An opto-acoustic cross-correlator in radar signal detection

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Introduction

The problem of obtaining a high distance resolution at the same time as a long range is familiar to all concerned with pulse radar systems design.

A long range requires a long period between transmitted signals and, in order to maintain an adequate signal-to-noise ratio, a high transmitted mean power; a high resolution needs a brief signal after reception. In short range, high resolution radars, this brief received signal is occasioned by a similarly brief transmission.

The peak transmitted power usually has some finite limit, due for example, to dielectric voltage breakdown in the aerial system and this limitation restricts the minimum ratio of the transmitted pulse length to the period between transmitted pulses for a given mean power. Thus in a simple pulse radar, the minimum pulse length, and hence the maximum resolution for a given range, is limited by the peak power capabilities of the system.

Pulse compression systems \[1\] are one way of avoiding this limitation. The emitted signal occupies a large bandwidth and is transmitted for a long time, for example as a swept frequency. This signal after reception is time compressed into a pulse of short duration \[\Delta T = 1/\Delta F\], where \(\Delta F\) is the signal bandwidth. Such systems require an accurate specification of the transmitted pulse and of the pulse compression network.

Correlation detection is an alternative method of obtaining this pulse compression which does not depend upon the exact nature of the transmitted signal, but merely upon the “goodness of fit” between a delayed replica of the transmitted signal and the returned echo from the target.

This article discusses correlation detection of signals in radar, shows the reason for the choice of an opto-acoustic system to perform the correlation, and discusses a novel, two delay line correlator in which one signal is time reversed. The opto-acoustic design considerations for a suitable system are briefly considered, the practical design discussed and its potential performance described.

Correlation detection

The cross-correlation function \(C(T_r)\) \[2\] is a measure of the degree of similarity of two functions of time, \(f_1(t)\) and \(f_2(t)\), and is the “long term average” of the product of all the coincident pairs of samples of the two functions one of which is delayed by the time \(T_r\) (\(T_r\) is in this case the time interval between the transmission of the radar signal and its reception) (fig. 1).

This may be written for an aperiodic function as:

\[
C(T_r) = \int_{-\infty}^{+\infty} f_1(t) f_2(t + T_r) \, dt.
\]

In the case of radar in which a pulse of finite duration \(T\) is transmitted, the signal may be taken as zero outside the time \(T\) and hence the integral outside of the pulse is also zero so that we may then write the integral:

\[
C(T_r) = \frac{1}{2T} \int_{-T}^{+T} f_1(t) f_2(t + T_r) \, dt. \quad \ldots \ldots \quad (1)
\]

In the proposed system the two waveforms have the same source, i.e. the transmitted pulse, but the received echo from the target will be diluted with noise and it would probably be more correct to replace \(f_2(t)\) with \(f_1(t - T_r) + f_3(t)\) where \(f_3(t)\) is the unwanted noise. Thus the integral becomes:

\[
C(T_r) = \frac{1}{2T} \int_{-T}^{+T} f_1(t) [f_1(t) + f_3(t + T_r)] \, dt.
\]


This process is a powerful means of recognizing a signal submerged in noise when one has a knowledge of the exact nature of the signal. It is thus of use in radar and sonar where these conditions frequently exist, and has noise suppression properties similar to those of the matched filter.

One simple application of this type of detection to a radar signal is shown in Fig. 2. The output from the transmitter is fed to the aerial and to a variable delay line of delay $T_d$. An echo received from the target after a time $T_r$ is correlated in the multiplier with this delayed signal, and the integration is achieved by the low pass filter. A maximum correlation occurs when $T_r = T_d$.

Fig. 3 illustrates a multi-channel correlation detector in which the signal delay is achieved by a tapped delay line, the number of taps corresponding to the number of discrete range elements required and the total delay to the maximum range, a signal at a given range resulting in an output from a particular element. In a typical radar with a range of 150 km and a desired resolution of 150 m the delay line will require 1000 taps and a total delay of 1 ms with a bandwidth of at least 1 MHz. This performance cannot at the moment be obtained in a single unit but it is conceivable that in the future an integrated circuit approach may prove practical.

