mean value of \( i_{g2} \) is about 25 mA and the energy dissipation in the second grid 15-20 W, which has to be removed by thermal radiation from the surface of the grid wires. The radiating area should be sufficiently large. Therefore the diameter of these wires cannot be less than 100 \( \mu \). In figs 3.4 and 3.5 the quite satisfactory characteristics of an experimental tube are given. The grid current \( i_{g2} \) is very small at maximum anode current. The dimensions of the gun were

\[
\begin{align*}
   d_1 &= d_2 = 100 \mu; \\
   a_1 &= a_2 = 1 \text{ mm}; \\
   l_{g2} - l_{g1} &= 1 \text{ mm}; \\
   l_{g1} &= 0.5 \text{ mm}.
\end{align*}
\]  

(3.8)

*) Personal communication.
Fig. 3.4. \((i_a, V_a)\) characteristics of experimental tube with aligned-grid gun.

Fig. 3.5. \((i_{a2}, V_a)\) characteristics of experimental tube with aligned-grid gun.

Rogers\(^{17}\) has made a large number of measurements on multigrid constructions of the same kind. The conclusions drawn from his results are in accordance with the above-mentioned system.

Although tubes with this type of gun operate quite well, these systems with wire grids had to be abandoned due to the fact that their focal spots are of very poor quality (fig. 3.6), small and with a very uneven electron distribution, which leads to a quick deterioration of the target area of the anode. This difficulty does not arise when the principles for an effective gun construction outlined by Pierce\(^{18}\) are applied.

### 3.3.2. The Pierce gun

Achievements in the microwave field created a demand for systems giving a converging electron beam of high intensity and uniform density. Pierce provided a very suitable solution for this problem. His starting point was the fact, already mentioned by Langmuir\(^{19}\), that Poisson's equation, describing the field in cases that the space charge cannot be neglected, can be resolved exactly in the case of rectilinear electron movement between parallel planes, coaxial cylinders and concentric spheres, only.

If, in the case of concentric spheres, we take for the cathode and the accelerat-
ing electrode those parts of the spheres that are cut out by a circular cone with its top in the centre of the spheres, then Pierce has shown that it is always possible to introduce an external field in such a way that the electron beam within the cone remains unchanged. This external field is created by the shape of the cathode electrode and the accelerating electrode (fig. 3.7). The shape of the electrodes can be calculated in simple cases only; in more complicated cases the solution has to be found by experimental means. Samuel 20) and Helm et al. 21) have made several calculations and electrolytic-trough measurements for different configurations. Starting from the requirements which the gun has to fulfil, the geometry of a gun can be found from the graphs and electrode shapes obtained by them.
3.3.2.1. Determining the size and shape of the Pierce gun

The following notation is used (fig. 3.8):

\[ \frac{I}{V^{3/2}} = 14.68 \times 10^{-6} \frac{1 - \cos \theta}{\alpha^2} \]  

(3.9)

In this relation the left-hand expression stands for the space-charge factor which may be considered as a measure of the quality of the gun; \( \alpha \) is a factor given by

\[ \alpha = \log \frac{R_{ac}}{R_c} - 0.3 \left( \log \frac{R_{ac}}{R_c} \right)^2 + 0.075 \left( \log \frac{R_{ac}}{R_c} \right)^3 + \ldots ; \]  

(3.10)

\( I \) is given by the required anode current, i.e. \( I = 0.3 \text{ A} \). The choice of \( V \) is still open but we want to keep \( V \) as low as possible. As an acceptable compromise we fix \( V \) at 1000 volts. From this follows

\[ \frac{I}{V^{3/2}} = 9.5 \times 10^{-6}. \]  

(3.11)

By giving \( \theta \) a distinct value we fix \( \alpha \) and \( R_{ac}/R_c \) too. Then all parameters in the gun have been established. The choice of \( \theta \) is determined by the consideration that the beam still has to be convergent after passing the aperture of the Pierce gun.

Fig. 3.8. Pierce-gun geometry.
accelerating electrode. This aperture acts as a diverging lens with a focal distance

\[ f = \frac{4V}{E_2 - E_1}, \tag{3.12} \]

where \( E_1 \) and \( E_2 \) stand for the potential gradient outside and inside the aperture, respectively.

