A flexible method for automatic reading of handwritten numerals

M. Beun

I. General description of the recognition method
II. Thinning procedure and determination of the special points

For the last twenty years or so, efforts have been made in various parts of the world to devise automatic methods of reading handwriting, so that large volumes of handwritten data for clerical processing can be fed direct to a computer. The present article describes an automatic method of reading numerals. In an experimental equipment using this method, which is based on a few simple principles, more than 91% of a collection of 10,000 numerals written by hand without any restriction were read correctly, some 3% were read wrongly and 6% were rejected as unrecognizable. Since the numerals were not written particularly well (they were obtained in a door-to-door survey) this result compares very well indeed with the results of other methods, and in fact the flexibility of the method allows further considerable improvement. The author comes to the conclusion, however, that the reliability in the automatic reading of forms with many numerals written on them (such as giro cards) will never be completely adequate unless the people who have to fill in the forms are prepared to give a little cooperation. In part I of the article the recognition system is described in general terms; in part II the author deals at greater length with some important details of the procedure.

I. General description of the recognition method

Introduction

Information to be fed into a computer is frequently presented in the form of handwriting. A familiar example in some countries is the giro card, which is filled in by the account holder, usually by hand. The problem is by no means limited, however, to the giro service or to the cheque and payment-transfer departments of banks. There are very many other applications of the computer, both administrative and scientific, in which the input consists of large quantities of handwritten data. At the present time the data are usually processed by hand to bring them into a form in which the computer can understand them. In the giro service, for example, many hundreds of punch operators are occupied in this kind of work. Mark sensing is another system that has been used, in which marks made in predetermined positions on documents can be directly read by the computer, either optically or magnetically. Filling in these marks presents too many problems, however, and they are also difficult for people to read back. There is clearly a need for a machine that can put data into a computer by reading handwriting directly.

For most practical purposes numerals are adequate. Giro services, for example, can manage with the

Ir M. Beun is with Philips Research Laboratories, Eindhoven.

[+] Part II of this article will appear in the next issue.
amount and two account numbers. For many other applications the information can be presented in a numerical code. In some countries the post office now asks people to add a numerical 'postcode' for the town to the address on the envelope. The considerable research that has already been done on optical character recognition has therefore usually been limited to this image is electronically transformed into a matrix of occupied and unoccupied positions (fig. 2). The occupied positions (black in fig. 2) reproduce the numeral, the unoccupied positions (grey) reproduce the surroundings. This matrix can easily be stored in the memory of a computer and is therefore very suitable for electronic processing.

Fig. 1. Example of the test material used in our experiments. Test subjects were asked to write any 70 numbers on a sheet of paper with printed lines enclosing spaces measuring 5 by 7 mm. Four of these sheets are shown here. The first was filled in by the author himself with carefully written numerals. These were read with no errors using the method described here and a very simple recognition procedure.

numerals, and even in many cases to stylized numerals. In our research we have concentrated on the automatic recognition of non-stylized numerals, our aim being to impose no restrictions whatever on the handwriting. The principle of our system will now be described, and it will then be compared with the considerable work by others in this field.

Principle of the recognition method

Fig. 1 gives an idea of the types of numerals used in our experiments. The only rule laid down is that the numerals must be written on a preprinted form, with no more than one numeral per space. Before applying the recognition procedure the numerals are put into a form in which they can readily be processed electronically. This is done by means of a television camera. The camera produces an image of the numeral, and no knowledge of computers is required for the understanding of this article. The reader need only think of a drawing of a matrix in which occupied and unoccupied positions are distinguished from each other in some way, for example by making the occupied positions black and the others white.

Very briefly, the recognition process takes place in the following phases. First the numeral is 'thinned' to a skeleton (skeletonized), i.e. reduced to a pattern that is nowhere thicker than one square. Next the end points and forks or junctions in this pattern are found, and then the numeral which the camera has read is determined from the shape of the skeleton, and in particular from the relative positions of these special points. In this last phase, the actual recognition process, a procedure is used that is drawn up after studying the characteristics of a large number of numerals written by test subjects. By continuously
checking this recognition procedure against numerals written by other people, and if necessary introducing new criteria, the accuracy can be steadily improved. This is what gives our method its flexibility\[1\].