![Fig. 2. Variable delay correlator for radar signal detection.](image)

The diagram shows the input $f_i(t)$ from the transmitter being fed to both the radar transmitting aerial $A_1$ and variable time delay $T_d$. The signal $f_i(t)$ from the target $O$ is fed from the receiving aerial $A_2$, after amplification, to a multiplier $K$, the other input to which is the output from the variable delay $T_d$. The multiplier output is fed through a low pass filter. The multiplier and low pass filter form the correlator. The whole range is scanned by varying the delay time.

![Fig. 3. Tapped delay correlator.](image)

This figure shows a multiple tapped delay, the output from each tap feeding a correlator $C$.

**Single-delay line correlator**

Fig. 4 shows one method of achieving this type of performance without the use of a multi-tapped line and individual multipliers. In this system a photo-elastic delay line replaces the tapped delay line.

In a photo-elastic delay line, incident light is modulated by the stress in the delay medium so that at any instant the complete stress distribution in the line is visible. (The physical mechanism used in this process will be described later but for the present we will assume that this is a linear process.) The photo-elastic delay line forms an infinitely tapped delay line in which the distance along the bar represents the time interval the stress wave has taken to reach a given point.

In the opto-acoustic cross-correlator the received signal modulates a light source which illuminates the photo-elastic delay line with the function $f_i(t)$. The light intensity is modulated a second time by the function $f(t)$ by the photo-elastic delay line, the time delay $T_d$ corresponding to a distance along the delay line. The multiplication process is this double modulation of the light and the integration is achieved by viewing over a period. The presence of a target at a given delay range will be indicated by a high mean light intensity at the point in the bar corresponding to that delay.

The distribution along the bar of the total quantity of light transmitted in any one correlation period is the function $C(s)$ in which the variable $s$ of position along the bar is equivalent to $T_d$ in equation (1). In the ultrasonic case when both signals $f_i(t)$ and $f(t)$ are modulated on a carrier of frequency $f_0$, the light intensity fluctuates at this frequency.
The spatial resolution (the reciprocal of the light spot length along the bar) is proportional to the signal bandwidth, and the time over which this correlation from a single target is formed is the time duration of the transmitted signal.

This is inconvenient for system application since a large number of integrating photo-detectors distributed along the bar would be required to identify the range of a target. If a "one lead" output were required, with the time of a signal corresponding to the range of a target, some sampling mechanism would be required to turn this parallel access into a time sequential system.

This conversion from parallel-to-sequential may be overcome by using electronic integration achieved by imaging the photo-elastic delay line on a storage tube and reading the stored signals by the scanning electron beam. This overcomes the photo-detector array problem.

In practice both of the signals modulating the light would be present on a carrier frequency and it would be necessary on the storage tube to resolve the equivalent spatial frequency corresponding to the acoustic wavelength of this carrier frequency. This may be several times the necessary resolution determined strictly from a bandwidth criterion and it is again not an ideal system, suffering in particular from storage tube non-linearity and high noise levels.

Two delay line correlator

The method under consideration at Mullard Research Laboratories [6] involves the use of two photo-elastic delay lines and is shown in fig. 5.

The transmitted signal is launched in one delay line which modulates the light with the function \( f_1(t - T_1) \). The received signal launched in the second line in the reverse direction modulates the light with \( f_2(t - T_2) \) and the product of these two light modulations is again a correlation, spatially distributed along the delay line, with the difference that the fluctuations in light intensity occur at a carrier frequency of \( 2f_0 \). The figure shows the modulation as a swept frequency, and it will be seen that only at one point on the delay lines do identical parts of the waveform correlate and that the correlation exists for the total signal duration: at all other points on the line the correlation will be small.