Since we want to keep \( V \) as low as possible, \( d \) has to be small. This again means a strong diverging lens action, and hence \( \theta \) must be large. Therefore we fix \( \theta \) at 30°. From the graphs given by Samuel it follows that

\[ \frac{R}{R_c} = 0.32. \tag{3.13} \]

We already know \( r_c = 5 \text{ mm} \), so we have

\[ R_c = 10 \text{ mm}; \ d = 3.2 \text{ mm}; \ R_{ac} = 6.8 \text{ mm}; \ 2r_{ac} = 6.8 \text{ mm}. \tag{3.14} \]

The maximum excavation of the cathode is \( h = 1.3 \text{ mm} \). From the data collected by Helm et al. it further follows that the beam just converges after passing the aperture in the accelerating electrode.

The above-mentioned results are only true to a certain degree because the electric field at the cathode surface is altered by the aperture. Moreover, the lens action decreases with increasing diameter of the aperture.

The dimensions are now roughly known. For determining the right shape of the electrodes we make use of the results of the trough measurements of Helm et al. However, because of their complicated character the electrode shapes given by them do not lend themselves to simple manufacture. Therefore the prescribed curvature is approximated by a more suitable shape differing from the original one as little as possible. It is a fortunate circumstance that these

---

Fig. 3.9. Focussing electrode; deflection 65°.
Fig. 3.10. Focussing electrode; deflection 52.5°.
Fig. 3.11. Accelerating electrode; round profile \((R_a = 6.8 \text{ mm})\).
Fig. 3.12. Accelerating electrode; straight profile.
deviations, provided they are not too large, have little influence on the performance of the gun. A number of experimental tubes have been made in which various approximations of the ideal profiles of focusing electrode (figs 3.9 and 3.10) and accelerating electrode (figs 3.11 and 3.12) are combined in order to find the most practicable solution.

The following remarks should be made in advance.

(1) In practice the diameter of the L cathode is 9.6 mm instead of 10 mm. The aperture of the focusing electrode is adapted to this reduction, while the other gun dimensions remain unchanged.

(2) To gain some insight into the influence of $R_c$ on the gun performance a cathode-surface radius greater than prescribed was tried out too, i.e. $R_c = 12.2$ mm corresponding to $h = 1$ mm.

(3) When assembling the gun the distance between upper level of the cathode and lower level of the focusing electrode is made 100 μ. With heated cathode this distance remains about the same.

(4) In order to enable the diameter of the electron beam to be measured in the experimental tubes the X-ray anode is replaced by a flat anode of molybdenum coated with a fluorescent powder and provided with a scale division.

We now have a number of possibilities, given by table 3-I. All the measurements were done with $i_a = 0.3$ A, the focusing electrode being earthed ($V_a = 0$). The results are given in table 3-II. In the last two columns the beam diameter on the anode is given at $V_a = 5$ kV and $V_a = 10$ kV, respectively.

From table 3-II can be seen that the combination $(B, \alpha, 1)$ comes out as the best one by far. Both experimental tubes provided with a cathode with $h = 1$ mm gave such poor results that the two other combinations were not made. The tube with the combination $(B, \beta, 1)$ dropped out owing to short-circuiting of the filament and was not assembled again.

In all further experiments the X-ray tubes were fitted with the selected gun, of which fig. 3.13 gives more details. The characteristics too are satisfactory as shown by the $(i_a, V_a)$ curves in fig. 3.14.

| TABLE 3-I |
|------------|-----------------|---|---|
| gun part   | shape           | code | fig. |
| cathode    | $h = 1$ mm $(R_c = 12.2$ mm) | A | 3.7 |
|            | $h = 1.3$ mm $(R_c = 9.6$ mm) | B | 3.7 |
| focusing electrode | deflection: 65° | $\alpha$ | 3.8 |
|            | deflection: 52-5° | $\beta$ | 3.9 |
| accelerating electrode | round profile $(R_a = 6.8$ mm) | 1 | 3.10 |
|            | straight profile (60°) | 2 | 3.11 |
TABLE 3-II

