Optical polarization effects in a gas laser

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Introduction

Any book on classical optics has an important chapter on the characteristics of polarized light and its propagation in physical media. Experiments with polarized light have also greatly contributed to the understanding of the structure of atoms, molecules and matter. The advent of the laser has brought about a revival of optics, mainly because it made available light sources of extremely high intensities and very high temporal and spatial coherence. This article deals with the interaction of laser light with the medium in which it is generated and in particular with the optical polarization effects produced by this interaction.

The first of the following three sections contains a general exposition of the main concepts of laser physics useful for the understanding of the rest of the article. The next section deals with the information that can be obtained from the experimental study of polarization effects in a gas laser. It reveals unorthodox optical behaviour of the generating medium and the section contains a phenomenological description in classical terms. The last section gives the basic elements of a quantitative description. It indicates how a quantum-mechanical theory leads to a detailed understanding of the observations and to some unique predictions which are confirmed by experiment.

General concepts

Lasers and masers are devices in which use is made of the amplification of electromagnetic waves that can be obtained when these waves pass through appropriately prepared gaseous, liquid or solid media. Decrease in the amplitude of an electromagnetic wave propagating through a medium is a well known phenomenon: the absorption coefficient \( \alpha \) of the medium is defined as the relative decrease per unit length \(-I \text{d}I/\text{d}x\) of the intensity \( I \) of the wave. An increase in amplitude and thus amplification can be described by assigning a negative value to the absorption coefficient of the medium. It is possible to create media that have this unusual property and the negative value of \( \alpha \) is then caused by the phenomenon of stimulated emission of radiation, which will be discussed later. The mechanism which makes the medium exhibit negative absorption is called the pumping mechanism. The energy required to amplify a wave must be transferred from the medium to the wave and therefore an external energy pump must be present to supply the energy to the medium.

Contrary to what the words LASER (Light Amplification by Stimulated Emission of Radiation) or MASER (Microwave Amplification by S E R) might suggest, any kind of electromagnetic wave can be amplified in principle in appropriately designed devices by stimulated emission. In practice amplification has been realized at many frequencies between those of low-frequency microwaves and short-wave ultra-violet light. The different media used for these purposes must have very specific properties and, generally speaking, a particular medium and its appropriate pumping mechanism will only provide amplification of electromagnetic waves in a small frequency range. This available spectral range differs greatly from case to case.

Once the possibility of amplification exists, the medium can also be used to generate the corresponding electromagnetic waves. Ever present noise can grow to a high intensity through amplification and here the principle of feedback can be used with advantage when the growing wave is reintroduced into the medium several times. The radiation generated in this way has a frequency somewhere in the available spectral range and can be extremely monochromatic, with a spectral purity many orders of magnitude better than that obtained in any other known way.

In the optical region, ranging from the far infra-red to ultra-violet, various gases pumped by a gas discharge offer a large variety of possible laser frequencies. The available spectral range around each frequency is usually very small. For instance, in the helium-neon laser which shows laser action at various frequencies in the red and infra-red the available range is of the order of 500 MHz at centre frequencies of about \( 3 \times 10^{14} \) Hz. There are many solids that show laser action in the optical region when pumped by a high-intensity pulsed light source. Examples are the ruby crystal or glass in which ions such as neodymium are present. Lasers are mostly used to generate light of high brilliance in a very small linewidth. Here gas lasers are unique sources

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of light with extreme temporal and spatial coherence, a characteristic that is made use of in holography. Solid-state lasers can generate light of extremely high power, especially if the energy is released in pulses as short as $10^{-8}$ to $10^{-12}$ seconds. During a short pulse powers as high as $10^9$ watts may be reached. Completely new effects can be observed if matter is irradiated with such a high power.

We shall not try to give a summary of all the different laser sources here. Instead we shall now turn to a discussion of the phenomenon of stimulated emission.

**Stimulated emission**

In quantum theory, absorption of electromagnetic waves is accounted for by the transition of an atom or molecule from a state (a) of lower energy into one of higher energy (b) under the influence of incident light. If the difference in energy of the two states is $\Delta E$, only light with a frequency $\nu_{ba} = \Delta E/h$ can be absorbed, where $h$ is Planck's constant. The probability that an atom initially in the lower state (a) jumps to (b) is proportional to the intensity $I$ of the light. The probability that an atom initially in state (b) jumps down to state (a) under influence of the light is also proportional to $I$ and, other things being equal, it is in fact equal to the earlier probability. In the case of a downward transition under influence of incident light the released energy is added to that of the incident light. This is the stimulated emission of radiation. If a unit volume contains $N_a$ atoms in state (a) and $N_b$ in (b), the decrease of intensity, $-I \frac{\partial I}{\partial x}$, will be proportional to $N_a - N_b$, i.e. amplification can only occur if $N_b > N_a$.

This can be more precisely formulated. In an element of volume the electric field $E = E_0 \cos \omega t$ of the light induces a dielectric polarization $P$ per unit volume, given by

$$P = P_{in} \cos \omega t + P_{out} \sin \omega t$$

with the same angular frequency $\omega = 2\pi v$ as that of the light. Both the component $P_{in}$ in phase with $E$ and the out-of-phase component $P_{out}$ are proportional to $E_0$ as well as to $N_a - N_b$. The part $P_{out}$ that lags by $\pi/2$ is positive if $N_a > N_b$, implying that the medium absorbs energy, while $P_{out} < 0$ if $N_a < N_b$ in which case the light wave is amplified. Both $P_{in}$ and $P_{out}$ depend on $\omega$. In particular $P_{out}$ as a function of $\omega$ is a bell-shaped curve with a maximum at $\omega = \omega_{ba} = \frac{\Delta E}{h}$ corresponding to the fact that for various reasons an atomic absorption line has a finite spectral width. It is precisely over this width that, for a negative absorption ($N_b > N_a$), amplification and laser action is possible. This width was called the available spectral range above. The value and the frequency dependence of $P_{in}$ determine respectively the refractive index and the dispersion of the medium, which, as is well known, can be considerable in the neighbourhood of an atomic transition.