However, consider the case of one signal in the bar reversed in time, fig. 6, which again shows two swept frequency signals propagating in opposite directions correlate as they pass the point \( b \). The correlation exists over the whole length of the pulse.
The situation is the reverse of the one previously described in that we have interchanged the “time” domain for the “space” domain. The integration now occurs as the result of summing the transmitted light at all points along the bar and the correlation coefficient now is a direct function of time, \( C(T) \). The correlation exists for a time which is short compared with the pulse length, approximately the reciprocal of the bandwidth of the signals being correlated.

This process has achieved directly the desired “one lead” output. The correlation is now detected by imaging the whole of the light passing through the bar on a photo-detector and the correlated signal will be present on a carrier frequency which is twice the carrier frequency used to modulate the light.

This system is a type of pulse compression system and may be used in any conventional pulse compression application. It furthermore has the advantage that, assuming the time reversal of one of the signals can be achieved, the exact detail of the transmitted pulse is of no importance (on a pulse compression system using linear FM a high degree of sweep linearity is required) either within the pulse or from pulse to pulse, since a given radar return is correlated only with the transmitted signal which occasioned it.

The photo-elastic process

When an isotropic transparent medium is stressed it becomes birefringent \(^{[7]}\) and the effective refraction index becomes dependent upon the direction of the stress and the polarization of the light. In the case of an extension the strains and hence refraction index changes lie along and normal to the direction of the originating stress. Hence a light ray polarized along one of these vectors will be advanced in phase relative to a light ray polarized along the other.

A shear strain may be resolved into an extension and a compression along axes at 45° to the direction of shear, a similar situation with the maximum and minimum phase differences occurring along these extension and compression axes (see fig. 7).

In acoustic delay lines the stress wave may be propagated in the medium in both the longitudinal (extension) and the shear mode of vibration. The medium must be transparent and of a low acoustic loss with preferably a high stress optical coefficient and a high refractive index. The material most suitable is undoubtedly fused quartz although both glass and water are possible under some circumstances. The preferred mode of propagation is shear; this is due to the lower shear velocity which increases the delay obtained in a given length by about 50% and also to the freedom of the shear mode of propagation from mode conversion at the delay line boundaries \(^{[8]}\) which would cause spurious signals.

Fig. 8 shows the birefringent process. Plane polarized light obtained by passing the incident light through a polarizer may be resolved into two equal orthogonal components in phase. Transmission of these through the birefringent material produces a phase difference so that the light is now elliptically polarized. Analysing this light by an analyser at right angles to the polarizer gives rise to a single component whose intensity is:

\[
I = \frac{1}{2} I_0(1 - \cos \Theta) = I_0 \sin^2 \frac{\Theta}{2}, \quad \ldots \quad (2)
\]

where \( \Theta \) is proportional to the length of the opto-acoustic interaction and to the stress \( \sigma \), which in the ultrasonic delay line is in turn proportional to the transducer drive voltage \( E \). This characteristic is shown.
in fig. 9. When the polarizer and analyser axes are orthogonal and the birefringent material is unstressed, no light is transmitted. Small stresses have a roughly square law stress/transmitted intensity characteristic so that a sinusoidal applied stress is frequency doubled (see $\sigma_1$ and $I_1$ in fig. 9).

A more linear mode of operation may be obtained by a static bias to the centre of this characteristic which corresponds to a phase difference of the two orthogonal components of $\pi/2$ ($\sigma_2$ and $I_2$ in fig. 9). This bias may be obtained by a static stress in the bar or, more commonly, by the use of a ‘quarter wave plate’ placed between the polarizer and analyser with its axes at $\pi/4$ to the direction of polarization. In this case the intensity is:

$$I = \frac{1}{2} I_0 (1 + \sin \theta)$$

and is linear to 5% for a range of intensity of $\pm \frac{1}{4} I_0$.

![Fig. 8. Detection of birefringence.](image)

Fig. 8. Detection of birefringence. Plane polarized light at 1 may be resolved into two orthogonal components, shown at 2. After transmission through the birefringent material N one component is phase delayed relative to the other, at 3. These two components combine to give elliptical polarization shown at 4. Transmission through an analyser A results in plane polarized light at 5 whose intensity is dependent upon the birefringence.

