An international conference held in London in 1939 recommended that such bodies as the “Comité Consultatif International Téléphonique”, the “International Electrotechnical Commission” and the “Union Internationale de Radiodiffusion”, as also the various national standardisation committees, should fix the concert pitch at a frequency of 440 c/s for the note A₃ of the middle octave — instead of the 435 c/s internationally accepted at Vienna in 1885 — and that this should be adhered to as closely as possible in all musical performances¹).

In practice the usual tuning forks are, it is true, tuned higher than 435 c/s but often differ, in the positive or the negative sense, several c/s from the figure of 440 c/s. Since there was no standard for the frequency of 440 c/s in this laboratory a method has been developed, as the first step in this direction, for measuring audio frequencies direct (i.e. by counting the number of oscillations in a time interval) and with extreme accuracy. With the aid of an apparatus working according to this method not only have tuning forks of 440 c/s been calibrated but also sets of 13 tuning forks, forming a tempered chromatic scale of one octave (fig. 1).

Fig. 1. Set of 13 tuning forks made in the Philips Laboratory. Together they form a tempered chromatic scale from c¹ to e², with a³ = 440 c/s. The reason for the cylindrical shape is explained in the last paragraph.

¹) See, e.g., Internationale Neufestsetzung des Stimmtones, Akustische Zeitschrift 4, 288, 1939.
Accuracy required of a tuning fork

The useful decay time of a normal tuning fork is 5 to 15 seconds. When the pitch of a musical instrument is compared with the tuning fork in the normal way one can usually hear beats. If, however, the difference in frequency is so small that there are no more than two or three beats during the decay time then they are hardly noticeable as such and one has reached the limit of the accuracy of the tuning fork. It serves no purpose, therefore, to require any greater precision than 0.2 c/s (one beat in 5 seconds), or in other words the frequency of the tuning fork has to lie between the limits of 439.8 and 440.2 c/s. (This degree of accuracy is obviously only required for musical instruments with a fixed tuning, such as the organ, the piano, the harp, etc.) The frequency, however, has to be determined with greater accuracy, because one must be quite sure of the last decimal in the said limits. The permissible measuring error is therefore 0.05 c/s, which at a frequency of 440 c/s amounts to a measuring accuracy of about 0.01%.

This implies that account must be taken of the temperature when measuring and when using the tuning fork in practice. The temperature coefficient of the frequency in the case of a steel tuning fork is in the order of $-1 \times 10^{-4}$ per °C, while for the forks made of a hard aluminium alloy illustrated in fig. 1 it is about $-2.5 \times 10^{-4}$ per °C.

Method of measuring

As is evident from the foregoing, for calibrating a tuning fork a more accurate method is needed than that where the tuning fork is compared with a standard frequency by ear. Furthermore, with frequency differences less than 0.1% the sign of the error can no longer be determined by ear.

The method of calibrating worked out by us briefly amounts to the following. The frequency of an auxiliary oscillator is matched with that of the tuning fork to be calibrated and then a synchronous clock is connected to the oscillator and the oscillations made by the auxiliary oscillator in a certain interval of time are counted. This time interval is measured with the aid of a calibrated pendulum timepiece.

Matching the auxiliary oscillator

Owing to the damped character of the vibration a tuning fork is not suitable for measuring the frequency with sufficient accuracy without some other aids. It is true that a tuning fork can be kept in vibration by electrical means, but then, unless a great deal of care is taken, the frequency at which it vibrates differs somewhat from the frequency of a fork vibrating freely, and it is just this free vibration that has to be measured accurately. What is needed for calibrating, therefore, is an auxiliary oscillator which, once it has been matched with the freely vibrating fork, continues to oscillate in that frequency for an unlimited length of time.

An RC oscillator consisting only of resistors, capacitors and amplifying valves is very suitable for this purpose. If such an oscillator is properly designed, when it has reached temperature equilibrium the relative frequency change in the audio-frequency range can be kept smaller than $10^{-5}$. (This degree of accuracy is achieved when the oscillator is cooled in a Dewar flask.)

The matching of the frequency of the oscillator with that of the tuning fork can be done with sufficient accuracy if a cathode-ray oscilloscope is employed, by applying the signal from the RC oscillator to one pair of plates and a signal with the frequency of the tuning fork to the other pair. The latter signal can be obtained by setting up the tuning fork in front of a microphone. There then appears on the screen of the oscilloscope a changing Lissajous figure which, when the frequencies are equal, becomes a stationary ellipse.

There is, however, a drawback to this method. The Lissajous figure cannot demonstrate whether the frequency deviation is positive or negative, hence the operator does not know in which direction to turn the oscillator tuning knob. If this is turned in the wrong direction the consequent time loss results in the tuning fork signal having decayed too far, and the fork has to be struck again.

To obviate this, the following method is adopted. The signal from an RC oscillator is applied direct to one pair of input terminals of the oscilloscope and also, after having been shifted $90\degree$ in phase in a network of resistors and capacitors, to the other pair of terminals. Given the right proportions of amplitude of the two signals, a circle is then produced on the screen of the oscilloscope.