Until now it has been assumed that $N_a$ and $N_b$ are given quantities. In a medium in thermodynamic equilibrium atomic levels of higher energy are always less populated than the lower ones, i.e. $N_a > N_b$. In a laser or maser a very specific pumping mechanism must be operative to achieve an inverted population $N_a < N_b$ for at least one pair of levels (a) and (b). We shall not discuss pumping mechanisms further here since each different type of laser or maser is pumped in its own particular way. We shall simply assume the presence of a pump that produces $A$ atoms in state (b) per unit volume and per unit time. Even in the absence of incident radiation, there are many processes such as atomic collisions which cause the atoms to leave state (b), e.g. for completely different lower-lying states. Such processes obviously hinder the building up of an inverted population. We shall assume the number of atoms disappearing from (b) in this way per unit time to be proportional to the number present in (b), i.e. to be given by $\gamma_b N_b$. Similarly we assume the rate of disappearance from (a) to be equal to $\gamma_a N_a$ caused by transitions to still lower energy levels. The latter transitions favour an inverted population between (a) and (b), but obviously can only be effective if (a) is not the very lowest state of the atom. Indeed in many practical laser systems the lowest laser level (a) is not the lowest state of the atom. For simplicity we assume $\gamma_a$ to be so large that under all circumstances $N_a$ remains negligible ($N_a \approx 0$). Finally there are transitions from (b) to (a) which provide the energy required for the amplification of the incident light wave. This stimulated transition rate is given by $c_1 (N_b - N_a) I$, i.e. is proportional to $N_b - N_a$ and the light intensity. With $N_b \approx 0$ we now have the following rate equation:

$$\frac{dN_b}{dt} = \Delta - \gamma_b N_b - c_1 N_b I.$$ (2)

For given $I$ the stationary excess population $N_b-N_a$ follows:

$$N_b - N_a \approx N_b = \Delta/(\gamma_b + c_1 I).$$ (3)

In all these equations $c_1$ still depends on the light frequency $\nu$.

From these elementary considerations two conclusions can be drawn. Firstly, the negative absorption coefficient which was proportional to $N_b - N_a$ is by (3) a function of $I$. Its absolute value decreases with increasing intensity $I$, an effect that is referred to as saturation of the medium. This means that an electromagnetic wave which is reintroduced again and again into the medium, for example with the aid of mirrors, does not continue to grow indefinitely. The decrease of ampli-
fication leads to stabilization of the intensity and the corresponding negative absorption coefficient at a value for which the amplification of the wave just compensates the ever present attenuation due to effects such as absorption in or transmission through the mirrors. We then have a laser operating as an oscillator stabilized by its intensity. Secondly, the dielectric polarizaton \( P \) induced in the medium, which was proportional to \( E_0 \) and \( N_{b0}-N_{a0} \), becomes a nonlinear function of \( E_0 \), since \( I \) in (3) is itself proportional to \( E_0^2 \). For small \( E_0 \), i.e. \( E_0^2 \ll \gamma_{pl}/c_1 \), a series expansion of \( P \) in ascending odd powers of \( E_0 \) will approximate the nonlinearity. Later in this article the nonlinearity will play an important role in situations where the vector character of the quantities \( E \) and \( P \) is relevant. 

In the foregoing we have not discussed spontaneous emission. The possibility of absorption and stimulated emission by transitions between the states \((a)\) and \((b)\) implies the existence of spontaneous emission of light of frequency \( \nu_{ba} \) by a spontaneous transition from \((b)\) to \((a)\) whenever an atom is in state \((b)\), irrespective of the presence of incident light. However, the spontaneous transition rate is proportional to \( N_a \) and the effect on equation (2) can be accounted for by an adjusted choice of \( \nu_{ba} \). Furthermore, spontaneously emitted light does not contribute to amplification of the incident wave. By definition, it is light that is not able to do so, either because of the direction in which it is emitted or because of its incoherence with the incident light.

Spontaneous emission is a source of noise in a stationary oscillating laser. We shall not discuss such effects in this article.

**Characteristics of the gas laser**

Since the effects that occur in a continuously operating gas laser will be discussed later in this article, the principal features of the device will now be reviewed. Consider a helium-neon laser oscillating at a wavelength \( \lambda_1 = c/n_i (\approx 1.15 \mu m) \), where \( c \) is the velocity of light. It consists of two parallel plane circular mirrors with a diameter \( d \) of a few mm at a distance \( L \) of about 10 cm with in between them a cylindrical tube containing a helium-neon gas mixture (90\% He; 10\% Ne; pressure 5 torr, i.e. \( \approx 10^{16} \) Ne atoms/cm\(^3\)). The mirrors have a large reflection coefficient \( R \approx 0.99 \). The gas mixture is "pumped" by an electrical gas discharge and as a consequence atomic collisions between He and Ne atoms build up an inverted population between a pair of energy levels \((b)\) and \((a)\) of the neon atoms, their energy difference being \( \Delta E = h \nu_{ba} \approx hc/\lambda_1 \). Characteristic values for \( N_b - N_a \) are \( 10^9/cm^3 \), i.e. only a very small fraction of the total number of atoms. The frequency dependence of the resulting unsaturated negative absorption coefficient is given by

\[
\alpha(\omega) = \alpha(\omega_{ba}) \exp \left[-\left(\omega - \omega_{ba}\right)^2/\gamma_{pl}^2\right],
\]

where \( \alpha(\omega_{ba}) < 0 \) and \( |\alpha(\omega_{ba})| \approx 10^{-3} \) cm\(^{-1}\). The spectral width of \( \alpha \), for which \( \gamma_{pl} \approx 10^9 \) s\(^{-1}\) is a measure, is caused by the Doppler effect, which will be discussed later.

The fractional gain in intensity of a light wave with angular frequency \( \omega_1 \) travelling back and forth between the mirrors equals \( |\alpha(\omega_1)| \times L \) for one transit over a distance \( L \) and amounts to about 1\%. On the other hand there is a fractional loss \( \cal L \) per round trip caused by transmission and absorption at the mirrors and by a very small part of the light beam spilling over the edges of the mirror. As long as \( 2L \times |\alpha(\omega_1)| > \cal L \) the beam undergoes net amplification. In the stationary oscillating situation saturation reduces the negative absorption coefficient at \( \omega = \omega_1 \) to such a value \( \alpha_s \) that its value satisfies

\[
2 |\alpha_s(\omega_1)| = \cal L / L.
\]

The ratio of the unsaturated to the saturated value is

\[
F = \alpha(\omega_1)/\alpha_s(\omega_1),
\]

and \( F - 1 \) is called the excess fraction above threshold. The greater \( F - 1 \), the greater the intensity \( I \) reached by the standing light wave in the stationary oscillator. As soon as \( 2|\alpha(\omega_2)| > \cal L / L \), there exists a range of frequencies around \( \omega_{ba} \) for which

\[
2|\alpha(\omega)| > \cal L / L.
\]

The exact frequency \( \omega_1 = 2\pi n_L \) at which the laser in fact will oscillate is now determined by the requirement that the light wave must constructively interfere with itself after one round trip of length \( 2L \). Ignoring the (frequency-independent) phase jumps at the reflecting mirrors one must then have:

\[
2L = pL = p \lambda_1 / n_i,
\]

where the mode number \( p \) is an integer of the order of \( 2 \times 10^5 \). Generally speaking there now exists a finite number of frequencies \( n_1 \) satisfying both (7) and (8), separated in frequency by intervals of \( \Delta n_1 = c/2L \). For \( L \approx 10 \) cm, \( \Delta n_1 \) is approximately equal to \( 1.5 \times 10^9 \) Hz, i.e. of the same order of magnitude as the spectral (Doppler) width of \( \alpha \). It is therefore possible to operate the laser at one single frequency \( n_1 \) and also to tune the value within the Doppler profile by precise adjustment of \( L \).

