Electric motors

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Types of electric motor

The function of an electric motor is to convert electrical energy into mechanical work. The electrical energy supplied is usually obtained from mechanical work applied simultaneously somewhere else in a conversion in the other direction by means of generators. Electric motors and generators are both referred to as electrical machines and are essentially the same in construction; in principle the same electrical machine can perform both functions.

In this article a description will be given of the fundamental principles of the operation of the electrical machine. Various types of machine will be considered, all of them types that can be applied as small electric motors. Here small motors are taken to be motors with a rating of 1 kilowatt or less.

Most electric motors carry out the energy conversion by way of a rotating movement — exceptions are the linear electric motor, in which the movement is a linear one, and the vibrator [1]. The rotating part of the machine is called the rotor, and the stationary part the stator. At least one of these is provided with one or more windings that are connected to the source of electric power. Depending on the type of source, the machine will be a d.c. motor or an a.c. motor.

In both cases the operation of the motor can often be described by starting with the Lorentz force — the force experienced by a conductor carrying a current in a magnetic field (see fig. 1). If we imagine that the magnetic field in a d.c. motor is stationary, then to ensure that the Lorentz force on a conductor at the circumference of the rotor has the correct direction during the whole of each revolution, the current must be regularly reversed. This is the function of the commutator, an essential feature of every d.c. motor [2]. The commutator is usually a mechanical device and generally consists of a cylinder formed by insulated conducting segments mounted on the shaft; the windings are connected to the segments and the current is fed to the segments by brushes (fig. 2).

A commutator of this type is subject to wear, particularly when there is sparking between brushes and segments. The sparking also causes radio-frequency interference. For these reasons there has been consider-

[2] An exception is the unipolar machine, but this is rarely used. A special form of this machine, which made use of superconductivity, has been described by J. Volger in: A dynamo for generating a persistent current in a superconducting circuit, Philips tech. Rev. 25, 16-19, 1963/64.
[3] See the article by R. Raes and J. Schellekens about a d.c. motor with speed control for a washing machine, to appear in the second issue on electric motors.
In a.c. machines there is no need for a commutator; use can be made here of the periodic change in the direction of the current in the a.c. mains. This means that a relation must exist between the frequency of the a.c. current and the speed of revolution of the machine. In synchronous machines the number of revolutions per second is proportional to the frequency of the current. Synchronous machines are therefore widely applied where constant speed is a first requirement. Then there is the group of asynchronous motors; these motors run at a speed a little lower than the 'synchronous' value, and the difference (the slip speed) increases with the load on the motor. Because of this difference in speed currents are induced in closed circuits in the rotor; this induction is essential to the operation of this type, which is therefore usually referred to as the induction motor. The induction motor is the most common type of a.c. motor.

In some applications the relation between the speed and the mains frequency is a nuisance. One way around the difficulty is to use a commutator, even though the supply is a.c. This is because a commutator motor in the correct kind of circuit can operate from a mains supply of any frequency — and not just if that fre-

<p>| Table I. Survey of the most important types of motor. The torque-speed characteristics show the variation of the electromagnetic torque $T_e$ as a function of the rotational speed $n$ of the motor. |
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quency is zero, as for a d.c. motor. In practice a motor to be used in this way has to be specially designed, and is called a universal motor. If such a motor is run from an a.c. supply the currents in the rotor and stator windings continually change sign simultaneously. This means that the interactive force, which is proportional to the product of the two currents, maintains the same sign.

The speed of a synchronous or asynchronous motor can also be varied over a wide range if it is supplied from a source of variable frequency. This requires the mains frequency to be converted to another frequency; there are rotary frequency converters (a motor driving a generator) and static (or electronic) frequency converters. In fact the operation of the commutator in a d.c. or universal motor can also be considered as a frequency conversion. This is why the combination of a synchronous motor and a frequency converter is found to behave in a similar way to a d.c. commutator motor under certain conditions.

Finally, there is the stepping motor [1]. This is essentially a motor of the synchronous type, supplied by electronically generated pulses of current. Each pulse produces a constant angular rotation. The number of angular rotations (the 'steps') is equal to the number of pulses applied, so that a digital signal is converted into a mechanical displacement. Stepping motors are used in applications such as numerically controlled machine tools.

