Parametric amplifiers for a radio-astronomy interferometer

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Microwave parametric amplifiers can now be designed that give diode noise temperatures not far above 10 K when cooled by liquid nitrogen. For applications such as radio astronomy, these devices are more practical than masers, which are more complex in themselves and require a more elaborate cryogenic system since they are operated at liquid-helium temperatures. The article below describes a cooled parametric-amplifier system for the radio interferometer at Defford, England.

Introduction

In radio astronomy it is often important to be able to locate a faint source of radio noise and measure its angular diameter. To detect such faint sources it is clearly desirable to have a low-noise input stage for the receiver system, and the angular diameter of the source can be measured by using an interferometer arrangement. A radio interferometer has two aerials, each with its own receiver system; the diameter of the source is derived by training the aerials on it and measuring the correlation of the output noise as a function of aerial spacing [1].

The radio noise that reaches the Earth from such sources is in the microwave range of frequencies, so the radio interferometer must be a microwave system. Since the background noise from outer space is very low indeed, a great deal may be gained by using a low-noise amplifier in the system. The low-noise amplifier first used in such systems was the maser [2], which can have an effective input noise temperature [3] of about 6 K. However, although the maser has such a low noise temperature, it has two great disadvantages. It can only be used with complicated and expensive support equipment, in particular the cryogenic equipment associated with the liquid-helium cooling; and its bandwidth can never be very large. The maser introduced the microwave systems engineer to the spectacular benefits of low-noise devices, and at the same time convinced him that there would be much to be said for a simpler and perhaps broad-band device, even if the noise temperature were a little higher.

In fact, work on such a device had begun in the mid 1950s. This was the varactor-diode parametric amplifier, based on the varactor or variable-capacitance diode [4][5]. Its reactive method of energy transfer [6] promised a low noise level and the varactor diode itself was tiny and very suitable for use in microwave circuits. Since those days there has been a good deal of progress and today the parametric amplifier has superseded the maser for almost all applications in which a low-noise microwave amplifier is required. Even with fairly simple tuned circuits the bandwidth of the parametric amplifier is better than that of the maser, and it can be increased further by using simple filter techniques [7]. The parametric amplifier can also give quite good noise performance at ordinary ambient temperatures. However, in applications where maser-like performance is required cooling is still necessary, but the temperatures required are usually not so low that liquid helium has to be used.

[1] This principle and its application to optical stars have been treated earlier in this journal: R. Hanbury Brown and A. Browne, The stellar interferometer at Narrabri, Australia, Philips tech. Rev. 27, 141-159, 1966.  
[3] The effective input noise temperature (often called the amplifier noise temperature) is the temperature at which the input termination must be held to produce an output noise power, per unit bandwidth, double that which would occur if the termination were cooled to absolute zero.  
[6] One of the first to recognize the low-noise potentialities of non-linear capacitances was A. van der Ziel, then working at Philips Research Laboratories. He published the classic paper: On the mixing properties of non-linear condensers, J. appl. Phys. 19, 999-1006, 1948.  
Fig. 1. One of the two aerials for the radio interferometer at Delford, near Malvern, England. To vary the length and direction of the base line each aerial can be moved along its own special double railway track: the two sets of tracks intersect at an angle of 67°. The two aerials are trained on a radio source and the noise received is amplified in a separate channel for each aerial. Receiver noise is kept low by using a varactor-diode parametric amplifier as the first stage. Measuring the correlation of the output noise as a function of the aerial spacing will give the angular diameter of the noise source. The tripod structure carries the extra reflector of the Cassegrain feed.
In the present article we shall describe the parametric amplifiers that have been developed to replace the masers in the Defford radio interferometer (fig. 1). There are two parametric amplifiers, one for each receiver channel, and they operate at a frequency of about 2.7 GHz. The amplifiers are cooled to 80 K by liquid nitrogen to bring the effective noise temperature of the receiver below the specified value of 35 K.

On the face of it, it would seem that a microwave receiver with a noise temperature of 35 K for its first stage is going to give a much worse performance than one with a maser first stage whose noise temperature is about 6 K [8]. However, we have to remember that there are noise contributions from the parts of the system that precede the first stage. In the Defford radio interferometer the aerial itself has an effective noise temperature of 20 K and the aerial feeder system has a noise temperature of 10 K. Noise contributions after the first amplifying stage will not be significant because of the gain of the first stage, so we can take the total system noise temperature to be 36 K with the maser and 65 K with the parametric amplifier.