The operations described can all be performed by a computer. Working with a computer has great advantages in the development of the recognition method, since it enables a quick test to be made of the effect of changes in the thinning procedure or in the recognition scheme. With a final version of the method and development of a machine for practical use, it could well be advantageous to design a special electronic system capable of performing the necessary operations faster than a computer. Since we are for the present only concerned with the development of the method, these questions fall outside the scope of this article.

Among the recognition methods that others have developed [2] there are some that work as follows. First, the numeral to be read is enclosed in the smallest possible rectangle, which is then divided up into a grid of smaller rectangles, and some of the characteristics of the line sections of the numeral in the various spaces of the rectangular grid are determined. These characteristics (e.g. the direction of the line section, or the curvature, or the presence of a junction) are compared with the characteristics of the corresponding

![Fig. 2. Matrix of occupied (black) and unoccupied (grey) positions, the occupied positions forming the numeral to be recognized. A matrix of this type is very suitable for storage in a computer memory and further electronic processing.](image)

line sections of a number of reference numerals. Next, by applying certain criteria, the figure is determined from the correlations thus found. Some of these methods are not very flexible and they are therefore generally used for stylized, or at least 'well written' numerals. The basis is usually the 'thick' numeral [3], but various methods of skeletonization have been reported, not only of handwritten numerals [4] but also of chromosomes [5]. In two cases an attempt has been made, in much the same way as in our method, to use the special points of the skeleton for the recognition of handwritten characters [6]; as far as we know, however, these investigations have not led to practical results. A machine for reading non-stylized numerals [7] has been developed in Japan; this is used for reading postcodes on letters. These postcodes consist of three numeral digits, and the machine reads 70% correctly and 2% wrongly, while 28% are rejected as unrecognizable.

The good results that we have obtained with our method are in no small part due first to the applications of a special thinning procedure that makes use of

\[1\] In this article it will be seen that in addition to the skeleton we sometimes had to use the unthinned numeral; most of the criteria that were then applied were derived from the work of E. de Boer and F. L. A. M. Thissen of this laboratory.


E. S. Deutsch, Thinning algorithms on rectangular, hexagonal, and triangular arrays, Comm. ACM 15, 827-837, 1972 (No. 9).


K. Mori, H. Genchi, S. Watanabe and S. Katsuragi, Microprogram controlled pattern processing in a handwritten mail reader-sorter, Pattern Recog. 2, 175-185, 1970.
the work of P. Saraga and D. J. Woollons\[8\] and secondly to the flexibility built into it by allowing new criteria to be added continuously to the recognition procedure.

The description of our method will now be continued, followed by an examination of the provisional results obtained in an experimental arrangement used for the recognition of several thousand numerals. Part I of the article includes an evaluation of these results, plus the findings of a small experiment designed to find out how accurately people could read the numerals.

1) break points, i.e. points that leave a gap in the line when they are removed;
2) end points, i.e. points that have only one neighbour;
3) loop points, i.e. points that give a loop if they are removed (the loop is formed by the four neighbouring points).

A skeleton therefore consists exclusively of break points, end points and loop points. The loop points are fairly rare; an example is the crossing point in fig. 5b.

Making a good skeleton is not so easy as it may seem, since the shape of the skeleton depends to a great extent on the thinning procedure adopted. We need a

![Fig. 3. The skeletonization process. The pattern of positions forming the figure (a) is thinned in a number of steps (b and c) by removing points around the edge of the numeral. Points that must not be removed are end points and points whose removal would cause a break in the pattern. The thinning process is continued until there are no more points that can be removed (d). The figure then remaining is the skeleton.](image)

**Skeletonization**

We start from the situation where the numeral is present in the form of a matrix with occupied and unoccupied positions. Fig. 3 shows how a skeleton is made by successively removing points on the edge of the original shape. This thinning process is continued until the skeleton only contains points belonging to one of the following categories:

- break points, i.e. points that leave a gap in the line when they are removed;
- end points, i.e. points that have only one neighbour;
- loop points, i.e. points that give a loop if they are removed (the loop is formed by the four neighbouring points).