In the foregoing it was more or less implied that the light wave was a plane wave travelling between the mirrors. This cannot be correct since the beam and the mirrors have a finite diameter \( d \) of a few millimetres. It is true that since \( \lambda/d \ll 1 \) and also \( d^2/L \gg 1 \) the de-
vations from a plane wave are small, but a much more precise analysis is required to show that the simplification to plane waves is adequate for our purpose. In the analysis the system of two small mirrors, which may be plane or slightly curved, at a distance \( L \) is considered as an open resonator for electromagnetic waves. As with the more familiar cavity resonator for microwaves, open resonators have a large number of (now closely spaced) eigenfrequencies each with a certain attenuation. The damping is considerable for empty open resonators. Under pumping conditions the medium deattenuates one or more of the corresponding modes with frequencies \( v_\ell \) satisfying (7) and a more refined version of (8).

Alternatively the optical system can be considered as a special type of Fabry-Perot interferometer. Because of the small but finite transmission of the mirrors the empty optical system shows the characteristic transmission maxima as a function of frequency for an external incident light beam parallel to the optical axis. For our purpose it is sufficient to assume that in a laser, oscillating at a frequency \( v_\ell \), there exists a well defined standing-wave pattern and that this pattern can be thought of as being composed of two plane waves travelling between the mirrors. The finite transmission of the mirrors allows a narrow and nearly plane-parallel beam to leave the system on which the experimental observations can be made.

**Doppler broadening**

For a proper understanding of a gas laser a discussion of the Doppler width \( \gamma_D \) in equation (4) is necessary. The spectral width \( \gamma_D \) of the positive or negative \( \alpha(\omega) \) seen by a propagating wave does not correspond to the width of the absorption or stimulated-emission line of an individual atom, but finds its origin in the velocity distribution of the atoms. If all atoms in states (a) and (b) were at rest a much narrower line \( \alpha(\omega) \) centred around \( \omega_{ba} \) would be obtained:

\[
\alpha(\omega) = A(N_a-N_b)/[(\omega-\omega_{ba})^2 + \gamma_{ab}^2]. \tag{9}
\]

Here the characteristic width \( \gamma_{ab} \) is much smaller than \( \gamma_D \) and also the shape of the line differs from that in equation (4). In (9) \( A > 0 \) and the notation makes explicit the proportionality of \( \alpha \) with \( N_a - N_b \).

If all atoms had a velocity \( v_\ell \), a wave propagating in the + \( z \)-direction would experience absorption centred around a displaced frequency \( \omega_{\ell z} \) in accordance with:

\[
\alpha_{\ell z}^+(\omega) = A(N_a-N_b)/[(\omega-\omega_{\ell z})^2 + \gamma_{ab}^2], \tag{10}
\]

with

\[
\omega_{\ell z} = \omega_{ba}(1 + v_\ell/c). \tag{11}
\]

This is an immediate consequence of the Doppler effect as seen by an observer moving with the atom. In a gas there is a distribution of atomic velocities \( v_\ell \), so that if \( n_a(v_\ell)\,dv_\ell \) is the number of atoms per unit volume in state (a) in the velocity interval between \( v_\ell \) and \( v_\ell + dv_\ell \) and similarly for atoms in (b), then

\[
\alpha^+(\omega) = A \int dv_\ell \left[ n_a(v_\ell) - n_b(v_\ell) \right] / [(\omega-\omega_{\ell z})^2 + \gamma_{ab}^2]. \tag{12}
\]

The particular frequency dependence of the unsaturated \( \alpha \) expressed by (4) now follows from (12) if, in the absence of saturation, the velocity distribution functions \( n_a(v_\ell) \) and \( n_b(v_\ell) \) are Maxwellian distributions for atoms of mass \( M \) and temperature \( T \). In this case \( \gamma_D^a \) can be identified by \( 2kT\omega_{ba}^2/Mc^2 \) where \( k \) is Boltzmann's constant.

The Doppler profile \( \alpha(\omega) \) now appears as a weighted superposition of many narrow lines of width \( \gamma_{ab} \). This width is called the homogeneous linewidth, since in contradistinction to the inhomogeneous Doppler width it cannot be further resolved into contributions from different classes of atoms. One cause of the homogeneous width is the finite lifetime of the levels, because of collision and spontaneous emission. In the helium-neon laser \( \gamma_{ab} \approx 2.10^7 \text{s}^{-1} \) and is indeed smaller than \( \gamma_D \).

The dielectric polarization \( P \) induced in a gaseous medium by the electric field \( E \) of a wave will also consist of the sum of the contributions from atoms in the various velocity intervals. This now greatly complicates the evaluation of saturation effects such as saturation of negative absorption or, more generally, the evaluation of the nonlinear relation between \( P \) and \( E \). For this a detailed knowledge is required of how an intense beam at frequency \( v_\ell \) affects the values of \( n_a(v_\ell) \) and \( n_b(v_\ell) \) for each velocity \( v_\ell \). The point here is that the beam travelling in the positive \( z \)-direction reduces the inverted population \( n_b - n_a \) of only those atoms for which \( \omega_{\ell z} \) in equation (11) approximately equals \( \omega_\ell \), i.e.

\[
| \omega_{\ell z} - \omega_\ell | \leq \gamma_{ab}. \tag{13}
\]

From (11) this means that only atoms for which

\[
v_\ell \approx c(\omega_\ell - \omega_{ba})/\omega_{ba} \tag{14}
\]

are affected by saturation. Similarly the beam returning in the negative \( z \)-direction only affects atoms with

\[
v_\ell \approx -c(\omega_\ell - \omega_{ba})/\omega_{ba}. \tag{15}
\]

Therefore, in a stationary oscillating laser two holes are burned in the inverted distribution \( n_b(v_\ell) - n_a(v_\ell) \).

A quantitative description can be obtained by setting up rate equations of the type of equations (2) and (3) for atoms in each velocity interval separately. As a consequence of hole burning \( \alpha(\omega) \) is reduced to its
saturated value $|a_0(\omega_1)|$; in addition both holes influence the refractive index experienced by each of the two waves. Very special effects occur when $\omega_1 \approx \omega_{ba}$ as in this case atoms with $v_a = 0$ are saturated by the two waves simultaneously, so that the laser intensity reached will be lower than it would otherwise have been. The phenomenon is known as the Lamb dip in the laser output at $\omega_1 \approx \omega_{ba}$.

In what follows the general concepts exposed so far will be made use of, but the attention will be focused on questions related to the vector character of the electric field in a light wave. A stationary oscillating laser normally produces light of a definite state of optical polarization. If the laser system has ideal cylindrical symmetry about its z-axis one does not expect the laser output to be optically polarized, in, say, the x-direction. Polarization in the y- or any other direction would be equally likely or for that matter any superposition of linear polarizations, e.g. elliptical polarization. It appears, both experimentally and theoretically, that these considerations are not at all complete and that the experimentally observed polarization phenomena are not exclusively due to deviations from ideal cylindrical symmetry but are also related to nonlinear properties of the medium under the influence of polarized light.