Motors of the various basic types listed here, except perhaps the stepping motor, were in existence long before the end of the nineteenth century [**]. Nevertheless, new developments are still under way. The main trends are:

- Improvement of the various characteristic features (the efficiency, the variation of torque with speed, the starting characteristic, etc.).
- Increasing the power/weight and power/volume ratios.
- Improving reliability and safety.
- Improving methods of motor control.

These developments are possible since improved magnetic and insulating materials are now available, computers can be used in design work, and new production techniques can be used, such as sintering of rotors. The availability of electronic components, in particular thyristors and transistors, has also increased the range of possibilities, especially in motor control.

We shall now examine the operation and characteristics of the most important types of motor in more detail, and derive an expression for the delivered torque. Table I shows a diagrammatic survey of the motor types examined in this article, with the page references. Types with an external rotor, sometimes used in small motors, are not described separately as they do not differ in principle from the types with an internal rotor. Since the synchronous motors to be discussed in this article are of simple configuration, we shall start with these. Asynchronous or induction motors and the commutator motors will then follow.

### I. Synchronous motors

The kinds of synchronous motor to be discussed here all have a rotor with no windings. This rotor can be a permanent magnet, in which case its magnetization will give it a preferential position with respect to the magnetic field produced by the stator windings. In another type, the reluctance motor, the rotor is not permanently magnetized, but is made from a material of high permeability; this rotor is not cylindrical and therefore has a preferential position for which the magnetic resistance — the reluctance — of the magnetic field is at a minimum. In both motors the stator windings excite a rotating magnetic field; the rotor tries to maintain the preferential position with respect to the field and rotates with it, provided that it is not prevented from doing so by too great a load on the shaft. In the third type of motor to be discussed, the hysteresis motor, there is also a rotating stator field. This induces a magnetization in the rotor, as in the reluctance motor. The rotor is now cylindrical, however, and made of a material that has considerable magnetic hysteresis. Because of this hysteresis the magnetization always lags behind the direction of the rotating stator field, thus giving a torque.

#### Synchronous motor with permanent-magnet rotor and cylindrical stator bore

Let us now pass on to a calculation of the torque operating on the rotor of a synchronous motor with a permanent-magnet rotor. We shall assume here that the motor is constructed as in the cross-sectional diagram of fig. 3. A cylindrical rotor can rotate in the cylindrical bore of the stator. The stator — usually laminated in practice to reduce eddy currents, i.e. made from a stack of thin stampings insulated from each other — carries two 'diametral' windings $I_1, I_1'$ and $2, 2'$. In fig. 3 these are shown in the air gap, but in practice

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[1] We intend to publish a separate article on this subject in the third issue on electric motors. (Ed.)

[**] We intend to include an article on the history of the electric motor in the third issue on electric motors. (Ed.)
they can be mounted in slots in the stator. The rotor is
homogeneously magnetized parallel to a diameter.

It is not difficult to calculate the forces experienced by
the sides of the coils in the magnetic field of the rotor
when a current flows. These forces taken together give a
torque on each coil. The reaction torque on the rotor
is then equal in magnitude to the sum of the torques
on the two coils but of opposite sense. The only com-
ponent of the magnetic field that contributes to the
torque is the radial component that crosses the air gap.
This component is therefore the only one considered
in the rest of the discussion. Similarly, the forces ex-
perienced by the coils from their own and one another’s
field will not be taken into account, since they do not
contribute to the torque.

![Fig. 3. Schematic cross-section of a synchronous motor with a
cylindrical permanent-magnet rotor (N north pole, S south pole).
The rotor rotates inside the cylindrical bore of a stator carrying
two ‘diametral’ coils 1,1’ and 2,2’. When current flows in the
coils in the magnetic field of the rotor they experience a torque;
a reaction torque $T_e$ of the same magnitude but opposite sign
then operates on the rotor. If suitable a.c. currents flow in the
coils and the rotor has the correct angular velocity, the torque
on the rotor then always has the same sign and the rotation is
maintained. The position of the rotor is given by the angle $\theta$;
a rotation in the direction of increasing $\theta$ is taken as positive.
The angle $\phi$ is the coordinate used in the calculation for position
on the stator circumference.