Now the sensitivity of the receiver system can be expressed in terms of a minimum detectable source temperature $\Delta T_s$, which is equal to $T_s/\sqrt{2B\tau}$, where $T_s$ is the receiver-system noise temperature, $B$ is the bandwidth of the receiver and $\tau$ is the post-detector integration time. The relation shows that when the parametric amplifier is used the post-detector integration time has to be increased by just over three times to keep the minimum detectable source temperature at the same value (assuming the same bandwidth). This is a disadvantage, but one that is more than outweighed by the advantages of a system that, because it requires no liquid helium, is simpler and less expensive to run.

The parametric amplifier

Let us begin by indicating the main elements of the varactor-diode parametric amplifier used in this system. The most vital element is the varactor diode. This is a $P-N$ junction that behaves as a capacitance that varies with applied voltage [8]. The varactor diode is driven by a local source called the pump, which applies a voltage at a high frequency $f_p$ across the diode. Under these conditions the diode appears to small signals as a time-varying capacitance. Pumped in this way, the diode forms the link between two circuits as shown schematically in fig. 2, where $C$ is the time-varying capacitance. For clarity the pump circuit is not shown. The circuit to the left is known as the signal circuit, because it is connected to the source $E$ of the signal at frequency $f_s$; the right-hand circuit, connected only to the diode, is known as the idler circuit. The reactance $X_s$ is provided to make the signal circuit resonant at the frequency $f_s$, and similarly the reactance $X_1$ is provided to resonate the idler circuit at frequency $f_1$. Band-pass filters $F_s$ and $F_1$ are also included to confine the signal- and idler-frequency currents to the appropriate circuits. Parametric amplification is achieved when $f_s + f_1 = f_p$, where $f_p$ is the pump frequency.

\[ f_s + f_1 = f_p \]  

then [4] [9] the capacitance $C$ varying at the pump frequency is able to transfer energy from the pump to appear at the frequency $f_s$ (or at $f_1$). The circuit will then function as an amplifier. The gain can be considered to arise because the circuit behaves as a negative resistance for an input signal of frequency $f_s$. A simple physical explanation of the action is given in the Appendix.

A circuit like the one shown in fig. 2 is not particularly suitable as it stands for use as an amplifier, since the output signal will appear at the same terminals as the input signal. It is much more convenient to include a circulator, which separates the incident and amplified signals, as shown in fig. 3. (Port 1 couples to port 2, port 2 to port 3, etc.). In this configuration, the negative resistance presented by the amplifier at the signal frequency gives a reflection coefficient greater than unity, and thus a gain since input and output signals are separated. The circulator also prevents the gain from being affected by variations in source impedance.

Since the method of energy transfer is based on a time variation of a reactance, which gives no thermal noise, it would appear that the amplifier should have a low input noise temperature. In fact, the diode is not quite a perfect reactance; there is a small resistive loss (the "spreading resistance") caused by the resistance of the ionic lattice to the movement of the electrons and holes. This spreading resistance is represented by $R_d$ in

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**Fig. 2.** Equivalent circuit of parametric amplifier. The left-hand circuit is the signal circuit and the right-hand one is the "idler" circuit. They are coupled by the varactor diode, which is driven by a pump signal to make it behave as a time-varying capacitance $C$. (The pump circuit is not shown.) Loss in the diode is represented by the spreading resistance $R_d$. The reactance $X_s$ is provided to make the signal circuit resonant at the frequency $f_s$, and reactance $X_1$ resonates the idler circuit at the frequency $f_1$. Band-pass filters $F_s$ and $F_1$ confine the currents to the appropriate circuits. Parametric amplification is achieved when $f_s + f_1 = f_p$, where $f_p$ is the pump frequency.
the schematic diagram of fig. 2. By carrying out a theoretical analysis in which sources of resistive noise are taken into account, it can be shown [9] that for high gain (say 10 dB or more) the effective input noise temperature \( T_{\text{amp}} \) of a parametric amplifier of this type is given by:

\[
T_{\text{amp}} = T_d \left[ \frac{1}{p} + \frac{f_s}{f_1} \left( 1 + \frac{1}{p} \right) \right].
\] (2)

Here \( T_d \) is the physical temperature of the diode and \( p = R_g/R_d \) where \( R_g \) is the source impedance at the signal frequency \( f_s \). The quantity \( p \) is known as the overcoupling ratio.