<table>
<thead>
<tr>
<th>combination</th>
<th>$V_{g2}$ (V)</th>
<th>$i_{g2}$ (mA)</th>
<th>beam diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(V_a = 5$ kV)</td>
<td>$(V_a = 10$ kV)</td>
<td></td>
</tr>
<tr>
<td>A, $\alpha$, 1</td>
<td>1300</td>
<td>64</td>
<td>18</td>
</tr>
<tr>
<td>A, $\beta$, 2</td>
<td>1500</td>
<td>108</td>
<td>19.5</td>
</tr>
<tr>
<td>B, $\alpha$, 1</td>
<td>1350</td>
<td>0.5</td>
<td>12</td>
</tr>
<tr>
<td>B, $\beta$, 2</td>
<td>1350</td>
<td>3</td>
<td>15</td>
</tr>
<tr>
<td>B, $\alpha$, 2</td>
<td>1350</td>
<td>14</td>
<td>14.5</td>
</tr>
<tr>
<td>B, $\beta$, 1</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Fig. 3.13. Dimensions of final electron-gun design.

Fig. 3.14. ($i_a$, $V_g$) characteristics of selected gun.

### 3.4. The X-ray tube

The complete construction of the gun is given in fig. 3.15. As will be mentioned in sec. 4.2, the gun as a whole is shielded by a surrounding cap to reduce stray capacitance. The structural material is chrome-iron. The cathode poles are of molybdenum, and are insulated from the cathode mass by ceramic sleeves.

As the anode is not required to meet any special demands we make use of an
available type from the industrial production of 250-kV therapy tubes. The essential dimensions of this anode are given in fig. 3.16. It is made of copper with a beryllium window to let the X-rays pass. Via a "Fernico" ring the anode is connected to the hard-glass tube envelope by a glass-to-metal seal. A photograph of the tube can be seen in fig. 3.17.

3.4.1. The focal spot

The focal spot of well-assembled tubes is fairly homogeneous. By its very nature it has an elliptical shape. The area of the true focal spot is about 140 mm\(^2\) in our case.

It may happen that the focal spot shows a "white" spot in its centre, which means that there are no electrons bombarding the anode in that area. This may have two causes:

(1) in tubes which contain a small amount of residual gases the centre of the cathode may be poisoned by ion bombardment;
(2) the field strength at the centre of the cathode surface is too low. This may occur if the cathode has been mounted too deeply. Increase of the potential of the accelerating electrode should give an improvement in that case.

The size of the focal spot depends among other factors on how deep the target plate is set into the anode cylinder. Having once arrived inside the anode cylinder the electron beam will diverge. As a result of the high electron speed
this effect is not so strong. Other possibilities of influencing the size of the focal spot exist to a limited extent only, because the dimensions of the gun and the distance of accelerating electrode to anode have been prescribed. A picture of the focal spot is given in fig. 3.18.
4. THE OTHER COMPONENTS OF THE APPARATUS

4.1. The electric circuitry

The considerations of chapter 2 sprang from the idea of an X-ray tube the anode current of which could be cut off by a suitable negative grid voltage. It will be obvious, however, that the anode current in the system as described in chapter 3 cannot be controlled effectively by variation of the voltage of the focusing electrode, owing to the fact that even rather large voltage differences have very little influence on the gun current.

The beam current can be controlled instead by variation of the accelerating-grid voltage. A block diagram of the circuitry is given in fig. 4.1. The anode of the X-ray tube and the high-tension coil are connected in line with the 20-kV generator. The accelerating electrode of the gun is connected with a 2-kV generator via a resistance $R_{a2}$. The purpose of this resistance is to reduce the accelerating-electrode voltage in such a way that this voltage is never larger than necessary for the anode current to be supplied at any instant. In this way the gun dissipation is considerably reduced. From the tube characteristics it can be concluded that a slightly negative value of the accelerating-electrode voltage entirely suffices to cut off the beam current of the gun. How the voltage variation of the accelerating electrode is effected, follows from the circuit diagram shown in fig. 4.2. A negative pulse originating from a pulse generator is amplified and the resulting positive pulse is applied to the control grid of a power tetrode. The whole system is at a negative potential level of 500 V. The tetrode is made conductive by the positive voltage pulse on the control grid. A large anode current then flows and the accelerating-electrode voltage of the X-ray tube is thus brought below zero.