![Fig. 9.](image)

Fig. 9. The light intensity $I$ transmitted by the birefringent medium is shown as a function of stress $\sigma$. An alternating stress $\sigma_1$ for the case when the polarizer and analyser axes are orthogonal and no quarter wave plate is included results in an output intensity $I_1$ which is frequency doubled. The addition of a quarter wave plate biases the characteristic to the centre point resulting in a stress $\sigma_2$ giving rise to a linear intensity modulation $I_2$.

**Ultrasonic transducers**

The ultrasonic transducer most frequently used in the construction of delay lines is a thin slice of piezoelectric quartz crystal fundamentally resonant in its thickness mode at the carrier frequency [8]. This transducer is bonded to the fused quartz medium by a conducting layer, which forms the front electrode of the crystal, and a rear electrode is deposited upon the back face of the crystal. The drive voltage is applied between these electrodes. Either shear or longitudinal modes of vibration may be generated depending upon the direction of the crystal axes in the slice.

The frequency response of such a transducer/delay medium combination is an arithmetic series of passbands at the odd harmonics of the crystal resonance, each passband having a fractional bandwidth of rather over 50% of the fundamental carrier frequency, so that for a passband of 10 MHz a band centre of about 20 MHz is required. Recently piezoelectric ceramics such as lead zirconate titanate (the Philips PXE range of materials) have found increasing use as delay line  


transducers at frequencies of up to 10 or 15 MHz. The advantage of these materials is in the very high electromechanical coupling coefficient \( k \) which for crystalline quartz is about 0.14 and for PXE3 is about 0.7. Since the power conversion efficiency is proportional to \( k^2 \) this forms a very significant improvement although the very high dielectric constant of this material (about 1500 compared with 4.5 for quartz) is in this case an embarrassment since this creates a very high transducer capacitance.

At fundamental frequencies above 15 MHz the grain size in the PXE material becomes comparable to the thickness and this material is no longer of use. A new ceramic, potassium sodium niobate (P.S.N.), has, however, a very much finer grain size, partially due to the hot pressing technique used in its manufacture, and also has a considerably lower dielectric constant of about 450 and a coupling coefficient of about 0.6.

It is also possible that a recently investigated single crystal ferroelectric, lithium niobate, large crystals of which have been successfully grown at Mullard Research Laboratories, will prove useful as a piezoelectric transducer. This material has a coupling coefficient of about 0.54 and a relative dielectric constant of about 80. Problems exist in bonding this material satisfactorily to the fused quartz delay medium.

In order to reduce the transducer capacitance which with these high dielectric constant materials may be as high as 10 000 pF, and thus raise the transducer impedance to a suitable level for convenient electrical drive, the transducer is subdivided and the sections are connected in series as shown in fig. 10. This reduces the capacitance by a factor \( r^2 \) where \( r \) is the number of sections.

**Ultrasonic path cross-dimensions**

The acoustic wavefront in the "near field" or "Fresnel" region is plane and here no significant beam spread occurs. For a frequency of about 20 MHz and a transducer cross-dimension of 1 cm, the near field extends to approximately 7 cm. Beyond this region (in the "far field") the wavefront gradually assumes a spherical form.

In order to avoid excessive beam spread in this far field which in a narrow bar would result in wavefront errors due to reflection from side walls and in a wide bar to excessive curvature of wavefront and loss of sound intensity, it is desirable to make the transducer many wavelengths wide. A minimum cross-dimension of 50 wavelengths (about 9 mm in a fused quartz medium at a frequency of 20 MHz) is typical.

**Light modulation efficiency**

For a constant depth of modulation the product of the drive power and the acoustic beam width must be constant.

Optical considerations, however, limit the width of the interaction since any light ray contributing fully to the effect must lie within two adjacent acoustic wavefronts. An oblique ray which crosses two or more sound wavefronts will have the modulation partially cancelled since it will pass through regions of positive and negative birefringence. Thus for a given light beam divergence \( \varphi \) and a given sound wavelength \( \lambda_s \) the maximum interaction width is limited to \( \lambda_s/\varphi \).