It would appear from (2) that all that has to be done to give a low value of input noise temperature is to make \( p \) high and \( f_s/f_1 \) low. However, \( p \) and \( f_s \) are already related: it can be shown [10] that for high gain

\[
f_s/f_1 (1 + p) = (yfe)^2,
\] (3)

where \( y \) is a measure of the capacitance variation of the diode defined by \( y = (C_{\text{max}} - C_{\text{min}})/2(C_{\text{max}} + C_{\text{min}}) \) and \( f_e \) is the cut-off frequency defined by \( f_e = 1/(2\pi C R_d) \). The relation (3) is derived by setting the effective negative resistance in the signal circuit at signal frequency equal to the total positive resistance, a condition which is met to a good approximation for high gain. From (2) and (3) it can be shown that there are theoretical optimum values for both \( p \) and \( f_s \), giving a minimum noise temperature for the amplifier. This is

\[
T_{\text{amp min}} = 2f_s T_d/yfe.
\] (4)

The quantity \( yfe \) is thus a figure of merit for the varactor diode, for it indicates the lowest input noise temperature that it will give in an "optimum" amplifier. The optimum idler frequency is equal to this figure of merit:

\[
f_1^{\text{opt}} = yfe.
\] (5)

![Fig. 3. A circulator is used to separate input and amplified signals. The circulator Circ directs the signal from the source \( S \) to the amplifier Amp via port 2 and returns the amplified signal via port 3 to the load \( R_n \), which has the same impedance \( R_s \) as the source.](image)

The best commercially available diodes (e.g. the Mullard CXY10) have a \( yfe \) of 40 GHz or more.

The power gain \( G \) and operating bandwidth \( B \) for a parametric amplifier with simple tuned circuits as in fig. 2 are related by:

\[
G^2B = 2/(B_s^{-1} + B_l^{-1}),
\] (6)

where \( B_s \) is the bandwidth of the unpumped signal circuit and \( B_l \) is the bandwidth of the unpumped idler circuit. As indicated in the Introduction, greater bandwidths can be achieved, at the same gain, by using simple filter techniques [7] [12].

### Design and performance of the amplifier system

#### Design of basic amplifier

As we saw earlier, the effective input noise temperature had to be less than 35 K for radio astronomy. The lowest operating temperature for a diode cooled by liquid nitrogen would be about 80 K, and the amplifier was to operate at about 2.7 GHz.

With \( yfe \) taken as 40 GHz, eq. (4) shows that the minimum noise temperature that can be obtained at this frequency from a diode cooled to 80 K is 10.5 K.\footnote{Design of basic amplifier.}

The relations (5) and (1) show that to achieve this minimum value the idler frequency should be about 40 GHz and the pump frequency should be about 43 GHz. However, it was not as easy to design a practical parametric amplifier with a noise temperature below 35 K as these figures might suggest. There was no convenient source that would deliver pump power (about 150 mW) at a frequency of 43 GHz, and because of the stray elements of the varactor diode it was inconvenient to provide an idler circuit resonant at 40 GHz. Moreover, the figure of 35 K for the noise temperature would have to include noise contributions from other parts of the parametric amplifier unit such as the circulator and the microwave feed paths. As we shall see below, the circulator alone can contribute some 10 K of noise.

The highest frequency for which there was a convenient pump source available was 40 GHz. As (1) shows, the idler frequency should then be about 37 GHz. In practice it was not possible to achieve this, but the

\footnote{See pp. 206-207 of the article by Aitchison [5].}
\footnote{C_{\text{max}} \text{ is usually taken as the capacitance at } 1 \text{ \mu A forward current and } C_{\text{min}} \text{ as the capacitance at } -6 \text{ V reverse bias. The cut-off frequency } f_e \text{ is probably best defined for the value of } C \text{ corresponding to zero bias, but manufacturers often quote cut-off frequencies for } -6 \text{ V reverse bias.}

CXY10 diode does have a resonance at about 27 GHz, due to stray inductance in series with the junction capacitance. Equations (2) and (3) show that the noise temperature of the resulting non-optimum amplifier will only be about 3 K higher than the minimum value of 10.5 K. From eq. (3) it can be shown that the overcoupling ratio should be about 20.

Some earlier work had already been done on a design principle that could be used with the CXY10 diode to provide resonant circuits for the signal, idler, and pump frequencies required. In this approach [9] [12] a second diode is introduced. The final design for the amplifiers for the radio interferometer is based upon this arrangement. We shall now look at the design of the amplifier with the aid of the schematic drawing shown in fig. 4.

The two diodes D are located inside the pump waveguide W with their top ends connected by the flat bar B, in opposite polarity. (An enlarged sketch of a single diode is shown on the right.) The centre of the bar is joined to the coaxial input system C1 by the post P. The lower end of each diode is connected to a coaxial circuit C6. The pump waveguide W is terminated in a matched load behind the diode structure, and there is a tunable filter in the pump waveguide (neither of these is visible in the figure). As the figure shows, the coaxial input system consists of a cascaded series of lengths of coaxial line l2 and l6, stepped in the way shown. At the top of the coaxial system there is a connection to one port of the circulator (port 2 of fig. 3).