The pulse generator has been designed in such a way that repetition frequency and pulse width can be controlled independently.
During the first exposure-rate measurements the X-ray output remained far below expectation. Upon closer examination the anode current during the discharge period was found to be much too low, as a result of the effect of the capacitance between anode and accelerating electrode.

This effect can be explained as follows (fig. 4.1). Let the capacitance between anode and accelerating grid be denoted by \( C_{ae} \) and the corresponding charge by \( q_{ae} \), then the circuit formed by \( C_{ae} \) and \( R_{y2} \) obeys the equation

\[
V_a = \frac{A_{ae}}{C_{ae}} + i_c R_{y2}.
\]

In our case

\[
i_c R_{y2} \ll V_a,
\]

which results in

\[
i_c = C_{ae} \frac{dV_a}{dt}.
\]

Substitution gives

\[
i_c = i_{ao} \frac{C}{C_{ae}} \cos \omega t \quad 0 < t < t_1,
\]

\[
i_c = i_{ao} \frac{C}{C_{ae}} \{ \cos \omega t - \cos (\omega t - \omega t_1) \} \quad t_1 < t < t_2.
\]

The voltage drop over \( R_{y2} \) is of the same shape as \( i_c \) and must be added to the desired accelerating-electrode voltage. The distortion current \( i_c \), the desired electrode voltage \( V_{y2} \), the distorted electrode voltage \( V_{od} \) and the resulting distorted anode current \( i_{ad} \) are given in fig. 4.3.

The anode-current drop can be diminished by reducing the value of \( R_{y2} \). This possibility has two serious drawbacks: overrating of the power tetrode and increase of the mean accelerating-electrode current. An obvious solution is the addition of a screening electrode between anode and accelerating electrode. Although such a measure means more complexity in gun construction, we
preferred this solution to a rather complicated electronic compensation device. In fig. 4.1 the screening grid has already been incorporated.

4.2. The induction coil

The primary requirements to be met by the coil are:

(a) a constant self-inductance of at least 50 H,
(b) low losses,
(c) electrical insulation up to 300 kV between the coil ends.

Though not essential in our experiments, small size and low weight are additional conditions for the realization of a practical X-ray apparatus.

The coil used is of the coreless cylindrical type, built up from a number of segments. Each segment consists of two flat coils wound in such a way that the wire connection between them is on the inside whereas the wire connection between two coils belonging to different segments is on the outside. In this way a regular potential distribution is secured. The segments are slipped on a bar of plexiglass which supports the coil.

Initially the coil body was made of plexiglass for the core and hard paper for the side plates. The dielectrical losses were so excessive, however, that the hard paper became carbonized. We therefore switched to “thermoplac” wire, which consists of a copper base covered with an inner layer of a ceramic material and an outer layer of plastic. With the help of a mould the two coils are wound on the base of a segment. After that the whole set is warmed up by conducting an electric current through the coil until the outer layers of the wire have melted.
together into one homogeneous mass. After cooling the mould is removed, leaving behind a self-supporting pair of coils. In applying this method one should see that the separate coils are not too thin; otherwise deformation might occur. Figure 4.4 gives the drawings of a segment base.

It is not altogether possible to predetermine the right value of the inductance. As shown in sec. 3.1, \( L \) (in H) should equal \( C \) (in pF) where \( L \) and \( C \) stand for the total inductance and the total capacitance, respectively. The capacitance of the X-ray tube is about 20 pF. The contribution of the coil depends on its construction. This difficulty was overcome by making an ample number of segments. By addition or removal of one or more segments the required value of the inductance can be obtained. The dimensions of each separate coil are:

- outer diameter : 13 cm,
- inner diameter : 4 cm,
- thickness : 6 mm,
- number of windings : 2000,
- resistance : 150 \( \Omega \).

A photograph of the complete coil is given in fig. 4.5.