In general \( \varphi \) can only be decreased at the expense of light flux. The requirements of the final signal-to-noise ratio and the maximum intensity of available light sources affect this consideration.

A typical interaction width for the system with a 20 MHz carrier and 10 MHz bandwidth would be about 2 cm for a maximum light divergence of 5 milliradians.

**Correlation system**

The correlation system is shown in fig. 11. A light source which is a 1 kW compact mercury arc is imaged upon a source slit by the condenser lens \( L \). Mirror \( M_1 \) collimates the light upon the photo-elastic delay lines \( D_1 \) and \( D_2 \). The polarizer \( P \) polarizes the light either along or normal to the bar in the case of shear waves, and the analysers \( A_1 \) and \( A_2 \) either parallel or

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\[ \text{[9]} \text{ R. W. Gibson, Solid ultrasonic delay lines, Ultrasonics 3, 49-61, 1965.} \]

\[ \text{[10]} \text{ C. M. van der Burgt, Transducer materials and isopaustic glasses for delay lines, Ultrasonics, 1967 (to be published).} \]

\[ \text{[11]} \text{ Transducers using this material have been made fundamentally resonant at frequencies as high as 75 MHz.} \]
orthogonal to this direction. The first analyser forms the polarizer for the second system.

The quarter wave plates $Q_1$, $Q_2$ must be correctly orientated as mentioned previously and must lie between the appropriate analyser and polarizer, although the order in which the light passes through the fused quartz bar and the quarter wave plate is not important. The light, after passing through the system, is imaged upon a photo-multiplier by $M_2$.

The signal-to-noise improvement for weak signals below noise level, as would be expected with a 1000 : 1 pulse compression, is 30 dB.

Time reversal

For this system to succeed, as mentioned previously, it is necessary to reverse one signal in time. It appears preferable to select the transmitted signal for this operation since its amplitude is less variable than an echo which may well have a total range of amplitudes of 60 dB.

Two methods have been proposed for this reversal which requires storage of the signal for a time equal to its duration, the first using a similar opto-acoustic system and the second a storage tube.

Fig. 12 illustrates the first system using lenses as the optical components although mirrors may equally well be used. The signal to be reversed is fed into a photo-elastic line which modulates the light intensity in a linear fashion as in the correlation system. The trans-
mitted light intensity through this system is then a replica of the stress pattern in the photo-elastic delay line and this light pattern is moving at the sonic velocity in the line.

The optical system forms a minified and inverted image of this light pattern upon the second delay line. The minification of this optical system is exactly 2:1 so that this optical image occupies half the length of the original pattern and moves at half the acoustic velocity. The second delay line is operated in the cut-off mode (fig. 9) and a single stress pulse whose total time duration is short in comparison with the reciprocal of screen area, the writing and reading spot will be defocussed in a radial direction by the use of a rotating quadrupole magnetic deflection (a means of producing radial astigmatism).

This latter system appears to be capable of producing an adequate signal-to-noise performance and resolution, and will probably be preferred to the former.

Range improvement

The system so far described will only correlate over a total time given by the difference between the total photo-elastic delay line length and pulse length. Thus

![Diagram of optical system for time reversal]

The bandwidth of the signal to be inverted is fed into it. This single stress pulse, moving at the full acoustic velocity, “interrogates” the moving light pattern, taking exactly the original pulse duration in which to accomplish this. The resulting total light intensity transmitted through this second photo-elastic line is a time reversed replica of the original input signal.

In the second system, the transmitted signal is recorded by writing it on a storage tube [12], and reversed by reading it backwards. In order to utilize a large area of the storage surface and hence enhance the signal-to-noise ratio of the system, it is proposed to write the signal in a spiral scan around the outer third of the storage screen. In order to use an even greater for a pulse length of 100 µs and a line length of 200 µs the correlation time will be 100 µs. Ideally the input signal will be delayed by a pulse duration prior to propagation in the photo-elastic line.