If a current $I_s(1)$ flows in the coil 1, then a side of
this coil of length $l$ (at right angles to the plane of the
drawing in fig. 3) will experience a Lorentz force
$F = I_s(1)N_1LB$, where $B$ is the magnetic flux density
in the air gap and $N_1$ is the number of turns in the coil.
Using $B$ everywhere for the radial component, as indicated above, then whatever the position of the rotor $F$
is always perpendicular to the plane of the coil. The
magnitude of $B$ varies sinusoidally along the circum-
ference of the rotor: $B = \hat{B}\cos \phi$, where $\phi$ is the
angle in radians with respect to the plane of symmetry
of the rotor. For an arbitrary angle $\theta$ of the rotor with
respect to the stator the variation of $B$ around the stator
is given by $B = \hat{B}\cos(\phi_{s}-\theta)$; here $\phi_{s}$ is the coor-
dinate along the circumference of the stator (see fig. 3).
The force on the side of the winding at $\phi_s = \pi/2$ is
then $F = I_s(1)N_1\hat{B}\sin \theta$; the force on the other side
(at $\phi_s = -\pi/2$) is equal in magnitude but of opposite
sense. The torque resulting from the two forces is $2\hat{a}F$,
where $2\hat{a}$ is the width across the coil. If the second wind-
owing shown in fig. 3 is not taken into account this is the
electromagnetic torque acting in the motor. With the
directions of current and magnetization of the rotor as
shown in fig. 3 the sense of the reaction torque on the
rotor is opposite to the direction of increasing $\theta$, i.e.
it is negative, so that for this torque we have:

$$T_e(1) = -2\hat{a}I_s(1)N_1\hat{B}\sin \theta. \quad (1)$$

The torque is zero in the two positions opposite each
other for which $\theta$ is equal to zero or $\pi$ radians. In the
first position the equilibrium is stable; for a small de-
ivation of the rotor it experiences a torque tending to
return it to this position. This is the preferential posi-
tion referred to above. In the second position the equi-
lbrium is unstable; the sense of the torque is such that
a small deviation will increase.

By making use of the coil 2, which is rotated by $\pi/2$
radians with respect to the first coil, the rotor can be
given another stable final position $\pi/2$ radians ahead of
or behind the first one. In this way the rotor can be
kept rotating in ‘steps’. This is the essential principle
of a stepping motor with a permanent-magnet rotor
and a ‘stepping angle’ of $\pi/2$ radians.

However, what is usually required is a steady con-
tinuous rotation of the rotor. This can be obtained by
supplying the coils 1 and 2 with sinusoidally varying
a.c. currents of equal magnitude and differing in phase
by $\pi/2$ radians. To calculate the torque in this case the
expressions for the two currents, $I_s(1) = \hat{i}_s\cos\omega t$ and
$I_s(2) = \hat{i}_s\sin\omega t$ (where $\omega$ is the angular frequency of
the a.c. currents and $t$ the time) can be substituted in
equation (1). Summing the torques produced by the
two coils will give the total torque $T_e$. If it is now further
assumed that the rotor rotates at a constant angular
velocity $\omega_r$, so that $\theta = \omega_r t + \theta_0$, then for the total
 torque we have:

$$T_e = -2\hat{a}\hat{i}_sN_1\hat{B}\sin\{(\omega_r - \omega)t + \theta_0\}. \quad (2)$$

The time-average value of this torque can only differ
from zero if $\omega_r$ is equal to $\omega$, i.e. if the rotor runs at
synchronous speed. This means that the motor will not
operate as a motor at any speed other than the syn-
chronous speed. It also means that the motor will not
start of its own accord, since the time average of the
torque at zero speed is also zero. This is an important
practical difficulty; to avoid it synchronous motors are
often provided with a second rotor of another type on
the same shaft and rotating in the same stator. The second rotor can be for example a squirrel-cage rotor, as used in induction motors (see below).

At the synchronous speed the torque is constant:

\[ T_e = -2a \phi_i N_s \beta \sin \theta. \] (3)

With the motor rotating at constant speed, the accelerating torque acting on the rotor is apparently zero. This means that the magnitude of the electromagnetic torque is exactly large enough to compensate the torques produced by the load and by friction. At a given current the only variable in (3) that could enable the motor to match the electromagnetic torque to the field. For motor operation the electromagnetic torque is exactly large enough to compensate the load.

\[ \text{Angular frequencies} \]

Since all the coils at various points of the stator circumference, all connected in series. In this case it is more convenient to replace the quantity \( N_s \phi_i N_s \) by the current-density distribution \( S_n(1) \), expressed in amperes per unit of length along the stator circumference. Since all the turns are in series, \( S_n(1) \) is equal to \( i_n(1) z_s(\phi) \), where \( z_s \) is the number of turns per unit of length. This quantity \( z_s \) is called the copper-density distribution. The sign of this function indicates whether the current at the location \( \phi \) flows in the positive or negative direction (\( \bullet \) or \( \circ \) in fig. 3) when a positive current is applied at the terminals.