Let us now try to explain how the arrangement corresponds to the schematic diagram of fig. 2. At first glance it seems as though it cannot do so, for there are two diodes. However, at the signal frequency the two diodes may be considered to form a single unit, since the bar connecting the diodes is much smaller than the wavelength, and the diodes are therefore effectively in parallel. The filter F6 is not present as a separate item; its function is achieved by making C resonant with Xs, which therefore has to be an inductance (see fig. 5a). An inductance L4 is provided by the bar-and-post structure, which also serves to couple the pump power into the two diodes. The tunable filter mentioned above (but not shown) is used to match the diode to the pump source.

At the idler frequency, the two diodes are again combined, but in such a way that they form a closed resonant loop (see fig. 5b), connected to the outside world only by the parametric action of the varying capacitance. In each diode the stray inductance L2 and the junction capacitance C give the series resonance mentioned above, so that the complete idler loop thus formed is resonant at about 27 GHz. The element F1 of fig. 2 therefore represents the filtering effect of the two simple tuned circuits that form the loop; X1 represents the reactance due to diode stray inductance. With identical diodes connected in opposite sense this idler circuit is balanced, i.e. no idler voltage will appear across the terminals AA, which correspond to the lower end of the post P in fig. 4. The diodes cannot be connected directly together since they are not small compared with the idler wavelength λi (≈ 1 cm) and so are connected instead by a transmission line that is half a wavelength long at the idler frequency. (A half-wave line has no transforming effect.) Physically, this transmission line is a microstrip line formed by the bar B and the lower wall of the waveguide.

Fig. 4. A schematic diagram of the diode circuits of the parametric amplifier. The diodes D are located in the pump waveguide W (axis into the drawing) and linked by the bar B. The diodes are connected to the bar in opposite sense and are connected to the signal input and output (i.e. to the circulator) by the inductive post P and the coaxial-line system C1. The stepped coaxial lines form a double quarter-wave transformer at the signal frequency f1 and a stop-filter at the idler frequency f2; l1 = λs/4 and l2 = λi/4 where λs is the signal wavelength and λi is the idler wavelength. The distance between the two diodes is made equal to λi/2. The coaxial circuits C6 are low-pass filters for the diode current monitoring circuit. An enlarged sketch of a single diode is shown on the right.
The coaxial structure $C_1$ has two functions: it transforms the input impedance to the correct value and also provides a low-pass filter that will pass the desired signal but reject signals at the idler frequency (such signals may arise if the diodes are not quite identical). We saw above that a single diode should be fed from a signal source whose impedance is about 20 times greater than the spreading resistance $R_d$. With two diodes effectively in parallel the source impedance should be about $10R_d$, and at the signal frequency the coaxial structure acts as a double quarter-wave transformer to transform the standard 50-ohm input impedance to the appropriate value. The line of length $l_1$, equal to a quarter of the signal wavelength $\lambda_s$, is the first stage of the transformer and the second stage is formed by the alternate low- and high-impedance lines each of length $l_2$. This second stage of the transformer also acts as the low-pass filter; it is designed with $l_2$ equal to a quarter of the idler wavelength and therefore rejects signals at the idler frequency but passes the desired signal.

Since the idler circuit is well decoupled from other circuits by the balanced arrangement and the low-pass filter it has quite a large bandwidth (about 3 GHz). This makes it easier to obtain a sufficient signal bandwidth (see eq. 6), and also makes it easier to stabilize the signal phase against pump-frequency variations. The coaxial circuits $C_2$ are also low-pass filters: they provide the connections to the monitoring circuit.

The amplifier-circulator unit

We saw above that a microwave circulator is used with the parametric amplifier to separate the input and output signals. The circulator characteristics of importance are the isolation between ports that should not be coupled, and the loss between coupled ports. If the isolation is too low there will be an appreciable reflected signal, and hence mismatch, at the input port. The loss in the circulator is undesirable because it reduces the signal and has the effect of increasing the effective input noise temperature of the receiver.

Both coaxial and waveguide circulators were available when the equipment was designed, but the one whose performance was most suitable was a waveguide version. This was a four-port device with an isolation of 40 dB and an insertion loss of 0.15 dB. With this degree of isolation the reflected signal at the input is sufficiently small (at 20 dB gain the voltage reflection coefficient is less than 0.13). A loss of 0.15 dB at room temperature corresponds to an increase in noise temperature of nearly 12 K, and this is acceptable in our system provided that the waveguide-to-coaxial transition that connects amplifier and circulator is cooled: The effect on the noise temperature of the various losses in the amplifier-circulator unit is discussed below. A schematic arrangement is shown in fig. 6.
Calculated noise performance of the amplifier system

Table I lists the physical temperature, loss and theoretical noise contribution [9] for each component of the parametric-amplifier system. (The noise contribution from the amplifier is of course calculated from eq. 2.) Adding up the noise contributions in the right-hand column shows that at a system gain of 23 dB the noise temperature of the amplifier system should be 31 K. Some of the components in the Table are listed twice since they are encountered by the signal both before and after amplification. The rather high operating temperature of 320 K for the uncooled components was chosen because of the heat generated by the pump klystron: this will be discussed later.