**Optical polarization states in laser interferometers**

As argued above there would be no constraint on the state of polarization of a mode in an isotropic, i.e. cylindrically symmetric, laser. In practice such an ideal situation does not exist. Even if the interferometer only consists of two parallel plane mirrors there will be a small anisotropy, caused for instance by imperfections in the reflecting films on the mirrors. Indeed, in an article [1] on the first helium-neon laser, which was of the Fabry-Perot type, the radiation was reported to be plane polarized in a direction apparently controlled by a linear pattern of fine cracks in the films. In the literature at that time the direction of polarization was generally assumed to the direction for which the interferometer shows minimum loss. Such an interpretation in terms of loss anisotropy would imply independence of the direction of polarization from tuning conditions.

Experiments at Philips Research Laboratories with a small Fabry-Perot helium-neon laser ($\lambda = 1.15 \mu m$) [2] have shown however that the polarization direction more or less abruptly changed by $\pi/2$ when the laser was tuned through the centre of the Doppler profile [3] [4]. This “polarization flip”, also reported by other workers, has forced us to discard the interpretation in terms of loss anisotropy. We have replaced it by one involving phase anisotropy of the interferometer, i.e. a different optical path length for different polarization states, and, in addition, anisotropy of the gaseous medium induced by saturation effects [5] [6] [7]. In the later treatment it will be shown how experimental evidence has necessitated the introduction of the concept of saturation-induced anisotropy and how it can be understood in terms of the atomic properties of the medium.

In order to see how interferometer anisotropy affects the polarization state of a mode, the basic repetitive properties of the nearly plane waves travelling back and forth between the mirrors will be recalled. In a stationary mode of oscillation the wave must, after one round trip, reproduce a) its phase b) its amplitude and c) its state of polarization. The mode number $p$ being given, condition (a) fixes the frequency of the wave. Condition (b) can only be met in the presence of an amplifying medium. Condition (c) will be reformulated with the aid of the complex amplitudes of the linearly polarized components $E_x$ and $E_y$, into which a general state-of-polarization can always be resolved. For instance, a purely imaginary ratio of $E_x$ and $E_y$ characterizes circular polarization, a purely real ratio plane polarization at an angle arctan $(E_y/E_x)$ to the x-axis.

Now let $E_x$ and $E_y$ be the complex amplitudes before a round trip and $E'_x$, $E'_y$ those after one round trip. Condition (c) then requires:

$$E'_y/E'_x = E_y/E_x.$$  \hspace{1cm} (16)

It is clear that relation (16) is independent of conditions (a) and (b). Indeed a phase factor or an amplitude factor common to $E'_y$ and $E'_x$ does not affect (16). Therefore, if the mirrors are the only anisotropic elements in the interferometer, the consequences of (16) can be studied with the empty interferometer. In this case there will be a linear relation between $E'$ and $E$, so that

$$E'_x - E_x = a_{11}E_x + a_{12}E_y$$
$$E'_y - E_y = a_{21}E_x + a_{22}E_y$$  \hspace{1cm} (17)

where the matrix elements $a_{ij}$ are complex quantities. The ratios $E'_y/E'_x$ that satisfy both (16) and (17) are given by

$$E'_y/E'_x = \frac{a_{22} - a_{11}}{2a_{21}} \pm \left(\frac{(a_{22} - a_{11})^2 + 4a_{11}}{2a_{21}}\right)^{1/2}$$  \hspace{1cm} (18)

and define two states of (generally) elliptical polarization, the “eigenstates of polarization” of the inter-

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ferometer [8][15]. They are fixed by the characteristics of the mirrors and are independent of frequency at least to the extent that these characteristics are frequency insensitive. The mirrors are isotropic if \( a_{22} - a_{11} = a_{12} = a_{21} = 0 \). In this case \( E_y/E_x \) in (18) is undefined, corresponding to a previous statement that there is then no constraint on the state of polarization. In practice \( a_{22} - a_{11}, a_{12} \) and \( a_{21} \) are small quantities of the order \( 10^{-4} \). However, it is the ratios of these small quantities which determine the eigenstates in accordance with (18).

Assuming again that in a stationary oscillating laser the role of the medium is to provide enough amplification to make up for the losses, it may be concluded that a laser should oscillate in one or both of the eigenstates of polarization determined by the empty interferometer. The frequencies corresponding to these states may be different as a result of the phase anisotropy of the mirrors. Since polarization-dependent phase jumps at the mirrors are of the order of the coefficients \( a_{ik} \), i.e. \( 10^{-4} \), and the mode number is of the order \( 2 \times 10^5 \), a relative frequency shift of the order of \( 10^{-9}/2\pi \), i.e. about 50 kHz is expected. The empty cavity loss corresponding to the two states of polarization may also be different as a result of loss anisotropy of the mirrors. In view of the fact that the 50 kHz frequency difference is much smaller than the inhomogeneous linewidth \( \gamma_{ab} \), so that the two eigenstates suffer saturation effects from a common “burned hole”, then only the eigenstate of polarization with lowest empty-cavity loss will actually be expected to appear.

Experimental evidence shows however that a laser does not necessarily oscillate in a polarization eigenstate of the empty interferometer and that the medium does not play the neutral role so far assumed. This will be discussed later. First of all the Poincaré representation of polarization states will be introduced.

The Poincaré sphere

A very useful way of visualizing polarization states was introduced by Poincaré in 1892. According to Poincaré a state is represented by a point on a sphere, its location being defined by a longitude \( 2\psi \) and a latitude \( 2\chi \) (see fig. 1). The correspondence between a state of polarization and a point \((2\psi, 2\chi)\) on the sphere is that \( \psi \) corresponds to the angle between the long axis of the polarization ellipse and the x-axis of the laser while \( \tan \chi \) is the axis ratio of the ellipse \((-1 \leq \tan \chi \leq 1; -\pi/4 \leq \chi \leq \pi/4)\). In particular, the two poles represent circular polarization with opposite sense of rotation, whereas the equator is the locus of plane polarizations. The eigenstates of polarization discussed before can therefore be represented by two points somewhere on the sphere. They are diametrically op-

![Fig. 1. Polarization ellipse (a) and its representation \( P \) on the Poincaré sphere (b); the longitude on the sphere is twice the azimuth \( \psi \) of the long axis and the latitude is \( 2\chi \) where \( \tan \chi = a/b \) \((-1 \leq \tan \chi \leq +1)\). In particular the “equator” \((2\chi = 0)\) is the locus of plane polarizations whereas the “poles” \((2\chi = \pm \pi/2)\) represent opposite circular polarizations.](image-url)
time the wave inside the laser has a frequency \( \nu \), an intensity \( I \) and a state of polarization such that the repetitive conditions (a), (b) and (c) of page 194 are not exactly satisfied. It stands to reason that then the change of \( E_y/E_x \) per round trip time is \( E'_y/E'_x = E_y/E_x \) and follows from equation (17), again under the assumption that only the mirrors affect the polarization state. The temporal change in the state of polarization at a given time then only depends on the state of polarization at that time.