A second correlator can be time shared with the first to obtain a complete 200 µs correlation period.

In order to extend the range still further the transmitted, reversed, signal can be read from the storage tube a number of times until the complete radar range has been covered, at which time it is erased. This period must, of course, be less than the repetition period of the radar.

System limitations

The system limitations arise from two main sources, the practical restrictions of the acoustic system and the restrictions due to the quantum nature of light.

The former restrict the length of the photo-elastic delay line to about 200 μs (about 75 cm for shear waves in fused quartz), so that pulses of about 100 μs duration may be correlated. The acoustic loss down such a line restricts the maximum frequency to typically 60 MHz, unless very low loss quartz is selected and the delay line is operated at an elevated temperature when perhaps 80 or even 100 MHz may be possible.

Assuming that an acoustic fractional bandwidth of 50% can be maintained, the typical figure leads to possible pulse compressions of 3000 : 1 or a signal-to-noise improvement of 35 dB.

The limitations of the system due to the quantum nature of light arise from the practical considerations of the total useful light flux emitted by the light source and the sensitivity and maximum light detection capability of the photo-detector.

Since the photo-detector must resolve twice the ultrasonic carrier frequency, it must have an acceptable frequency response at around 60 MHz. This fact and the required high sensitivity and low noise requirements suggest a photo-multiplier as the only presently useful photo-detector.

The total useful light flux and the photo-detector sensitivity together form the lower boundary for a detected signal. The upper boundary is formed by the combination of this total light flux and the maximum light detectable by the photo-detector.

Photo-cathodes typically are limited to maximum photo-electron currents of about 1 μA/cm², corresponding to 10⁶ electrons per cycle of carrier at the maximum frequency 2(f₀ + Δf) of 60 MHz, which approximates to a noise figure of 50 dB. If the minimum detectable signal from the equipment is set at 0 dB signal-to-noise level, the maximum dynamic range will be the full 50 dB.

If the equipment has a pulse compression ratio of 1000 : 1 (e.g. 10 MHz bandwidth and 100 μs pulse duration) the input signal-to-noise ratio to achieve this maximum sensitivity of 0 dB output level will be -30 dB and saturation will occur for input signals of greater than 20 dB above noise level.

The light source used for this equipment is a 1 kW compact source mercury arc and the light is filtered to pass only the 0.405 μm and 0.436 μm lines. A gas laser would form a more suitable light source from most points of view, but at present the need for high light output at the peak of the photo-cathode spectral response makes such a laser very expensive, and an adequate performance can be obtained from the mercury arc.

A significant improvement could perhaps be made with the use of a semiconductor photo-detector which has a much higher quantum efficiency than a photocathode, but problems of internally generated noise in the semiconductor and the lack of a suitable internal gain mechanism at present prevent this.

Conclusion

The correlation system described has several advantages over other pulse compression systems in that accurate pulse compressors and expanders are not required and that the radar output need not be accurately specified.

Some improvements in the components, e.g. light source, photo-detector, and the success of present work on ultrasonic transducer development could produce an attractive device. Its major limitation will then be in the physical size of the equipment, making its use possible only in situations where space is not at a premium.

Summary. Correlation detection has long been known to form an ideal means of extracting a reflected radar signal from noise. Another property of the correlation detector is that a long, broadband signal can be time-compressed to provide a short collapsed pulse. Hitherto the realizations of this type of detector have suffered from a great complexity. The proposed system, although bulky, using large optical components, is essentially simple. It uses two photo-elastic delay lines to carry the transmitted and received signals. The correlation process requires the multiplication of coincident samples of the two signals and this product is formed by the double modulation of light passing sequentially through these two delay lines. In the general case of this system, one signal requires reversing in time and means for achieving this are described. System bandwidth of 10 MHz and correlation times of about 100 μs are possible giving a signal-to-noise improvement of 30 dB and a pulse compression ratio of 1000 : 1.