A skeleton therefore consists exclusively of break points, end points and loop points. The loop points are fairly rare; an example is the crossing point in fig. 5b.

Making a good skeleton is not so easy as it may seem, since the shape of the skeleton depends to a great extent on the thinning procedure adopted. We need a
The shape of the skeleton also depends, of course, on the initial pattern, and thus on the way in which it is formed from the television picture. Since we only have occupied and unoccupied positions in the matrix, the grey tints will be lost from the television picture; above a certain threshold value they will be regarded as black, and below it as white. The choice of this threshold value may affect the shape of the skeleton produced from the pattern. In our investigation this threshold was given a fixed value, but a better result would be expected if the threshold value were allowed to adjust itself automatically to the blackness distribution in the television picture.

We have successfully developed a thinning procedure that produces very useful skeletons (see fig. 5a, b and c). Even these skeletons may sometimes have a 'tail', but this always corresponds to a distinct bulging of the unthinned numeral, as can be seen in the top right-hand corner of fig. 5c. The details of our thinning procedure will be given in part II of this article, which will also give a description of the method of determining the special points of the skeleton.

A simple recognition procedure

To recognize numerals of the types shown in fig. 6a it is sufficient to use the simple recognition procedure of fig. 6b. In this the numerals are first divided into groups corresponding to the number of end points found in the skeleton — this number will be denoted by $E$ — and to the number of fork points — denoted by the letter $F$. If such a group comprises different

![Fig. 4. Example of a skeleton that cannot be used for the recognition procedure because it has a number of 'tails' that have nothing to do with the basic shape of the figure to be thinned. The picture shown here was made by the line printer of the computer used for the skeletonization process. The dots indicate the points of the original figure that were removed during the skeletonization. After the print-out, the skeleton points were joined up by hand to make the skeleton visible.](image1)

![Fig. 5. Three examples of skeletons that are suitable for our recognition procedure. The figures were traced out in the same way as in fig. 4 by the line printer, but in addition the unoccupied positions of the matrix have now been marked with an O. After printing, the skeleton points were marked by hand and the edge of the original figure was drawn in. Crossings (four-junctions) in the original numeral usually degenerate into two three-junctions (fork points) (a); if the skeleton does have a four-junction (b) it is counted as two fork points. The skeleton of (c) has a tail at the top on the right that corresponds to a marked bulge in the pattern.](image2)

[8] See the first article of note [4].
[9] See the first article of note [2].
(E,F)-GROUP   TYPE OF NUMERAL   PROCEDURE   RECOGNIZED AS

(0,0)   0   0

(0,2)   8     8

(1,1)   6     9

Is e above f?  yes → 6
  no → 9

(2,0)   1     2     3

Is W equal to H?  yes → 1
  no → 2

2 3 5 7

Is e₁ on the right-hand edge of the frame?  yes → 5
  no → 2

2 3 7

Is e₂ on the right-hand edge of the frame?  yes → 2
  no → 3

3 7

Is e₂ on the lower edge of the frame?  yes → 7
  no → 3

(3,1)   2     4     5

3 4 7

Is f in the upper third of the right-hand half of the rectangle?  yes → 7
  no → 3

2 3 4 5

Is f in the lower third of the rectangle?  yes → 2
  no → 4

3 4 5

Is f to the right of the three end points?  yes → 3
  no → 4

4 5

Are e₁ and e₂ above f?  yes → 4
  no → 5

5

Is e₁ on the right-hand edge of the frame?  yes → 5
  no → rejected

Fig. 6. a) Collection of carefully written numerals, for some of which two widely used types are included. This is of course only a very small selection from the many possible ways of writing these numerals. b) Procedure for recognizing numerals of the types in (a). The numerals are first classified by the number of end points (E) and the number of fork points (F). If a numeral belongs to an (E,F) group that includes more than one numeral, a subsidiary recognition procedure is used to distinguish between these numerals.
In addition to $E$ and $F$ there is a third and equally obvious characteristic of numerals; this is the number of closed loops. We do not use this characteristic in our method, but that does not mean that we neglect any essential information, for there is a simple relation between the quantities $E$ and $F$, the number of loops $L$ and the number of separate sections $S$ in the figure. This relation, which is easy to verify, is:

$$F - E = 2(L - S).$$

Normally the pattern (the numeral) is in one piece, so that $S = 1$. Once we have determined $F$ and $E$, the difference $L - S$ is then a fixed quantity, but not $L$ and $S$ themselves. This is easily understood, for if a closed loop is added as a new section, $F$ and $E$ do not change, but $L$ and $S$ are both increased by 1.