This makes possible the construction of trajectories on the Poincaré sphere. One such trajectory is shown in fig. 2. In general it starts from the eigenstate of polarization with highest empty-cavity loss and ends at the state of lowest loss, in agreement with the idea that the latter state will be found in the stationary laser. The length of the arrows in the flow field in fig. 2 indicates the magnitude of the time derivative. It is zero at the unstable point of departure and at the stable final position. In the case of an interferometer with pure loss anisotropy the trajectories are semicircles directly connecting the (opposite) eigenstates; in the case of pure phase anisotropy they are closed circles perpendicular to the axis connecting the two eigenstates. Such a case will be encountered in the next section (fig. 8).

A crucial check on the validity of the considerations given so far is provided by an experiment [3] [7] [5] in which a mirror is used that on reflection converts part of one circular component of the incident light into circularly polarized light of the opposite sense. Such a mirror does not exist, but its function can be realized by making use of the light transmitted through a normal isotropic mirror \( M_1 \) in such a way that a fraction is reflected back into the interferometer by means of an additional external mirror \( M_3 \). The arrangement also involves a polarizer and a \( \lambda/4 \) plate between \( M_1 \) and \( M_3 \) as shown in fig. 3.

For such an artificially anisotropic mirror \( a_{12} = ia_{11} = - ia_{22} = a_{21} \), so that from (18) the two eigenstates of polarization degenerate into a single one at the pole for which \( E_y/E_x = i \). The flow pattern corresponding to this case is given in fig. 4. It can be shown that the meridian trajectory crosses the equator at a longitude \( 2\varphi_0 = 4\pi z_{3M}/\lambda + \text{constant} \), where \( z_{3M} \) is the axial position of mirror \( M_3 \). This position determines the arguments of the matrix elements \( a_{1k} \).

The important point now is that experimentally a laser provided with this reflection arrangement oscillates in a nearly plane-polarized state with azimuth \( \psi \) depending on \( z_{3M} \) through the relation \( \psi = 2\pi z_{3M}/\lambda + \text{constant} \). This experimental fact is interesting from various points of view. First of all it is an example of an arrangement in which the plane of polarization of an intense light source changes periodically as a distance.
(2M) varies. Such an arrangement is of technical importance in that it allows one to construct an apparatus that counts and measures displacements electronically in complete wavelengths and fractions. It is also a surprising fact since, as can be seen in fig. 4, there is no point near the equator where the time derivative of the polarization state might be expected to be zero. It is therefore reasonable to conclude that not only the interferometer but also the gaseous medium itself plays a role in determining the polarization state.

To show this more clearly we ought to mention an additional observation. The polarization state of the laser is slightly elliptical, i.e. represented by a point slightly off the equator in the direction of the expected circularly polarized eigenstate. The deviation decreases as the excess fraction F - 1 above threshold increases. All this strongly points to an effect due to nonlinear properties of the medium. It is as if a strong additional “force” is present driving the polarization state towards the equator, this force increasing with excess fraction F - 1 and saturation effects increase. The force would be proportional to the light intensity in the medium. On the Poincaré sphere it would be represented by arrows pointing from the poles to the equator, the arrows being zero at the poles and at the equator (see fig. 5). The last two statements follow from the symmetry: the medium might show preference for plane polarization but it should not discriminate either between different values of ψ or between the two possible senses of rotation.

Within this hypothesis it can be seen that the laser will find a stationary state of oscillation A in fig. 4 on the meridian trajectory where the arrows due to the mirror anisotropy and the saturation-induced medium anisotropy just compensate each other. A second stationary state diametrically opposite to A is easily seen to be unstable. The 2M-dependence of the longitude 2ψ is then also explained.

A more precise analysis given in the next section confirms that this is essentially a correct interpretation but only part of the story. It will be shown that saturation-induced anisotropy also gives rise to driving forces with a component parallel to the equator on Poincaré’s sphere as depicted by the flow pattern in fig. 6. They are caused by nonlinear dispersive effects in the medium. The superposition of the flow patterns of figs. 5 and 6 yields the net flow pattern resulting from saturation effects and is shown in fig. 7. In the crucial experiment discussed above the superposition causes a displacement of the stationary point A.

It is of interest to note that the existence of the net saturation-induced flow pattern was postulated to account logically for the polarization-flip effect men-

\[ \text{[7]} \quad \text{H. de Lang, G. Bouwhuis and E. T. Ferguson, Physics Letters 19, 482, 1965.} \]
tioned earlier in this article, before a detailed quantum-mechanical theory of nonlinear polarization effects was available. We shall return to a discussion of the polarization flip in the next section.

Theoretical treatment

In a more quantitative description of polarization effects the electromagnetic field in the laser is thought of as a standing wave $\sin kz$, with angular frequency $\omega$ and an amplitude and phase slowly varying in time $[8] [9] [10] [11]$. In particular for the electric field

$$E(z,t) = \frac{1}{2} [E(t)e^{-i\omega t} + E^*(t)e^{i\omega t}] \sin kz; \quad (L = \pi n/k). \quad (20)$$

Here $E(t)$ is a slowly varying complex variable. The aim is to find the equations of motion for the two-dimensional vector $E_x(t), E_y(t)$. Since the mirrors are discrete elements in the laser, such a description is not immediately applicable. As long as the effect of the mirrors on the cavity waves is only small for each round trip, it can however be spread out over the cavity length so that the purely sinusoidal $z$-dependence in (20) is guaranteed. The spreading-out procedure consists in formally assigning anisotropic dielectric and lossy properties to vacuum in such a way that the polarization-dependent optical path length and the anisotropic losses caused by the mirrors are accounted for in the mean. The total dielectric polarization $P(z,t)$ at any point is then the sum of a fictitious polarization in vacuum and the actual dielectric polarization of the medium.

From Maxwell's equations for nearly plane waves where $\varepsilon_0$ is the permittivity of free space,

$$k^2E(z,t) + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} E(z,t) + \frac{\partial^2 P(z,t)}{\varepsilon_0 c^2 \partial t^2} = 0. \quad (21)$$

On applying equation (20) the second derivative of $E(t)e^{i\omega t}$ gives three terms. By choosing $\omega = ck$, $k^2E(t)e^{i\omega t}$ is cancelled, the mixed derivative is retained and the term in $\partial^2 E/\partial t^2$ is neglected because of the assumed slow variation of $E(t)$. If for $P(z,t)$ an expression analogous to (20) is used, the vector equation of motion is obtained in what is known as the "adiabatic" approximation:

$$\frac{2i\omega \partial E(t)}{c^2} = \frac{\omega^2 P(t)}{\varepsilon_0 c^2}, \quad (22)$$

together with its complex conjugate. In deriving (22) the largest term arising from the second derivative of $P(z,t)$ is retained.