<table>
<thead>
<tr>
<th>Component</th>
<th>Physical temperature K</th>
<th>Loss dB</th>
<th>Noise contribution K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input waveguide</td>
<td>320</td>
<td>0.02</td>
<td>2.25</td>
</tr>
<tr>
<td>Circulator (input)</td>
<td>320</td>
<td>0.15</td>
<td>11.8</td>
</tr>
<tr>
<td>Download</td>
<td>80</td>
<td>0.05</td>
<td>1.34</td>
</tr>
<tr>
<td>Transition</td>
<td>80</td>
<td>0.10</td>
<td>2.3</td>
</tr>
<tr>
<td>Amplifier</td>
<td>80</td>
<td>—</td>
<td>13</td>
</tr>
<tr>
<td>Transition</td>
<td>80</td>
<td>0.10</td>
<td>0.004</td>
</tr>
<tr>
<td>Upload</td>
<td>80</td>
<td>0.05</td>
<td>0.002</td>
</tr>
<tr>
<td>Circulator (output)</td>
<td>320</td>
<td>0.30</td>
<td>0.09</td>
</tr>
<tr>
<td>Transition</td>
<td>320</td>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>Coaxial output/lead</td>
<td>320</td>
<td>0.3</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Total noise temperature 31 K

Fig. 7. Sectional drawing and top view of the cryogenic system. Outer diameter 31.8 cm, inner container-diameter 10.2 cm and length 58.4 cm. D is a standard 34-litre liquid-nitrogen dewar made of stainless steel. The amplifier A is mounted in an atmosphere of helium gas inside the sealed container K, whose upper part is made of glass-fibre bonded resin to reduce the nitrogen boil-off rate. The lower part C is made of copper, and is joined to the upper part by steel flanges F with a gold O-ring vacuum seal. T waveguide-to-coaxial transition. Wp signal waveguide: this has very thin walls of copper-plated stainless steel. St vacuum seal of thin plastic sheet mounted across the signal waveguide. The pump waveguide Wp is made from silver-plated nickel, and has a sliding vacuum seal Ss to allow for differential contraction. M remotely adjusted matching unit to match pump power to the diodes. To keep down the boil-off rate plugs P1 and P2 of expanded PVC are mounted beneath the top plate Q; the horizontal lines represent annular aluminium-foil radiation shields. The thermal connection between the liquid nitrogen and the amplifier is formed by the copper base C, the copper fingers \( V \) and the diaphragm spring \( Di \), which allows for differential contraction. Further heat transfer is also provided by the helium gas. The baseplate nitrogen-vacuum seal \( S3 \) is demountable, with a locking sealant, so that the amplifier can be removed from the inner container.

On top of the inner container: \( Wp \): pump waveguide; \( Ws \): signal waveguide, \( M \): pump-match control, \( E \): helium safety-valve, \( B \): diode bias connector, \( I \): helium inlet valve. In the filling process the inner container is initially evacuated through \( I \), then flushed with helium gas and evacuated again. Finally it is filled with helium gas at a pressure of about 1.1 atm. With the helium supply still connected via \( I \) the dewar is then filled with liquid nitrogen through \( Ws \). When the fill is complete \( I \) is closed and the helium supply disconnected. \( N3 \): sensing point for liquid-nitrogen level; this can be used with an automatic nitrogen-replenishing system. Gaseous nitrogen can escape through the nitrogen vents \( N3 \), which have heated release valves. Switch \( S \) will connect a meter into the varactor-diode circuit, which is short-circuited under operating conditions.
Thermal considerations

The cryogenic system

The cryogenic system, which contains the cooled components of the amplifier complex, is shown in fig. 7. To keep the volume occupied by the cooled components as small as possible, the waveguide-to-coaxial transition $T$ is of the “end-fire” type \[13\]. The amplifier $A$ and the signal and pump waveguides are suspended from a top plate $Q$ and are housed in a thermally designed sealed container $K$, which is filled with helium gas. The container keeps the microwave components free of water vapour and liquid nitrogen, which would introduce losses and also variations in performance as the nitrogen level changed.