We shall now deal in turn with the various groups (see fig. 6b).

$$E = 0, F = 0 \ (\text{group } 0,0).$$

Only a nought can possibly give a skeleton of this kind. The noughts are thus recognized (since here we are only concerned with the carefully written noughts of fig. 6a).

$$E = 0, F = 2 \ (\text{group } 0,2).$$

This applies only to the eights, which are thus recognized with this group.

$$E = 1, F = 1 \ (\text{group } 1,1).$$

This applies to the sixes and the nines. A simple and sufficient criterion for making the correct choice is found in the relative positions of the end point and the fork. If the end point $e$ is higher than the fork $f$, we then have a six, otherwise we have a nine.

$$E = 2, F = 0 \ (\text{group } 2,0).$$

This group includes the one, and also the two, the three, the five and the seven of the top row in fig. 6a, none of which have a fork. To distinguish between these numerals we first count the total number of points (squares) forming the skeleton. We call this number $W$ (for weight). Next we count the number of rows that the skeleton covers in the matrix, and we call this number $H$ (for height). For all ones we have: $W = H$, but this does not apply to the other numerals. We have thus eliminated the ones.

To select from the other possibilities, we number the end points in the sequence in which they are encountered when the matrix is scanned row by row, beginning with the top row and moving from left to right. The end points are given the symbols $e_1$ and $e_2$. We then enclose the skeleton in a rectangle formed from the horizontal and vertical lines that touch the skeleton. If $e_1$ is at the right-hand edge of this frame, the numeral must evidently be the five from the top row of fig. 6a; if, however, $e_2$ falls here, then the numeral is a two. If neither $e_1$ nor $e_2$ are at the right-hand edge of the frame, the decision is between the three and the seven. Because of the limitation to the types of numeral given in fig. 6a, we need in this case only look to see whether $e_2$ is at the bottom edge of the frame. If this is so we then have a seven, otherwise it is a three.

$$E = 3, F = 1 \ (\text{group } 3,1).$$

This group includes the two fours, and also the two, the three, the five and the seven of the lower row in fig. 6a. We look first to see whether $f$ (the fork) lies in the right-hand half of the upper third of the rectangle. If it does, then we have a seven. Next we see whether $f$ is in the lower third of the rectangle. In this way we identify the twos. We then see whether the three end points are all on the left of the fork, which enables us to recognize the threes. We find the fours by seeing whether $e_1$ and $e_2$ both lie above $f$. Any numeral that is still not classified can only be a five. We have included a further safety criterion here, however, to make sure that $e_1$ does in fact lie on the right-hand edge before a five is decided upon. If it does not, the numeral is rejected as unrecognizable.

In the procedure of fig. 6b all numerals are rejected that have different numbers of end points and forks than those that correspond to one of the five combinations above. If the only numerals presented are of the types included in fig. 6a as in the original requirement, no numeral will be rejected, and the safety criterion is then superfluous. Our purpose in adding it was simply to introduce the concept of 'safety criterion', since in general a procedure for recognizing numerals written without restrictions can contain a large number of such safety criteria.