The tuning fork to be calibrated is set up front in a microphone, the amplified signal from which

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2) Other authors arrive at the same tolerance another way: H. J. von Braunmühl and O. Schubert, Ein neuer elektrischer Stimmtongeber für 440 Hz, Akustische Zeitschrift 6, 299-308, 1941.


4) For this and the other method to be described later it is an advantage to use an oscilloscope with two amplifiers, one for each pair of plates, such as type GM 3159 or GM 5655 (Philips Techn. Rev. 9, 202-210, 1947, and 11, 111-115, 1949 (No. 4)).
is used for modulating the beam current of the cathode-ray tube, such that the beam current is allowed to pass only during the positive half cycles of the microphone signal. Thus only half a circle is seen on the screen (fig. 2). When the oscillator frequency is exactly equal to the frequency of the tuning fork this half circle is stationary, but when there is a difference in frequency it rotates. The best method is to arrange the apparatus so that the tuning knob of the oscillator has to be turned clockwise to increase the frequency and that the half circle rotates clockwise if the oscillator frequency is too high and anti-clockwise if it is too low. Matching is then done very quickly, the vernier control of the oscillator being turned in the direction opposite to that in which the half circle is moving, until the latter is stationary. (A half circle has been chosen here for the sake of simplicity, but this is not essential for matching the frequencies. One can also work with the half of any ellipse, since the phase shift in the network need not be precisely 90°.)

If a voltage source with the standard frequency is available (e.g. an electrically driven tuning fork, or the 440 c/s signal as regularly transmitted by the B.B.C. for tuning purposes), then the tuning-fork frequency that is to be determined can be compared directly with the standard frequency by means of the set-up shown in fig. 2, replacing the RC oscillator by the standard frequency signal. When this method is followed a rotating half-circle is again seen, and from the number of revolutions it makes in a certain time it is easy to derive the difference in frequency (the sign is indicated by the direction of rotation). Thus one measures, as it were, a fraction of a beat. The accuracy is proportional to the duration of the observation, being already very great at the duration of the decay time. If, for instance, the half circle makes \( \frac{1}{2} \) revolution in 12.5 seconds then the frequency difference is 0.02 c/s.

**Measuring the oscillator frequency with the aid of a clock**

A synchronous clock with a seconds hand is then connected to the RC oscillator matched to the tuning fork. In our case a clock was used running at the right speed when connected to a signal of 1000 c/s. If it is fed with a frequency \( f \) during a period of time \( T_1 \) and in that time its reading changes by an amount \( T_2 \) then

\[
f = \frac{T_2}{T_1} \times 1000 \text{ Hz.}
\]

\( T_1 \) is measured with a good pendulum timepiece checked before and after the measurement with radio time signals, which thus form the basis of the calibration. Since the seconds hand of this clock moves forward 1 second at a time, and in fact the two clock readings cannot be taken simultaneously with sufficient accuracy, a stop-watch is used, this being started when the seconds hand of the pendulum timepiece arrives at a certain dial mark, say 60. From that moment onwards the watch forms, as it were, an extension piece of the pendulum clock could be made smaller than one second per

**Fig. 2. Circuit for matching the frequency of an RC oscillator (RC) to that of the tuning fork to be calibrated (S).** The signal from the oscillator is applied direct to the input terminals \( I \) of the cathode-ray oscilloscope \( O \) and, via a 90° phase-shifting network of resistors and capacitors \( (R_1, C_1, R_2, C_2) \), to the input terminals \( II \). The beam current of the cathode-ray tube is modulated by the output from an amplifier \( (A) \) to which is connected a microphone \( (M) \) picking up the sound from the tuning fork.

As indicated in the diagram, \( R_1 \) may be the resistor already contained in the oscilloscope between the terminals \( II \).

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\( ^8 \) This method has been used before, for measuring the pitch of musical instruments during a musical performance (Balth. van der Pol and C. J. Addink, Philips Techn. Rev. 4, 205-210, 1939, and Wireless World 44, 441-442, 1939). The oscillator — in that case a calibrated string with electrically sustained vibration and variable through a small frequency range — could thereby, within one second, be made equal in frequency to the note \( a^2 \) in the music to within 0.2 c/s. For calibrating a tuning fork, however, a string variable in frequency is not accurate enough.
24 hours, i.e. less than $1/10^4$, so that no correction need be made for this. Neither is any correction necessary for the movement of the stop-watch, since for the purpose of this test it runs for such a short time that any error can be ignored.

The reading errors at the beginning and at the end of the time $T_1$ being additive, when a stop-watch is used that can be read accurately to within 0.2 s the duration of $T_1$ is known accurately to within 0.4 s. Since, as deduced above, the measurement with a tuning fork of about 440 c/s has to be accurate within about 0.01%, the duration of $T_1$ has to be at least $0.4/10^4 = 4000$ s, thus about one hour. (With a stop-watch recording time to within 0.1 s the duration of $T_1$ need be only half an hour.)