4.3. The experimental set-up

In fig. 4.6 the arrangement is shown as far as the X-ray tube and the induction coil are concerned. The latter has been mounted vertically in a cylindrical oil-
filled tank of insulating material. The low-voltage output to the 20-kV generator can be seen just above the second ring from below. The X-ray tube has also been mounted vertically in a cylindrical vessel of hard glass. The anode side of the X-ray tube points downwards. In the picture, in order to improve visual penetration, the glass vessel has not yet been filled with oil. Its diameter is smaller than the coil-tank diameter, so that it is possible to lower the glass vessel partly into the coil tank. In this way a good view is obtained of the tube during operation, and at the same time excellent electrical insulation is obtained. Moreover, the cathode and grid connections can easily be made.

The circulation of the anode-cooling oil is effected by a standard pumping unit normally used for high-voltage therapy tubes. The connecting oil tubes are made of polyesterol.
The X-ray tube, the induction coil and the 20-kV generator have been placed together in a lead-walled room for high-voltage and radiation protection. Observation of the interior is possible through lead-glass windows. The X-rays can leave the room through a small aperture.
5. MEASUREMENTS AND CONCLUSIONS

The evaluation of the X-ray-tube voltage together with the spectral-energy distribution and the exposure rate of the emitted X-radiation are the main subjects of this chapter.

5.1. Measurement of high-voltage and spectral-energy distribution

5.1.1. Introduction

It will be clear that the measurement of the peak voltage of the X-ray tube by one of the well-known direct methods, i.e. spark gap and high-ohmic resistor, was impossible in our case for the simple reason that any system of measurement in direct contact with the anode circuit would alter the value of \( C \). We were therefore compelled to look for an acceptable indirect method. The obvious method was to make use of the Duane-Hunt wavelength limit of the continuous X-ray spectrum emitted by the anode with the aid of a crystal spectrometer. The principle of the method is shown in fig. 5.1. From the X-ray beam that leaves the anode a small rectangular beam is separated by collimator slits. A part of this narrow beam is reflected by a crystal. By rotating the crystal a relation is obtained between the intensity in a distinct direction and the corresponding wavelength. For a given set of lattice planes a discrete wavelength corresponds to each angle of reflection. The part of the primary beam that is not reflected passes through the crystal and is intercepted in a lead cone.

![Fig. 5.1. Principle of X-ray detection.](image)

First of all we wish to obtain an impression of the magnitude of the angles of reflection. The wavelengths corresponding to a peak tension of 300 and 100 kV, respectively, are

\[
\lambda_1 = 0.0413 \text{ Å}; \quad \lambda_2 = 0.124 \text{ Å}. \tag{5.1}
\]
From Bragg's relation,

\[ n\lambda = 2d \sin \varphi, \]

\( \varphi \) can be found if \( n \) and \( d \) are known. With the quartz crystal used \((d = 3.343 \text{ Å}; n = 1)\) we find

\[ 2 \varphi_1 = 42'28''; \quad 2 \varphi_2 = 2'7'30''. \] (5.2)

Thus for 200 kV difference in voltage a difference of about 1°25'' in angle reflection can be expected. This situation makes it necessary to employ a very narrow beam and a relatively long distance from reflecting crystal to detector.

There are various possibilities of detection:

1. By photographic means. Although this method can give exact results, it is not acceptable because of the long exposure time.
2. A G.M. counter might in principle be used. It is true that the resolving power is restricted, but arrangements might be made to record the repetition frequency of the X-ray pulses. In that case the indication remains constant while the continuous X-ray spectrum is being scanned, and drops to zero in the vicinity of the wavelength limit. The sensitivity of this method will depend especially on the efficiency of the G.M. counter for 250-kV quanta and the number of these quanta still reaching the G.M. tube during each discharge.
3. Scintillation crystal with photomultiplier tube. The high resolving power opens in principle a number of possibilities of recording. In addition intensity measurements can be made by using a suitable crystal. Thus we can find:
   a. the peak tension of the X-ray tube by determining the angle at which the intensity falls off to zero, and
   b. the spectral-energy distribution by scanning the diffraction pattern with the scintillation counter.

For the last reason in particular we preferred the scintillation counter. In the next section a description of the spectrometer will be given.

5.1.2. Description of the spectrometer

The basic of the instrument is a heavy iron bar (fig. 5.2). The collimators are situated at one end of the bar. The reflection crystal is held on an adjustable support. This crystal holder is coupled mechanically to the scintillation counter to ensure synchronous rotation of the latter with an angular velocity twice that of the crystal. The rotation is accomplished by a small synchronous motor. The unreflected part of the primary beam is intercepted by a small lead case provided with a removable stop.