If the recognition procedure described here is applied to an arbitrary set of numerals, the result will be a number of correct readings, a number of errors (substitutions) and a number of reject. All the numbers in the first block in fig. 1 were correctly recognized, but these 70 numerals were written with the procedure of fig. 6 in mind. It does however show that the automatic recognition of handwritten numerals would be a fairly simple matter if the people that write them could be persuaded to observe a few simple rules. We shall return to this point later on in the article.
The success of our method is largely due to the subdivision into groups before the actual recognition begins. The only characteristics used for this classification are the number of end points and forks. The recognition of arbitrarily written numerals requires a complicated scheme for each of these groups, to be able to distinguish the many numerals belonging to them. We could increase the number of groups by using various other appropriate characteristics of numerals in the classification, such as the presence of a 'tail', etc. The groups would then be smaller and the subprocedures probably simpler. Perhaps a better result might be obtainable in this way for the whole system than with a classification based only on special points. The possibilities of extending the number of groups in such a manner have not yet been fully examined.

Refining a recognition procedure

In developing a recognition procedure for numerals written without restrictions we start from a simple procedure like the one described in the previous section. This is applied to a number of numerals, some of which will be read wrongly or rejected. We now devise some new criteria to enable these numerals to be properly recognized as well, and add these criteria to the procedure. We then apply this modified procedure to a new group of numerals, and again introduce modifications that will permit all these new numerals to be recognized. Since this process is repeated a few times the procedure will gradually improve, but at the same time it will become more complicated. In making adjustments we will undoubtedly devise criteria that could more usefully have been added to the procedure earlier. When it becomes obvious that the whole procedure is becoming impossibly complicated, the time has come to throw it into the wastepaper basket and then, armed with the experience gained, to start on a new one. The new procedure will be more effective than the old one, and at first simpler, but after some time this procedure will also have to be discarded to make way for yet another one. This discarding is one of the most important but also one of the most difficult processes in the development of a recognition procedure.

The adjustment process will be explained with a single example, showing at the same time that the use of the skeleton by no means implies a limitation to the end points and the forks, but that in fact other special points can also usefully be employed for recognition. We consider the nine and the nought from fig. 7. On attempting to recognize these numerals with the aid of the simple procedure described in the previous section, we see that they both belong to the group (1,1) and both are classified as nine, which is of course unacceptable. We now scan the matrix row by row, beginning with the row on which e lies and then moving up to the row on which f lies. We look for the first row on which only one skeleton point lies, and once we have found that point we mark it as the special point p. In fig. 7a the point p coincides with e, in fig. 7b we find no point p. Next we scan the matrix again, now moving downwards row by row beginning with the row below f, and we go no further than the row with e. Here again we look for the first row with only one skeleton point; if such a point exists we call it q. We find such a point q in fig. 7a, but not in fig. 7b. In the further process of recognition we can now make use of the existence or non-existence of the points p and q, and of their positions. To solve the problem of fig. 7 it is sufficient to ask whether a point p exists. If it does, we have a nine (the numeral then has a tail projecting downwards), otherwise we have a nought. With this extension the
recognition procedure is still far from perfect, but it is slightly improved.

To recognize a numeral we have so far confined ourselves to the characteristics of the skeleton, but this is not possible in all cases. It regularly happens that the typical features of the original numeral get partly or completely lost in the skeleton. Let us take one example. Fig. 8 shows a nine whose skeleton, owing to a slight discontinuity on the right-hand side at the top in the ‘thick’ numeral, has the same basic shape as a five. A choice between these two numerals, purely on the grounds of the skeleton, is very difficult in this case; we must therefore use the thick numeral. We could for example add a criterion that enables us to decide, from the size of the break in the loop in the thick numeral, whether it is a five or a nine.

An unfortunate aspect sometimes encountered in the development of a recognition procedure is that an improvement in one point makes another point worse. It is therefore dangerous to let the procedure include a type of numeral that occurs infrequently if it at all resembles a frequently used type of another numeral (inclusion of the nine from fig. 8 endangers the recognition of the five). It is best to delay this until very many numerals (e.g. hundreds of thousands), of widely different origin have been tested.

Provisional results

In total, more than 25 000 written numerals have been used in our investigations. Eighty to ninety per cent of these numerals present no problems with our method. The remainder show a wide variety of less conventional shapes, some of which were encountered very infrequently, others more often, although they were then usually from the same person. From the limited number of numerals available to us we were not able to conclude anything about the frequencies with which these aberrant forms of numeral will occur in practice. Consequently it is not possible to say exact what percentage of correctly recognized numerals will be obtainable with our method. Nevertheless some general conclusions can be drawn about the practical usefulness of the method.