Equation (22) contains only slowly varying quantities. In order to solve it the dependence of $P(t)$ on $E(t)$ must be known. $P(t)$ consists of the sum of three terms.

The first of these terms is the fictitious vacuum polarization $P_v$. It is linearly related with $E$ by means of a two-dimensional tensor $\tilde{A}$:

$$P_v = e_0 \tilde{A}E. \quad (23)$$

The dielectric permittivity $e_0$ of free space has been added to make $\tilde{A}$ dimensionless. The real part of $\tilde{A}$ must be chosen so as to account for the phase anisotropy of the mirrors, the imaginary part for the anisotropic losses $[12]$. Remembering that equation (17) was...
the change in $E$ per round-trip time $2L/c$ we find using (22) the following relation between $A_y$ and $a_y$:

$$a_y = -i\omega A_y/e.$$  

(24)

Since $a_y$ was of the order $10^{-4}$, $A_y$ is of the order $10^{-10}$.

The second term is the dielectric polarization $P_1$ of the medium as far as it is linearly dependent on $E$, i.e. characteristic for the unsaturated medium. In view of the isotropy of the unsaturated medium we have:

$$P_1 = S_1\epsilon_0 E,$$  

(25)

where $S_1$ is a complex, strongly frequency-dependent scalar. Its (positive) imaginary part is proportional to $E(t)$ and one factor $e^{iwt}$.

The third term is the nonlinear dielectric polarization $P_3$ of the medium, occurring in the first term of a series expansion of $P(z,t)$ in odd powers of $E(z,t)$ describing nonlinear effects. Remembering that $P(t)$ is the part of $P(z,t)$ approximately proportional to $\exp i\omega t$ and that an expression involving the third power of $E(z,t)$ produces terms proportional to $\exp i\omega t$ only if two factors $E(t)$ and one factor $e^{iwt}$ (i.e. $E^3$) occur, then

$$P_3 = \epsilon_0^2 S_2(E\cdot E*)E + \epsilon_0^2 S_3 (E\cdot E*)E*. $$  

(27)

The special form of (27) involving two complex scalars $S_2$ and $S_3$ is dictated by the requirement that the expression should be invariant for the choice of the coordinate axes $x$ and $y$, i.e. by the cylindrical symmetry of the laser medium. Note the absence of any quadratic terms in $E$ and $E*$. This is a consequence of the invariance symmetry of the medium.

The equation of motion now reads:

$$2i\frac{dE}{\omega \, dt} = \hat{A}E + S_1E + S_2\epsilon_0 (E\cdot E*)E + S_3\epsilon_0 (E\cdot E*)E*.  $$  

(28)

It is instructive to observe that at any time the right-hand-side vector can be resolved into a vector $P_{\parallel}/\epsilon_0$ parallel to $E$ and a vector $P_{\perp}/\epsilon_0$ perpendicular to $E$. Only the latter can cause a change in state of polarization. In other words the terms with $S_1$ and $S_2$ do not contribute to such a change. The component $P_{\parallel}$ to which all four terms in (28) contribute causes a change

in intensity by means of its out-of-phase part $P_{\parallel,\text{out}}$ and a correction to the frequency $\omega$ by its in-phase part $P_{\parallel,\text{in}}$.

With the aid of equations (19) and (28) the rate of change of the polarization-state parameters $\psi$ and $\chi$ can now be calculated. It is found that:

$$\frac{1}{\omega} \frac{d\psi}{dt} = \Gamma_1(\psi, \chi) + \text{Im}(S_2)(I/4) \sin \psi, $$  

(29a)

$$\frac{1}{\omega} \frac{d\chi}{dt} = \Gamma_2(\psi, \chi) - \text{Re}(S_3)(I/4) \sin \chi. $$  

(29b)

Here $\Gamma_1$ and $\Gamma_2$ depend on the elements of the matrix $\hat{A}$, and describe cavity anisotropies. Moreover $I$ is proportional to the intensity and is defined by

$$I = \epsilon_0(E\cdot E*). $$  

(30)

The time derivative of $I$ itself also follows from (28):

$$\frac{1}{\omega} \frac{dI}{dt} = \Gamma_3(\psi, \chi) + \text{Im}(S_1) + I \left[ \text{Im}(S_2 + S_3) - \text{Im}(S_3) \sin^2 \psi \right]. $$  

(31)

Here $\Gamma_3$ depends on the lossy part of the matrix $\hat{A}$. It reduces to a negative scalar for isotropic cavities. It equals $\epsilon/2oL$ times the cavity loss $L$ per round trip and its order of magnitude is therefore $10^{-3}$. In an anisotropic cavity the average empty-cavity attenuation $N_{av} = \text{Im}(A_{11} + A_{22})/2 < 0$ always predominates over the $(\psi, \chi)$-dependent part of $\Gamma_3$. In a laser $N_{av}$ is over-compensated by the positive $\text{Im}(S_1)$ representing the unsaturated deactivation of the medium. The excess fraction $F - 1$, which in a typical case amounts to about 10% is given by

$$F - 1 = \frac{|\text{Im}(S_3)|}{|\Gamma_{\text{av}}|}, $$  

(32)

the value of $\text{Im}(S_3)$ to be taken at the appropriate frequency $\omega$. Finally, the term in (31) proportional to $I$ is always negative and represents gain reduction by saturation, allowing stationary laser oscillation as discussed in an earlier section. Its order of magnitude is $(F-1)|\Gamma_{\text{av}}|$, i.e. $10^{-9}$. From this it follows that saturation effects are an order of magnitude larger, in the experiments discussed in this article, than interferometer anisotropies.

The terms proportional to $I$ in (29) represent the "driving forces" on the state of polarization due to saturation-induced anisotropy. In particular the term containing $\text{Im}(S_3)$ defines the flow pattern of fig. 5 and the term containing $\text{Re}(S_3)$ defines the pattern of fig. 6: their superposition is shown in fig. 7.

The stationary state in which a laser will eventually oscillate is determined by those values of $I, \psi$ and $\chi$ for

[12] Strictly speaking this is only true for reflection arrangements in which no magnetic materials or magnetic effects are involved. In this case the matrix $\hat{A}$ is symmetric. In the general case, the Hermitean part of $\hat{A}$ describes phase effects and the anti-hermitean part loss effects.
which all time derivatives in (29) and (31) are zero. These equations are quite complex because of their interrelation. In many practical cases, i.e. for not too small values of $F - 1$, saturation-induced effects are predominant. In the case of the helium-neon laser at $\lambda = 1.152 \ \mu m$ this means that $\chi$ is small, so that $I$ can be taken to be practically constant. Its value follows from (31) with $dI/dt = 0$ and $\chi = 0$:

$$I \approx \frac{\text{Im}(S_1) - |r_{01}|}{|\text{Im}(S_2 + S_3)|} \approx \frac{(F - 1) \text{Im}(S_1)}{F |\text{Im}(S_2 + S_3)|}.$$ \hspace{1cm} (33)

A constant value of $I$ greatly simplifies the discussion of (29).