Higher harmonics

So far it has been assumed that the magnetic field originating from the rotor is sinusoidally distributed along its circumference. This is usually only approximately true. Terms of higher order are therefore present; these give rise to fluctuations in the torque at every revolution of the rotor, which interfere with the even running of the motor.

These fluctuations can be counteracted by using distributed windings 1 and 2 instead of the diametral coils 1 and 2. A distributed winding consists of a number of coils at various points of the stator circumference, all connected in series. In this case it is more convenient to replace the quantity \( N_s i_s N_s \beta \) by the current-density distribution \( S_n(3) \), expressed in amperes per unit of length along the stator circumference. Since all the turns are in series, \( S_n(3) \) is equal to \( i_n(3) z_s(\phi) \), where \( z_s \) is the number of turns per unit of length. This quantity \( z_s \) is called the copper-density distribution. The sign of this function indicates whether the current at the location \( \phi \) flows in the positive or negative direction (\( \circ \) or \( \circ \) in fig. 3) when a positive current is applied at the terminals.

Fig. 4. Schematic cross-section of a synchronous motor in which the two windings 1,1' and 2,2' each consist of three coils. The stator has twelve equidistant slots for the coils. This arrangement is used to give an approximation to a sinusoidal current-density distribution.

twelve slots, in which there are two windings, each consisting of three coils. Usually two to five coils per winding are considered sufficient, with equal numbers of turns in each coil. This approach gives a considerable reduction in the torque fluctuation.

In many cases a distributed winding in which \( S_n(1) \) is distributed sinusoidally around the stator circumference is ideal. As we saw above, a uniform torque can be obtained with a purely sinusoidal distribution of the rotor flux density and a non-sinusoidal current-density distribution. An even torque is also obtained with a sinusoidal current-density distribution and a magnetic flux density that is not sinusoidally distributed, since the current \( i \) and the magnetic flux density \( B \) are equivalent factors in the expression \( F = liB \) for the Lorentz force.

In practice only an approximation to a sinusoidal current-density distribution is attempted. The coils of the winding are usually arranged in equidistant slots in the stator circumference. Fig. 4 shows a stator with...
The variation of the current-density distribution \( S_{s13} \) of stator winding 1 around the stator circumference of the motor shown in fig. 4. The other curve shows a possible curve for the rotor flux density \( B_r \) around the stator circumference. The angle \( \alpha \) characterizes this trapezoidal curve and the angle \( \beta \) represents the width of the slots.

Fig. 5. The variation of the current-density distribution \( S_{s13} \) of stator winding 1 around the stator circumference of the motor shown in fig. 4. The other curve shows a possible curve for the rotor flux density \( B_r \) around the stator circumference. The angle \( \alpha \) characterizes this trapezoidal curve and the angle \( \beta \) represents the width of the slots.

The difference of the stator of the current-density distribution \( S_{s13} \), applying to stator coil 1, for the motor of fig. 4. The 'current-density pulses' have the width of the slot opening and are reckoned as positive when the current flows 'into the paper' in fig. 4. The other curve shown in fig. 5 is an example of a possible non-sinusoidal flux-density distribution. A general method for the calculation of the torque on stator coil \( \{s\} \) for the motor of fig. 4. The current-density distribution in terms of a series in \( \sin(\phi_s) \) and \( \cos(\phi_s) \) is applicable, is then given by

\[
T_e = n f \int \left( S_{s11} + S_{s23} \right) B_d \, d\phi_s
\]

For a purely sinusoidally distributed winding \( z_{s3}, z_{s6} \) and higher terms are zero. This cannot be achieved for all the terms. However, some degree of compensation for this can be obtained by choosing a distribution for the rotor flux that will make the remaining troublesome terms in the series for \( B_r \) very small. With the configuration shown in fig. 5, for \( \alpha = \pi/3 \) and \( \beta = \pi/60 \) radians:

\[
\begin{align*}
|z_{s0}| &= 0.37, & |z_{s6}| &= 0.27, & |z_{s7}| &= 0.27 \\
|B_{s0}| &= 0, & |B_{s3}| &= 0.04, & |B_{s7}| &= 0.02
\end{align*}
\]

The amplitudes of the terms in \( 3\theta_0, 5\theta_0 \) and \( 7\theta_0 \) are thus respectively \( 0\% \), \( 1\% \) and \( 0.5\% \) of the maximum value of the first term.