During our investigations various recognition schemes were developed. The most extensive scheme now available at Philips Research Laboratories for the recognition of arbitrary numerals was designed from more than 15 000 handwritten numerals. Most of these were placed at our disposal by the Dr Neher Laboratory of the Netherlands Post Office; the remainder came from forms that were filled in by colleagues and by visitors to demonstrations. The recognition scheme was checked against 10 000 numerals that were not involved in the development of the scheme. These numerals also came from the Dr Neher Laboratory. This test material was obtained in a door-to-door survey, in which people were asked to fill in a particular series of numerals in spaces on a printed form. With this method it was known what numeral the writer intended each time. No restrictions were imposed on the handwriting, and since the people filling in the forms had no personal stake in the results, the average quality of the handwriting was fairly poor (see fig. 9). The result of the test is presented in Table 1; it can be seen that 91.37% of the numerals were recognized, 2.67% were read wrongly (confused with another numeral) and 5.96% were rejected as unrecognizable.

Table 1. Results of automatic reading of 10 000 numerals. Each space shows how many numerals which the writer meant to be read as the numeral on the left were read as the numeral above. The last column contains the numbers of rejected numerals; the numbers of numerals read correctly appear along the diagonal.

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Correctly Read: 9137 numerals (91.37%)
Incorrectly Read: 267 numerals (2.67%)
Rejected: 596 numerals (5.96%)
The origin of the numerals has been described at some length because in judging the performance of a machine for recognizing handwriting it is of the utmost importance to know exactly what test material was used. This seems obvious but we have found by personal experience how easy it is to deceive oneself and others in this kind of experiment. In fact, a character-recognition system can only be adequately judged when it has been used in practice for millions of numerals.

Evaluation of the results

A result of not quite 92% of numerals recognized is of course far from sufficient for practical purposes. By including large numbers of new numerals in our investigations we would certainly be able to increase the percentage quite considerably, but the effort this costs increases with the improvement achieved. In our opinion it will never be possible to make the step from 99 to 100%. To see whether there is any point in seeking further improvement, we have tried to determine what a particular percentage of correctly recognized numerals means in practice.

Reading forms

In general, the forms that have to be read have not one but several numerals written on them. The requirement then is of course that all the numerals are recognized. If however the individual numerals are not recognized with 100% certainty, then the percentage of forms read correctly will decrease rapidly as the number of numerals written on them increases. For example, if the percentage of numerals recognized is 98, then 90% of forms with 5 numerals will be correctly read, but only 78% of forms with 12 numerals. To read correctly 99% of forms with 12 numerals (the average on Dutch giro cards) it will be necessary to recognize no less than 99.9% of the individual numerals. This seems an impossible task; even a human reader will not be able to reach this percentage if he has no means of making any comparison between the numerals written on a form — which the machine does not do.

The various percentages are calculated in the following way. A form on which a number of numerals are written is only correctly read if all the numerals are correctly recognized. It is rejected if one or more numerals are rejected and it is wrongly read if one or more numerals are wrongly read and no numeral is rejected. If we take the proportion of correctly read numerals as $c_n$, the proportion rejected as $r_n$, and the proportion read in error as $e_n$, we can find the corresponding proportions $c_r$, $r_r$ and $e_r$ for forms with $m$ numerals from:

$$c_r = c_n^m,$$

$$r_r = 1 - (1 - r_n)^m$$

and

$$e_r = 1 - (c_n + r_n).$$

Fig. 10 gives a plot of the curve

$$y = 1 - (1 - x)^m$$

for a number of values of $m$. Given $c_n$ and $r_n$ we can easily read the proportions $c_r$, $e_r$ and $r_r$ from this graph.

In making these calculations we assumed that the numerals that were difficult to recognize were randomly distributed over
the forms. It is not unreasonable to expect, however, that in practice the rejected and wrongly read numerals will tend to be concentrated on cards filled in by people who write carelessly. This would make the number of wrongly read forms much lower than follows from the calculation. In fact, however, it turned out that this was not the case at all — at any rate with the procedure used for the test.