Of the various components of the spectrometer only the container of both the scintillation crystal and the photomultiplier tube is given in detail in fig. 5.3.
The container is made of brass with an inner sheet of 0.5 cm lead for the absorption of stray radiation. The removable front part of the box is provided with a lead diaphragm with an opening of $10 \times 1$ mm$^2$, which is covered with black paper in order to make the box impervious to light. A felt ring between container and cover serves the same purpose. Attached to the container is a smaller box in which a part of the electronic circuit is accommodated.

The scintillation crystal is enclosed in a plexiglass case. A layer of MgO be-

![Diagram of spectrometer and scintillation counter]

Fig. 5.2. Outline of spectrometer.

Fig. 5.3. Longitudinal section of scintillation counter.
between the crystal and the side walls facilitates good reflectivity. On the side of the multiplier tube a silicon oil is used with the same index of refraction as plexiglass.

As mentioned before, quartz was chosen for monochromator, the (101) plane acting as reflecting plane \( (d = 3.343 \text{ Å}) \).

The choice of the scintillation crystal was determined by our wish to make energy measurements. For that reason an NaI(Tl) crystal was used with dimensions \( 2 \times 1 \times 1 \text{ cm}^3 \). Brucker gives the values of \( \mu/\rho \) at quantum energies up to \( 200 \text{ keV} \). By extrapolation we find at \( 250 \text{ keV} \): \( \mu/\rho = 0.125 \). As \( \rho = 3.667 \) the energy adsorption of \( 250 \text{-keV quanta} \) amounts to \( 60\% \) for \( 2 \text{ cm crystal length} \).

The photomultiplier employed is an E.M.I. tube type no. 5659. It has its photocathode on the inside of the glass wall at the top of the tube. The spectral sensitivity attains a maximum at \( 4000 \text{ Å} \) which conforms well with the emission line at \( 4100 \text{ Å} \) of NaI.

5.1.3. Principles of measurement

During each period of discharge of the X-ray tube a complete spectrum is emitted. The position of the scintillation counter determines the energy of the detected X-ray photons that are selected from this spectrum. The pulses corresponding to high-energy photons are only found at the beginning of a discharge. The longer the wavelength the greater the part of the period in which this wavelength is emitted and its pulses recorded. This is illustrated in fig. 5.4, which gives a random distribution of pulses at a fixed wavelength. The isolated pulses are due to natural background radiation.

\[ \text{Fig. 5.4. Random distribution of pulses at a fixed wavelength.} \]

There are various methods of recording the pulses. Firstly, we may count the number of X-ray photons per wavelength by counting the separate pulses. This method, however, makes very severe demands on the decay time of the scintillation crystal and the resolving power of the electronic circuit.

Instead of measuring the separate photons we can also measure the total amount of energy for a discrete wavelength by integration of the pulses over a certain period \( ^{22} \). In our first solution we made the time constant about \( 100 \mu \text{s} \) so that the pulses were integrated during a single discharge. The linear amplified output was applied to a capacitor which discharged through a mA meter (fig. 5.5).
At a well-defined repetition rate of the discharges the indication of the mA meter was proportional to the total photon energy at a fixed wavelength. This method, however, is restricted to pulsed X-rays, and there is the additional requirement that the dead time should be long in comparison with the pulse time.

To avoid this disadvantage we adapted another solution in which the pulses are integrated over a much longer period, i.e. a time constant of 1 second. The resulting signal is passed to a recorder. As the output impedance of the photomultiplier tube is very high and the input impedance of the recorder rather low, an extra step has to be inserted comprising an electrometer tube. The circuit diagram is given in fig. 5.6, and fig. 5.7 gives a picture of the spectrometer and the recording apparatus.