Purely sinusoidal current-density wave the magnetic field wave excited around the stator circumference (the rotating field) is also sinusoidal, provided that the air gap is uniform or the rotor consists of a material whose permeability is equal or nearly equal to that of air (e.g. permanently magnetized ferroxdure). The flux-density wave \( B_{s3} \) is then always displaced in phase by \( \pi/2 \) radians with respect to the current-density wave \( S_{s3} \).

A special case arises in a synchronous motor when only one stator winding is energized, say winding 1. The conditions for a current-density wave of constant amplitude and angular velocity are then no longer satisfied. The fundamental of the current-density distribution then has the form \(-i_{s(1)} z_{s1} \sin(\omega t - \phi_0)\) and is zero for \( \phi_0 = 0 \) and \( \phi_0 = \pi \). This 'standing' wave can however be resolved into two travelling waves rotating in opposite directions:

\[
-i_{s(1)} z_{s1} \sin(\omega t - \phi_0) = \frac{1}{2} i_{s(1)} z_{s1} \sin(\omega t - \phi_0) + \frac{1}{2} i_{s(1)} z_{s1} \sin(-\omega t - \phi_0).
\]
The motor — it is called a single-phase motor — can thus rotate clockwise or anticlockwise at the synchronous speed. However, because of the current-density wave in the opposite sense the torque is not constant, but pulsates at twice the mains frequency.

**Multipole machines**

In all the machines so far discussed the air-gap field has a single pair of poles, and they are therefore known as two-pole rotating-field machines; the angular velocity of the rotating field is equal to the angular frequency \( \omega \) of the supply mains.

A stator can also be wound in such a way that the air-gap field has four poles; alternate north and south poles are then obtained around the stator with \( \pi/2 \) radians between poles. Fig. 7 shows how the stator windings are arranged in the four-pole version of the motor of fig. 3. Winding 2 is now displaced by \( \pi/4 \) radians with respect to winding 1, which means that the angular velocity of the rotating field, and therefore the synchronous angular velocity of the motor, is equal to half the angular frequency of the mains. The rotor is magnetized in such a way that it also has four poles.

This provides a way of reducing the speed of a synchronous motor by a factor of two. The procedure can be extended to give six-pole and eight-pole motors, or in general terms \( 2p \)-pole motors, where \( p \) is the number of pairs of poles.

**Supply from a constant-voltage source**

In all the calculations given above for the torque the situation has been simplified by the assumption that the magnitude of the stator currents is given. In practice this is rarely the case. The motor is almost always connected to a source of supply whose voltage at the terminals is given; the magnitude of the currents then varies with the load on the motor.

To derive an expression for the torque in terms of the terminal voltage instead of the stator currents we must calculate the currents that flow for a given voltage at the terminals. The voltage induced in the stator winding by the alternating magnetic fields is an important quantity in these calculations. Part of this voltage will be related to the magnetic field produced by the stator winding itself and will depend upon the inductance \( L_s \) of the winding; there is also a magnetic field produced by the rotor that will induce a voltage in the stator winding when the rotor is rotating. A useful representation is given by the equivalent circuit shown in fig. 8 for a stator winding. This equivalent circuit corresponds to the equation in complex notation:

\[
V_s = (R_s + j\omega L_s)I_s + E_{sr}. \tag{4}
\]

Here \( V_s, I_s \) and \( E_{sr} \) are complex numbers whose moduli are equal to the r.m.s. values of the terminal voltage, the current and the motional e.m.f. or 'speed voltage' induced by the rotor field respectively, and with arguments equal to the appropriate phase angles. \( R_s \) is the resistance of the stator winding, and as previously \( \omega \) is the angular frequency of the stator winding, and as previously \( \omega \) is the angular frequency of the supply mains.