A human reading test

To get some idea of the accuracy with which people can read the numerals used in our investigations, 35 people were subjected to a reading test. In this test 840 numerals were used taken at random from the group of 10000 numerals with which we tested our recognition method. Some of these 840 numerals are shown in fig. 9. The figure indicates the method of presentation; the subjects could not see which numerals had originally appeared together on the same form, so that they were not able to recognize numerals by comparing them with other numerals in the same handwriting — a possibility that does not occur in our recognition method either. Fig. 11 gives the result of the test; on average 98% of the numerals were correctly read, 1.3% were read wrongly and 0.7% were rejected. The same 840 numerals were read by the machine (using the same recognition scheme as tested above) and the percentages found were 91.7, 2.6 and 5.7 (point M in fig. 11).

The result of the reading test shows that the machine does not as yet rival the capability of a human reader. In comparing man and machine, however, we should bear in mind that the results of the reading test are somewhat biased by the fact that in doubtful cases a human reader can guess, which the machine does not do. With the percentages found in the reading test with human readers, only 78% of forms with 12 numerals are read correctly, 13.5% wrongly and 8.5% are rejected (see the numerical example in fig. 10). This does not of course mean that people are unable to read giro forms; on average the numerals will be written more clearly than our test numerals were, but the most important point is that people can compare the numerals on a form.

Provisional conclusions

Without wishing to draw any final conclusions from the above, it does seem that the results clearly indicate that the problem of reading giro cards entirely by machine — the problem always in mind during the investigation — cannot be solved simply by further improvement of the recognition procedure for individual numerals. It is hardly likely that the performance of the machine will ever be better than the average result of our 35 test subjects, and even this result gives little reason for optimism. This does not imply, however, that the method in its present form is useless. In the giro service in the Netherlands all cards are pro-

![Fig. 11. Result of a reading test in which 840 numerals were read by 35 people. The numerals were taken at random from the material used to test our method (see fig. 9). The number of rejected numerals R is plotted along the horizontal axis, and the number of numerals read wrongly E is plotted vertically. The lines at 45° give the proportion of correctly read numerals c%. Each point gives the result for one person; a circle around a point indicates that two people scored the same result. An approximate relationship between the number of rejects and errors can be obtained from the points (see the solid curve). For comparison the figure also shows the result given by our recognition procedure with these 840 numerals (point M.)](image-url)
alternatives, which may occur with numerals like 1 and 7. All this, however, is still rather speculative. As far as we can see the situation at present, there is no likelihood as yet of faultless automatic recognition of numerals written without restriction, and a little cooperation from the public will be needed to obtain complete automation.

Where an automatic reader is to be used exclusively within a closed organization, for example only for the administration departments of a firm, the handwriting can then be subjected to constraints designed to ensure that the automatic reader will recognize all numerals correctly. This can be done by choosing types of numerals that are suited to the writing habits of the people in the department, and programming the machine with a recognition procedure that recognizes all these types of numerals. If the reader now rejects numerals, or makes errors, it means that someone has not kept to the rules. The machine has then been adapted to the users. One can of course adopt the opposite reasoning and design an automatic reader that recognizes only a limited number of types of numeral. The users must adapt themselves to the reader but the recognition procedure can be made so obvious that the users can easily understand it and can easily learn how to write the numerals.

At Mullard Research Laboratories, which are affiliated with us, a simple recognition procedure based on our ideas has been developed for use in a character-recognition system. This system (the Philips X 1300), which is primarily intended for reading standard (OCR) characters, is being manufactured by the M.E.L. Equipment Co., Ltd., a part of the British Philips Group.

A system imposing constraints on the freedom of writing could also be devised for giro services. The constraints imposed in such a case, however, would have to be kept to the minimum. The principal requirement would be that forms on which the writing did not follow the rules would have to be rejected by the automatic readers. This could perhaps be arranged by providing the giro card with a special space in which the account holder makes a mark to show that he has observed the rules and agrees to automatic reading. All the cards are then read automatically, and the automatically read numerals from the marked cards only are used in the giro records. The numerals from the unmarked cards are also read by human readers and only their results are used in the giro processes. However, the automatically read numerals are given on the account holder's statement. In this way the account holder has a chance of becoming familiar with automatic reading before consenting to the use of the system. In return for the care required from the account holder he could be offered some recompense, such as faster handling of his payment orders.