5.1.4. Measurement of high voltage

An example of a spectrum is given by curve C in fig. 5.8. It was derived from the originally recorded curve A from which the stray-radiation curve B was subtracted. This background-intensity curve was obtained by a point-to-point measurement with the quartz crystal removed.
Since the wavelengths of the K\(\alpha\) and K\(\beta\) lines are known, the limit wavelength \(\lambda_0\) can be directly determined and from the limit wavelength the high voltage can be easily deduced. The accuracy of the limit-wavelength evaluation depends on the resolving power, which again is influenced by the horizontal divergence of the X-ray beam and by the fluctuations in number of the X-ray quanta. For obvious reasons we cannot decrease both factors simultaneously. A compromise has been found at a slit width of 0.1 mm.

The horizontal divergence is approximately given by

\[
2\alpha = 0.001
\]

or, with Bragg's relation,

\[
\Delta \alpha = 2 d \alpha = 0.006 \text{ Å}.
\]

In consequence the inaccuracy amounts to 0.006 Å. This is the distance over which the signal fades out to zero. In fact the error is much smaller, since there is no reason for the curve to turn off at the end. Including the influence of the fluctuations the total deviation will therefore not exceed 0.002 Å, which corresponds to an error of 4% at 250 kV.
5.1.5. Measurement of the spectral-energy distribution

At small angles the resolving power of the spectrometer amounts to 0.003 Å, corresponding to a linear displacement of the detector over a distance of 0.36 mm. Since the scanning speed is 1 mm/min the required time is about 22 seconds. The time constant of 1 s has therefore no influence on the resolving power. The same can be said of the speed of the recorder pen, which takes 1 s for full-scale deflection.

In order to compare the X-radiation quality of our generator with that of a constant-voltage apparatus we measured the spectral-energy distribution of the X-radiation for both voltage forms at 250 kV. The results are given in fig. 5.9. Due to the fact that the measuring range of the spectrometer is limited to about 0.25 Å we had to normalize the curves by reducing them to equal maximum intensity of the continuous spectrum. The curves quite clearly show the much higher amount of hard radiation in the case of constant voltage.

We should point out that the above-mentioned spectral-energy distribution is by no means identical with the spectrum of the X-radiation emitted by the
anode. The original spectrum has been transformed by a number of absorption and transmutation processes. Our energy curves are quite useful, however, for a qualitative comparison.

5.2. Conclusions

Now that the experiments described in the preceding chapters have demonstrated the practicability of the principle outlined in sec. 1.3, i.e. the possibility of designing a high-current grid-controlled X-ray tube comprising an L cathode, the question remains as to its usefulness.

We expected beforehand that our system would be an inferior X-ray source compared with most common types of X-ray generators. This has already been shown qualitatively in sec. 5.1.5. It is demonstrated again quantitatively by an exposure-rate measurement the results of which are given in fig. 5.10. The conditions of measurement were:

\[ V_p = 240 \text{ kV}, \]
\[ i_m = 0.14 \text{ A}, \]
\[ V_{ao} = 20 \text{ kV}, \]
\[ f = 1300 \text{ c/s}, \]
\[ \tau = 39 \mu s. \]

From these figures it can be deduced that the effective current during the discharge period amounts to 14.2 mA while the total anode dissipation is 2.8 kW. The inherent tube filter consists of 10 mm hard glass, 20 mm transformer oil, and 1 mm Be, giving a Cu equivalence of 0.9 mm Cu for absorption. From fig. 5.10 an exposure rate of 32 R/min at 50 cm or 11.4 R/min kW can be deduced. These data should be compared with those for a d.c. generator which
are approximately 70 R/min at 14 mA or 21 R/min kW at 50 cm, using the
same tube filter. For a generator that has a potential pulsating between 0
and 240 kV these figures are approximately 40 R/min or 16 R/min kW, respec-
tively. The low output of the described system has to be attributed to the un-
favourable shape of the high-voltage pulse. If it would be possible to improve
this shape, the output of the system could be considerably increased.

The application of the described system would, due to its compactness, be
particularly attractive where small dimensions and the absence of high-voltage
cables are simultaneously wanted or where a pulsating X-radiation is desired.
REFERENCES

1) W. C. Röntgen, Ann. Phys. 1, 64, 1898.
2) E. O. Lawrence and N. E. Edlefsen, Science 72, 376, 1930.
3) R. Wideröe, Arch. f. Electrot. 21, 387, 1928.
4) D. W. Kerst, Physiol. Rev. 60, 47, 1941.