The container must introduce only negligible thermal loss to the outside while providing a good thermal connection between the amplifier and the liquid nitrogen. The thermal loss is minimized by making the upper part of the container of glass-fibre bonded resin, and good thermal contact between the amplifier and the liquid nitrogen is ensured by making the lower part $C$ of the container of copper. The resin tube and the copper section are joined by a pair of special stainless-steel flanges $F$, one brazed to the copper and one bonded to the resin. Between the two flanges, which are bolted together, there is a gold O-ring to form the vacuum seal. The seal is unaffected by repeated temperature variations between $77 \text{ K}$ and $300 \text{ K}$. Good thermal contact between the amplifier and the copper wall is ensured by copper fingers $V$ and a diaphragm spring $Di$ which allows for differential contraction. Further heat transfer is provided by the helium gas inside the container.

A view of the end-fire transition and the amplifier assembly is shown in fig. 8.

The waveguide feed $W_p$ for the 29.5 GHz pump signal (internal dimensions $3.56 \times 7.12 \text{ mm}$) is electroformed from nickel and plated inside with $5 \mu \text{m}$ of copper to give good electrical conduction. The larger signal-waveguide feed ($3.404 \times 7.214 \text{ cm inside}$) could not be made in this way, however, since the thermal conduction would then be too large. This waveguide is fabricated from $0.038 \text{ mm}$ stainless steel sheet in two halves, which are welded together. This waveguide is also plated inside with $5 \mu \text{m}$ of copper. There is a row of six holes along the centre of one broad face of this waveguide to prevent it from collapsing when the vacuum is released during the filling process (see caption to fig. 7). Fig. 9 shows a photograph of the sealed container and the amplifier complex.

The sealed container is suspended in liquid nitrogen contained in a stainless-steel dewar of standard dimensions. To reduce the boil-off rate the underside of the dewar top plate is lined with a 15 cm thick layer of expanded polyvinyl chloride (PVC). Heated nitrogen vents are included in the top plate to prevent a dangerous build-up of pressure. A safety valve (10 lbs/in$^2$, i.e. about $0.7 \text{ kg/cm}^2$) is also fitted for the helium gas in the sealed container.

In the design of the cryogenic system great care was taken to allow differential contraction of the various components while keeping the structure rigid enough.

to give stable operation. The design was also arranged so that the number of demountable seals immersed in liquid nitrogen was as small as possible.

A single filling of liquid nitrogen will keep the system adequately cooled for 100 hours when the dewar is kept vertical. However, the complete cooled amplifier system is rigidly attached to the aerial, and may therefore tilt through ±45°. Under these conditions a single filling will keep the system cooled for 72 hours.

An automatic nitrogen-replenishing system has also been added which can be used if there is a need for a long period of continuous operation. It also allows the amplifier system to be operated more easily at the primary focus of the aerial.

*The temperature-stabilized enclosure*

The temperature of the varactor circuits is stabilized at about 80 K by the liquid nitrogen, but to achieve the stability necessary in interferometry the temperatures of the pump klystron and the circulator also have to be stabilized. These components are therefore housed in a temperature-stabilized enclosure mounted on top of the dewar.

The temperature-stabilized unit is shown in the photograph of fig. 10. Since the klystron gives out nearly 60 watts of heat it is convenient to stabilize the enclosure to a temperature above the highest ambient value. This is done by using air blowers with heaters that are automatically controlled by a mercury-contact thermometer.

The outer surface of the unit, through which the heat is dissipated, is finished in a white enamel paint that has an emissivity of about 0.8. The unit measures 50 × 37.5 × 25 cm. A weather-proofing cover, finished with the same paint, is also available: this enables the stabilizer-dewar unit to be used out of doors.
Performance

The main details of the performance of the amplifier system are given in Table I.

The noise temperature was measured by the Y-factor method, used earlier with the masers \(^2\) that the parametric amplifiers replace. In this method matched loads at room temperature and at 77 K are connected in turn to the input and the difference in noise output is recorded. In use, however, the noise performance of the system is a function of the aerial match: there is a room-temperature matched load at the fourth port of the circulator, and any mismatch in the input circuit will therefore cause noise to be reflected back into the system.

The phase stability was measured by means of a phase bridge and the results are shown graphically in fig. 11. The measurements were made for a wide range of ambient temperatures and indicate a stability of better than \(\pm 2^\circ\) per day \(^{14}\).

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Appendix: Physical explanation of parametric amplification

The explanation of parametric amplification given some years ago in this journal \(^{15}\) was limited to a rather simple case. Here we shall extend that explanation a little further in the way indicated by H. Mooijweer \(^{16}\).