All the constants in (28) can in principle be measured. The mirror anisotropies can be calibrated and the values of $S_1$, $S_2$ and $S_3$ can be determined by a careful study of the dependence of the stationary polarization states on the intensity. The quantities $S_1$, $S_2$ and $S_3$ can also be calculated with the aid of a quantum-mechanical treatment of the response of the medium to an electromagnetic field $^{101}$. Such a calculation also gives the frequency dependence: the imaginary (absorptive) parts of $S_1$, $S_2$ and $S_3$ are even functions of $(\omega - \omega_{ba})$, the real (dispersive) parts are odd functions of $(\omega - \omega_{ba})$, where $\omega_{ba}$ is the centre of the Doppler profile.

**Interpretation of some results**

The theory will now be applied to explain the polarization-flip effect mentioned earlier, on the basis of a small phase anisotropy of the mirrors, due for instance to stress birefringence in the mirror coating, and saturation-induced anisotropy. Without loss of generality we may take the $x$-axis as the anisotropy axis of the cavity, so that the real matrix $\hat{A}$ reduces to $A_{11} = - A_{22} = a$, $A_{12} = A_{21} = 0$. In that case one deduces:

$$I_1 = \frac{1}{2} a \sin 2\psi, \quad I_2 = - \frac{1}{2} a \cos 2\psi \sin 2\chi, \quad I_3 = 0.$$ \hspace{1cm} (34)

The corresponding flow pattern is given by the circles in fig. 8. The presence in equation (29a) of the term with $\text{Im}(S_3)$, assumed to be positive, drives the polarization state towards the equator and the eventual stationary state will be found to be plane polarized at $\chi = 0$, $2\psi = 0 \ or \ \pi$. In fact for these values $d\chi/dt$ and $d\psi/dt$ are zero. Apparently there are two solutions, the solution $\psi = 0$ being plane polarized in the $x$-direction, the one with $\psi = \pi/2$ in the $y$-direction. The corresponding stationary values $I_0$ of the intensity are identical, in agreement with the absence of loss anisotropy in the cavity. The question is which solution is stable.

In order to investigate this, consider the superposition of the flow pattern from interferometer anisotropy (fig. 8) and that from saturation-induced anisotropy (fig. 7) with $I$ given by (33). The resulting superposition is shown in fig. 9 for a small region around the stationary point ($\chi = 0, \psi = 0$). Looking on to the sphere around the other stationary point ($\chi = 0, 2\psi = \pi$) fig. 10 is obtained. The picture with the converging flow lines corresponds to the stable state, the diverging flow lines indicate instability.

Inspection of the figures 9 and 10 shows that the inclination of the saturation-induced flow lines with respect to the meridian (see fig. 7) plays an essential role in the considerations given above. The inclination exists because of the term with $\text{Re}(S_3)$ in equation (29b). A change in sign of this term would cause an inclination in the opposite direction, thereby interchanging the stable and unstable points. This is none other than the polarization flip; since $\text{Re}(S_3)$ is an odd function of $(\omega - \omega_{ba})$ tuning of the laser from $\omega > \omega_{ba}$ to $\omega < \omega_{ba}$ brings about this change of sign and makes the laser flip from one state of plane polarization to the other with an abrupt change in azimuth by $\pi/2$.

There exist interesting effects in the case $\omega = \omega_{ba}$ where $\text{Re}(S_3) = 0$. A discussion of these effects, bistability and hysteresis, can also be given on the basis of the general theory.

The analysis leading to equation (28) can be extended to the case in which a constant external magnetic field $B$ is applied. Limiting the situation to for instance an axial field, i.e. in the $z$-direction, additional terms are obtained. For small $B$ the main effect is on the relation between $P_1$ and $E_1$, notwithstanding the simultaneous appearance of $B$-dependent terms in the nonlinear part of the response of the medium. The main effect is the
Faraday rotation in the medium: the magnetic field causes a difference between the propagation velocities of circularly polarized light of opposite sense of rotation. In addition there may occur a difference in (negative) absorption coefficient, which will be ignored.

In this situation $S_1$ is no longer a scalar and (25) becomes

$$P_1 = \vec{b} \cdot E,$$

where $\vec{b}$ is an antisymmetric tensor ($b_{12} = -b_{21}$) with $b_{11} = b_{22}$. In particular $b_{12}$, for small $B$, is proportional to $B$. Its imaginary part $\beta$ describes the Faraday effect.

The flow pattern corresponding to this type of linear-medium anisotropy consists of circles parallel to the equator with a unique sense of rotation (see fig. 11). In (29b) an additional term appears:

$$\frac{1}{\omega} \frac{d\psi}{dt} = -\frac{\beta}{2}.$$  \hfill (36)

The stationary states of an isotropic laser in a magnetic field are clearly the two poles on the Poincaré sphere, each one corresponding to circular polarization. The presence of the saturation-induced anisotropy given in fig. 7 makes these points unstable. In fact there is no stable state of polarization. The laser polarization state will eventually perform a uniform rotation along the equator. It will show plane polarization, the azimuth being a linear function of time. Seen through a
tropy, such as shown in fig. 8. The equations (29) with
the additional term (36), combined with (34) then have
an approximate solution determined by

$$\chi \approx [2 \text{Im}(S_3)]^{-1} a \sin 2\psi \ll 1,$$  
(37a)

$$\frac{1}{\omega} \frac{d\psi}{dt} \approx - [2 \text{Im}(S_3)]^{-1} a \text{Re}(S_3) \sin 2\psi - \frac{1}{2} \beta,$$  
(37b)

if saturation anisotropy is dominant, i.e. if both \(\text{Im}(S_3)\)
and \(\text{Re}(S_3)\) are much larger than \(a/I\) and \(\beta/I\). The
solution (37a) says that the state of polarization proceeds
along an ecliptic passing through the eigenstates
cavity anisotropy and slightly inclined towards
the equator (fig. 12). Equation (37b) says that on this
ecliptic the dispersive saturation-induced anisotropy
\(\text{Re}(S_3)\) periodically makes itself felt, alternately:
creasing and decreasing the “Faraday” velocity \(\frac{1}{2} \omega \beta\).

As a result the motion along the ecliptic is no longer
uniform. While the intensity \(I\) remains practically con-
stant the output observed through a polarizer will be
a non-sinusoidal function of time (fig. 13).

For sufficiently large phase anisotropy \(a\), or suf-
ciently small \(\beta\), the continuous motion around the
ecliptic will be broken up and a stationary state
\(\frac{d\psi}{dt} = 0\) becomes possible. The laser then exhibits

polarizer a periodic output will be observed with a modu-
lation frequency proportional to \(B\) which amounts to
50 kHz for \(B = 1\) gauss, corresponding to \(\beta\) of the
order \(10^{-10}\) for \(B = 1\) gauss. In this situation the equa-
torial flow line in fig. 11 acquires a very real physical
significance.