A high degree of certainty that a form on which the writing does not follow the rules will be rejected can be obtained by combining the recognition system with a number-entry system proposed by L. van der Toorn [10]. To illustrate this system let us take as an example a case in which the number 2503 is to be filled in. A strip of spaces like that shown in fig. 12 is used; first we look for the space containing the numerals 2 and 3. These numerals indicate that a 2 may be entered in this space, or a 3, but no other numeral. The first digit of our number is therefore written down in this space. Next we look along the strip — going to the right — to find the first space in which we can place the 5 of our number and so on. While the numbers remain easily readable for people, the task of the machine is lightened enormously, since it now no longer has to decide from ten choices but only two. The location of the space indicates which pair it is. We can now use a recognition procedure that is simpler and more effective; doubtful cases such as 1 and 7, which increase the number of rejects in a normal scheme, do not now arise.

As an illustration we shall first combine Van der Toorn's entry system with the recognition scheme which we tested above. With the 10 000 figures used for this test the percentage of wrongly read numerals falls from 2.67% to 0.21%. This may be counted in Table I, where we now take only the confusions of 0 and 1, 2 and 3, etc. as errors, and all the others as rejects. The number of errors is indeed very small, but there are now more rejects (8.42%). Table II gives the result for a recognition procedure adapted to the number-entry method. Although it was still in a very primitive state of development it gave much better results. There are now 0.75% errors and only 1.76% rejects.

The system works even better if each space is marked with one
numeral instead of two. When the machine reads a numeral, then the locations alone of the space containing the numeral gives unambiguous identification of the numeral, so that the recognition is given the nature of a verification. This is a combination of two entirely different methods of recognition; a method that classifies by location (in the same way as in a marksensing system), and a method that classifies by shape.

A serious difficulty with this version of the number-entry system is that twice as many spaces are needed for the same number of digits. Even when each space is marked with two numerals, it is difficult enough to find room for the strip on

forms in current use; using only one numeral per space it would be impossible.

It can easily be seen that Van der Toorn's system meets the requirement that the machine will reject forms filled in wrongly. If no notice is taken of the rules for filling in the numerals, or they are not understood, the chance is extremely small that one of the two permitted numerals will be read in each of the squares. If more numerals than we have available at the moment could be used for making the recognition scheme, it would be possible to reduce the percentages of errors and rejects very considerably, and the system could then become a really practical proposition.

Summary. A method is described for the automatic recognition of freely handwritten numerals. The numerals are scanned one by one with a television camera, and the television picture is then electronically converted into a matrix of occupied and unoccupied positions. This matrix is very suitable for processing in a computer or in electronic equipment specially designed for the purpose. The numeral is first 'thinned' to a skeleton that is nowhere thicker than one matrix position. The end points and forks in the skeleton are then determined. These special points form the basis of the recognition procedure that is used to classify the numeral, and is based on a large number of test numerals. In certain cases tests must be made on the unthinned numeral. The system described has great flexibility since the recognition procedure can continuously be elaborated upon by the addition of new criteria. A recognition procedure drawn up from more than 15,000 numerals was checked against 10,000 other numerals; 91.37% of these were recognized, 2.67 were wrongly read and 5.96% were rejected. A reading test is described in which 35 people had to read a number of numerals taken at random from the test group of 10,000. Without being able to compare numerals written in the same handwriting, they were able to read 98% of the numerals correctly. Since this is not sufficient for reading forms with say 12 numerals, like giro cards, and since it is not to be expected that a character-recognition system will perform better than a human reader, it is concluded that if forms are to be read entirely by such a system, either the method must be extended, e.g. by building in a memory function for comparing numerals, or constraints must be imposed on the freedom with which the numerals are written. Some possibilities of devising a practical system are indicated, both for a closed organization and for the general public.