A picture of the complete calibrating apparatus is given in fig. 3.

Some readings taken during a practical calibration are given below.

The watch was started when the pendulum clock showed 2 hrs 37' 0" and stopped when the seconds hand of the synchronous clock passed a given mark of the dial, viz. when this clock showed 3 hrs 55' 35". The stop-watch then showed 7.2". Thus at the moment that the synchronous clock showed 3 hrs 55' 35" (the beginning of $T_2$) the time shown by the pendulum clock was 2 hrs 37' 2" (the beginning of $T_1$). After about one hour, at 3 hrs 31' 0" on the pendulum clock, the stop-watch was started again and then stopped when the synchronous clock showed 4 hrs 19' 20" (end of $T_2$). The stop-watch then showed 7.8" and was thus stopped at the time 3 hrs 31' 7.8" on the pendulum clock (end of $T_1$).

Thus we find: $T_1 = 3$ hrs 31' 7.8" - 2 hrs 37' 2" = 3240.6", and $T_2 = 4$ hrs 19' 20" - 3 hrs 55' 35" = 1425". Thus $f = (1425/3240.6) \times 1000$ c/s = 339.73 c/s.

To minimize any inaccuracy due to errors in reading, the start and the finish of $T_1$ and $T_2$ are repeated twice; of the three readings thus obtained the identical ones are usually correct.

**Correction of the tuning fork**

If the tuning fork tested is found to deviate from the desired frequency then one will want to correct it. If the frequency is too low this can be done easily; the exact frequency can be approached very closely by removing a little material from the end of the prongs of the fork. This has to be done in such a way however, that no difference in the lengths of the prongs is introduced, for the slightest difference considerably increases the damping. For this reason, bifurcated rotationally cylindrical tuning forks were adopted (fig. 1). The prongs can be shortened as required by stopping the saw cut temporarily with, for example, a strip of

Fig. 3. Complete apparatus for calibrating tuning forks. S the tuning fork to be calibrated, M microphone, $A_1$ and $A_2$ amplifiers together forming the amplifier $A$ of fig. 2, O oscilloscope, RC auxiliary oscillator, P pendulum timepiece, F synchronous clock, H stop-watch.
of brass, and then shaving the end face in a lathe.

Once the frequency $f$ of the tuning fork has been measured, then using the formula giving the frequency as a function of the dimensions and material constants, an accurate calculation of the requisite modification can be made. The formula is $^6$):

$$f = \frac{K^2v}{(l + l_0)^2},$$

where $K^2$ is a factor dependent upon the shape of the tuning fork, $v$ is the velocity of sound in the material of the fork, $l$ the length of the prongs and $l_0$ a constant, which in the present case is small compared with $l$ (a few %). The length of the prongs is taken to be the distance from the vibration node to the tip; with the form described above the vibration node lies on one level with the centre of the bore at the bottom of the saw cut.

After the prongs of a fork having too low a frequency have been shortened by the amount calculated according to this formula, the remaining frequency deviation is usually less than the tolerance allowed ($\pm 0.2$ c/s).

If the tuning fork has a frequency slightly too high it is still possible to correct it, by making the prongs a little thinner at the level of the vibration node. It can hardly be calculated in advance, however, how much material has to be removed, and furthermore the appearance of the tuning fork is thereby spoiled. For this reason the uncorrected tuning forks are always given a frequency slightly too low.

This type of fork is made in a lathe and a milling machine. A rod is turned to produce the required diameters for the prongs and the stem, and symmetry is assured. A hole is now drilled at a right angle through the axis of the rod. A slot, in line with the drilled hole, is cut along the axis of the rod on a milling machine. It is obvious that this is a much simpler process than that of making the usual rectangular cross section fork.

So as to have a light and stainless product a hard aluminium alloy was chosen instead of steel, although it has a somewhat greater absolute temperature coefficient.

A disadvantage due to the shape and the material chosen is that the fork has a weaker tone and a shorter vibration time than the usual fork when placed on a sounding board. However, this fork gives better results when held close to the ear, especially at low frequencies, owing to the relatively larger radiating surface of the prongs.

Summary. A method is described by means of which tuning forks can be calibrated without requiring an acoustic standard frequency. With the aid of a cathode-ray oscilloscope the frequency of an RC oscillator is matched to that of the tuning fork to be calibrated. Connected to this oscillator is a synchronous clock, the seconds hand of which would make one revolution per minute if the frequency were 1000 c/s. If, after an interval $T_1$, this clock shows a difference in reading $T_2$ then the frequency sought is $f = (T_2/T_1) \cdot 1000$ c/s. The duration of the interval $T_1$ is measured with the aid of a pendulum timepiece the movement of which is checked before and after the measurement with radio time signals.

If the frequency found is lower than that required then the latter can be very closely approximated by shortening slightly the prongs of the tuning fork. A special, cylindrically shaped tuning fork is described, with which this correction can be made easily on a lathe, thus retaining the required symmetry.