Interesting effects occur when, in addition to a mag-
netic field, there is a small interferometer-phase aniso-

Fig. 11. Flow pattern due to Faraday rotation in the laser me-
dium as caused by a longitudinal magnetic field.

Fig. 12. Periodic motion of mode polarization along an ecliptic. The component flow patterns
are those of figs. 7, 8 and 11.
nearly plane polarization with an azimuth \( \psi \) sensitively depending on the the ratio \( \beta/a \). All these effects have been observed and interpreted \([3] [4] [5] [9] [10] [11]\). In actual fact these experiments with a magnetic field were the ones that first made us introduce the concept of saturation-induced anisotropy.

Quantum-mechanical considerations

The physics behind saturation-induced anisotropy lies in the behaviour of the individual atoms, which can only be properly described in quantum-mechanical terms. Nevertheless it is possible to make clear what is actually going on. To do this let us consider an atom of which both the upper state (b) and the lower (a) have angular-momentum quantum number \( j = \frac{1}{2} \). This is not the situation in the helium-neon laser discussed previously, but we shall return to this later. Both states are twofold degenerate and the sublevels can be distinguished by the projection of \( j \) on the \( z \)-axis, i.e. one has \( a_\uparrow \) and \( a_\downarrow \) and the other \( b_\uparrow \) and \( b_\downarrow \). It is a matter of simple symmetry considerations that optical transitions by light propagating in the \( z \)-direction are only possible between the pair \( a_\uparrow \leftrightarrow b_\uparrow \) and \( a_\downarrow \leftrightarrow b_\downarrow \), the former only interacting with, say, the clockwise-rotating circular component, the latter pair only with the anticlockwise circular component of the light. The (negative) absorption coefficients of the clockwise and anticlockwise components are therefore determined by the relative overpopulation of \( b_\uparrow \) over \( a_\uparrow \) and of \( b_\downarrow \) over \( a_\downarrow \) respectively.

Suppose that at some initial time an elliptical state of polarization is present, in which the clockwise circular component has a larger amplitude than the anticlockwise one. Saturation then causes a greater reduction in overpopulation of the \( a_\uparrow \leftrightarrow b_\downarrow \) pair than in the other pair. As a result the anticlockwise component will grow relative to the clockwise component, causing the elliptical polarization to become more plane polarized. This, in essence, is the explanation of the equation-directed saturation-induced driving force on the Poincaré sphere for a \( f - j = \frac{1}{2} \) transition.

In the laser with \( \lambda = 1.152 \ \mu m \) a transition between a threefold-degenerate level with \( j = 1 \) and a fivefold-degenerate level with \( j = 2 \) is operative. We have therefore worked out a detailed theory \([8]\) for calculating the values of \( S_1, S_2 \) and \( S_3 \) for any possible set of integer or half integer \( j \) values \((j,j')\) satisfying the selection rule \( j' - j = 0 \) or 1.

It is not unexpected that the values of \( S_1, S_2 \) and \( S_3 \) turn out to be proportional to \( N_b - N_a \). Unfortunately this is a quantity that depends strongly on experimental conditions and it cannot even be directly measured. The ratios, such as \( S_2/S_1, S_3/S_1, \) or \( \text{Im}(S_3)/\text{Im}(S_1) \), will turn out to be proportional to \( F_j/\bar{F}_{j\text{avg}} \). Therefore \( \text{Im}(S_3) \) can be expressed in terms of \( P \) and \( \bar{F}_{j\text{avg}} \). The detailed calculation gives

\[
\text{Im}(S_3) = F_j/\bar{F}_{j\text{avg}} \| p_{ab}^2 \left[ (\omega - \omega_{ab})^2 + 2\gamma_{ab}^2 \right] \left[ \gamma_{ab}^2 + \gamma_{ab}^2 \right] g(j),
\]

where \( p_{ab} \) is the electric-dipole matrix element between the upper and lower states and \( g(j) \) is defined by

\[
g(j) = -\frac{1}{5} (j - 1) (j + 2) \quad \text{for a } j \leftrightarrow j \text{ transition} \\
g(j) = +\frac{2}{5} j(j + 2) \quad \text{for a } j \leftrightarrow j + 1 \text{ transition}
\]

Equations (38) and (39) are particularly interesting for the following reasons. Until now it has been implicitly assumed that \( \text{Im}(S_3) \) was a positive quantity, thus providing saturation-induced preference for plane polarization. Indeed, (39) shows that for the neon \( \lambda = 1.152 \ \mu m \) transition \( j = 1 \leftrightarrow j = 2 \) transition, as well as the \( j = \frac{1}{2} \leftrightarrow j = \frac{1}{2} \) transition considered above, \( \text{Im}(S_3) \) is positive. However, this is not always the case. From (39) the rules are \([8]\):

\[
\text{Im}(S_3) > 0 \text{ for } \frac{1}{2} \leftrightarrow \frac{1}{2} \text{ and all } j \leftrightarrow j + 1 (j > 0),
\]

\[
\text{Im}(S_3) < 0 \text{ for } j \leftrightarrow j (j > 1),
\]

\[
\text{Im}(S_3) = 0 \text{ for } 0 \leftrightarrow 1 \text{ and } 1 \leftrightarrow 1.
\]
This striking result of the theory predicts three possible types of saturation-induced anisotropy. It may either lead to preference for plane polarization (case 40a), or to preference for circular polarization (case 40b), i.e. equal preference for the poles on the Poincaré sphere. Finally saturation-induced anisotropy may be absent (case 40c), in which case the dispersive part Re(S₃) also vanishes.

The theoretical predictions induced us to investigate a \( j = 2 \leftrightarrow j = 2 \) transition in the helium-neon laser, which is operative at a wavelength of \( \lambda = 1.207 \ \mu \text{m} \). The experiments did indeed show that in this case a strong preference for circular polarization exists and that bistability corresponding to the two poles on Poincaré's sphere occurs [13] [5].

Experiments [13] [8] on the \( \lambda = 1.523 \ \mu \text{m} \) \( j = 1 \leftrightarrow j = 0 \) transition however did not confirm the neutral behaviour predicted by theory. Instead a clear preference for circular polarization was found, though of a much smaller magnitude than for the cases of type (40b). This residual effect could be explained [14] [15] by a mechanism not considered so far, the relaxation mechanism between the levels of a degenerate \( (j = 1) \) state. We could show that, if the nature of the operative collision processes is such that angular momentum relaxation of the degenerate level is faster than quadrupole relaxation, the observed preference is obtained.

It is of interest to note that, although in a quite different context and in a different atom, a difference between the two relaxation rates has already been found by other workers.

In this article it has been the aim of the authors to build up what is in their view a coherent picture of the subject. Full reference to important work of other workers in the field can be found in the articles quoted.

Prof. Casimir presenting an award to a group of girl participants in the Final of the European Contest for Young Scientists and Inventors, an annual event arranged by Philips and held at Eindhoven in the Evoluon.