We consider a simple lossless \(LC\) circuit (fig. 12) in which there is a current of frequency \(f = 1/2\pi\sqrt{LC}\). The charge on the capacitor and hence the voltage across it then vary sinusoidally with time. The capacitance is the reactive element that is varied, and it is assumed that the capacitance variation shown in fig. 12 is obtained by moving the plates of the capacitor. Every time that the charge reaches a maximum value, the plates are pulled sharply apart, and they are moved together again at the zero points of the charge curve. The first movement requires work to be supplied, but the second requires no work. There is therefore an overall flow of energy to the circuit in each period of the oscillation: this energy goes to aid the oscillation. The amplitude of the

<table>
<thead>
<tr>
<th>Signal centre frequency</th>
<th>2.695 GHz (11.12 cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gain</td>
<td>23 dB</td>
</tr>
<tr>
<td>Effective input noise temperature</td>
<td>31 ± 3 K</td>
</tr>
<tr>
<td>3 dB bandwidth</td>
<td>40 MHz</td>
</tr>
<tr>
<td>Input voltage reflection coefficient</td>
<td>&lt; 0.13</td>
</tr>
<tr>
<td>Gain stability</td>
<td>± 0.25 dB/hr</td>
</tr>
<tr>
<td>Phase stability</td>
<td>± 2°/day</td>
</tr>
<tr>
<td>Pump frequency</td>
<td>29.5 GHz</td>
</tr>
<tr>
<td>Pump power</td>
<td>150 mW</td>
</tr>
<tr>
<td>Operating temperature</td>
<td>77 K</td>
</tr>
<tr>
<td>Operational time (vertical)</td>
<td>100 hours</td>
</tr>
<tr>
<td>Operational time (with tilting through ±45°)</td>
<td>72 hours</td>
</tr>
</tbody>
</table>

Fig. 10. Photograph of the temperature-stabilized enclosure. Air is sucked over the components by the main blower \(B_1\), then forced over the low-thermal-capacity heaters \(H_1\) between the two skins of the case. The klystron \(K_1\) is blown by the small blower \(B_2\); the sensor \(S\), which is a mercury-contact thermometer, is situated between \(B_2\) and the klystron. The anti-condensation heaters \(H_2\) are switched on when the amplifier is switched to standby.

Fig. 11. Phase stability of the parametric-amplifier system over three days' continuous operation. \(A\) indicates nitrogen fill by pouring. At \(B\) there was a disturbance in the measuring system. \(C\) marks the start of automatic nitrogen filling. The phase stability is \(± 2^\circ\) per day.

\(^{14}\) The phase-stability information was made available by Mr. G. Moule of the Royal Radar Establishment, Malvern, England.

\(^{15}\) B. Bolle\-é and G. de Vries, Experiments in the field of parametric amplification, Philips tech. Rev. 21, 47-51, 1959/60.

voltage across the capacitance consequently increases in steps, since the charge tends to remain constant when the capacitance is altered because of the presence of the inductance. The effect is like a sort of "negative damping". In this way spontaneous oscillations in the circuit can grow from a small disturbance because of the periodic variation in capacitance (the pumping action) at a frequency equal to twice the resonant frequency \( f \) of the circuit. This implies that a small signal introduced into the circuit at its resonant frequency will also grow in magnitude, i.e. be amplified.

For a circuit with losses, the situation would have been much the same except that some of the energy supplied by pumping would have been dissipated in the losses.

**Fig. 12.** Parametric excitation in a simple LC circuit. The capacitance \( C \) is the reactance that is varied; it is assumed that the variation is caused by moving the plates apart and together. 

- **a)** Original variation with time of the voltage \( V \) and charge \( Q \) on the capacitance. 
- **b)** Periodic variation of the capacitance \( C \). 
- **c)** Increase in the circuit energy \( E \) caused by the pumping action. 
- **d)** Growth of the voltage \( V \) across the capacitance ("negative damping").

If the phase \( \phi \) of the pumping signal is varied with respect to the situation shown in fig. 12, \( \phi = 0 \) the energy transfer is reduced \(^9\) and can even be negative for \( 45^\circ < \phi < 135^\circ \). Now if the frequency of an input signal deviates a little from half the pump frequency the effect is the same as a continuous change in phase: there will be an amplitude modulation of the amplified signal. In many applications it is desirable to amplify a fairly complicated circuit. Instead of a single oscillatory circuit with periodically varying capacitance or inductance, a circuit is used that has two (or more) resonant frequencies \( f_1 \) and \( f_2 \), with the coupling at the periodic reactance. It is found that in this configuration the phase condition described above is no longer present. We shall explain this with the aid of fig. 13. Again, it is assumed that the circuit is lossless and that \( C \) is the reactive element that is varied.

This difficulty can be avoided by using a slightly more complicated circuit. Instead of a single oscillatory circuit with periodically varying capacitance or inductance, a circuit is used that has two (or more) resonant frequencies \( f_1 \) and \( f_2 \), with the coupling at the periodic reactance. It is found that in this configuration the phase condition described above is no longer present. We shall explain this with the aid of fig. 13. Again, it is assumed that the circuit is lossless and that \( C \) is the reactive element that is varied. It is also assumed that one loop only supports current at frequency \( f_1 \) and the other loop only supports current at frequency \( f_2 \). The total charge on the capacitance then consists of a component \( Q_1 = Q_0 \sin \omega_1 t \) and a component \( Q_2 = Q_0 \sin \omega_2 t \), where \( \omega_1 \) and \( \omega_2 \) are the angular frequencies corresponding to \( f_1 \) and \( f_2 \).

For convenience we have assumed that the amplitudes of the two components are equal, but this is not essential. The total charge is then \( Q = Q_1 + Q_2 = 2Q_0 \sin \omega t \cos \frac{1}{2} (\omega_1 - \omega_2) t \), an oscillation of frequency \( \frac{1}{2} (\omega_1 + \omega_2) \) modulated in amplitude at a frequency \( \frac{1}{2} (\omega_1 - \omega_2) \). (The carrier is suppressed.)

The pump source again provides a capacitance variation in the form of a square wave. The capacitance is increased at the zeros of \( \cos \omega t \) and reduced half-way between the zeros. This means that the pump frequency is now equal to \( f_p = f_1 + f_2 \). Moreover, the energy flow from the pump source to the capacitance is no longer the same at every decrease in capacitance because of the amplitude-modulated character of the charge. However, it is never negative, since the capacitance is always increased at the zeros of the charge curve.

Once again, spontaneous oscillations can arise and grow through the periodic variation of a reactive element in the circuit. It seems however that the phase condition should remain the same as for the earlier situation. In fact, this is not the case: in a practical amplifier only the current varying at frequency \( f_1 \) is supplied (as the signal); the current of frequency \( f_2 \) only arises from the effect of the pumping action on the impressed signal, as a mixing product across the varying reactance. This current generated in this way at \( f_2 \) then has the correct phase automatically. If the wrong phase were to appear, it would be quickly damped out anyway, since this energy would be extracted from the circuit by the pumping action. The mixing product, the charge or current varying at \( f_2 - f_1 = f_0 \), is the idler signal.

In this arrangement, with the phase condition no longer relevant, we can choose the frequencies \( f_1 \) and \( f_2 \) far enough apart to separate them with filters.

The arrangement with the two resonant circuits can be considered as a more general case; the arrangement with the single resonant circuit and \( f_1 = f_2 = \frac{1}{2} f_0 \) is therefore referred to as the "degenerate" case.

In an ordinary LC circuit with loss represented by a resistance \( R \) in the loop a disturbance will give rise to a damped waveform of the form \( \exp (-R/2L) t \cos (\omega t + \phi) \). But in the pumped
Fig. 13. Parametric excitation in a circuit with two resonant frequencies. a) Component $Q_1$ of the charge varying at frequency $f_1$. b) Component $Q_2$ of the charge varying at frequency $f_2$. c) Total charge $Q_1 + Q_2$ on the varying capacitance. d) Periodic variation of the capacitance $C$. e) Energy increase in the circuit, due to the pumping action.

In the circuits we have been considering here, the oscillatory waveform grows with the pumping action (before it is eventually limited by non-linearity). Now assuming that the waveform grows exponentially, the coefficient of the exponential would have to be positive — which would imply an effective negative resistance. In analysis it is frequently convenient to consider the gain action as being due to the introduction of a negative resistance into a circuit.

The pumping action in the above simple description was assumed to be a square-wave variation of capacitance, caused by mechanical action. In practice, of course, the time variation of capacitance is obtained by driving a varactor diode with a large sinusoidal voltage.

Summary. A parametric-amplifier system has been developed by Mullard Research Laboratories for the radio-astronomy interferometer at Delford, England. Two of these systems replace the two masers previously used. A very stable and sensitive receiver system is required in radio astronomy, and such requirements can be met by using a varactor-diode parametric amplifier, cooled with liquid nitrogen as the first receiver stage. A brief recapitulation of parametric-amplifier principles is followed by an account of the design approach of a two-diode amplifier and a description of the complete system. Special attention is given to the effect of loss and other imperfections in the signal-feed system, and the resulting noise degradation is evaluated. Stability is ensured by careful temperature stabilization of the uncooled microwave components. The amplifiers operate at 2.695 GHz and are pumped at 29.5 GHz. The noise temperature is $31 \pm 3$ K and the 3 dB bandwidth for 23 dB peak gain is 40 MHz. Gain stability is $\pm 0.25$ dB over 1 hour, $\pm 0.5$ dB over 24 hours, and the phase stability is $\pm 2^\circ$ over 24 hours. The system will operate for 100 hours on one filling of nitrogen, and automatic replenishment can also be used.
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