Equation (4) gives the desired relation between applied voltage and current; if we introduce the parameter \( \lambda = |E_{sr}|/V_s \) then \( I_s \) can be expressed in explicit terms:

\[
I_s = \frac{V_s}{R_s + j\omega L_s} \left[ 1 - \lambda e^{j(\theta_0 + \pi/2)} \right]. \tag{5}
\]

The parameter \( \lambda \) has a constant value for a synchronously operating motor; for a terminal voltage of given amplitude and frequency \( \lambda \) depends only on the construction of the motor. When the rotor is stationary \( \lambda \) in equation (5) is equal to zero; the stator current then takes the value corresponding to the impedance \( R_s + j\omega L_s \) of the stator winding. The angle \( \theta_0 + \pi/2 \) is the phase angle of the voltage induced by the rotor field with respect to the applied voltage; \( \theta_0 \) is the rotor position at the instant when the voltage across the winding reaches its maximum value. As we saw earlier, \( \theta_0 \) varies with the load on the motor; equation (5) shows how \( I_s \) then varies.
The equations (4) and (5) apply to each stator winding separately. The calculation of the torque, for example for the motor shown in fig. 3 with two windings, is most readily performed by setting up a power balance. The mechanical power $T_e \omega$ can then be set equal to the electrical power taken up by the two stator windings, less the ohmic losses in the windings.

The power balance can be expressed in complex notation by:

$$T_e \omega = 2 \text{Re}(I_s \ast V_{s1} - I_s \ast I_s R_s) = 2 \text{Re}(I_s \ast E_{s1}). \quad (6)$$

Here the asterisk indicates the complex conjugate. The power balance is the same for both stator windings; it is therefore sufficient to take the electrical power for a single stator winding and double it.

With the aid of the two equations (4) and (5) the current $I_s$ and the speed voltage $E_{s1}$ can be expressed in terms of the terminal voltage $V_s$ in (6). Returning now to real quantities and writing $V_s$ for the r.m.s. value of the terminal voltage, then from the power balance:

$$T_e = \frac{2}{\omega} \frac{\lambda V_s^2}{R_s + \omega L_s} \{\cos(\theta_0 + \pi/2 + \gamma_s) - \lambda \cos \gamma_s\} \quad (7)$$

Here $\gamma_s = \arctan \omega L_s / R_s$, the phase angle of the stator impedance.

If $R_s$ is much smaller than $\omega L_s$ the stator resistance can be neglected in the expression for the torque. A convenient expression for the torque at a given r.m.s. value $V_s$ of the terminal voltage can then be obtained from the power balance:

$$T_e = -\frac{2 \lambda V_s^2}{\omega^2 L_s} \cos \theta_0. \quad (7')$$

For motor operation $T_e$ is positive and therefore

$$-\frac{3\pi}{2} < \theta_0 < -\frac{\pi}{2};$$

for stable operation however $\theta_0$ must not be smaller than $-\pi$, since as soon as the motor falls behind by more than $\pi$ radians the torque decreases and the motor falls out of step. Note that the angle $\theta_0$ is the 'phase angle' of the rotor with respect to the a.c. voltage across winding 1, whereas in (3) $\theta_0$ is a phase angle with respect to the current in winding 1.

Synchronous motors with non-uniform air gap

There are also synchronous motors that do not have a cylindrical stator bore but have salient poles (fig. 9a). In these motors the rotor can be subject to a torque even when there is no current in the stator winding. This is because it tries to take up a position in which the reluctance (the 'magnetic resistance') to the flux of the permanent magnet is at a minimum. The torque involved here is a stator-reluctance torque. In a motor like the one in fig. 9a this makes no contribution to the energy conversion, since on rotation its average value is equal to zero.

If the rotor is non-cylindrical (fig. 9b) instead of the stator and also has a magnetic permeability greater than that of air, then a rotor-reluctance torque can be produced under the influence of the stator field. The operation of the reluctance motor is based upon this torque.

In the treatment of both types of motor it is probably best to start from energy considerations, since we can then obtain some insight into their characteristics without previous knowledge of the complicated variation of the magnetic field. Also, a direct calculation of the electromagnetic forces acting on the stator in the case of fig. 9a would not only have to include the Lorentz force on the conductors of the stator winding: it would also have to include the forces operating on the stator core. Such a calculation would be a complicated exercise. To make the energy considerations as general as possible it will be assumed that we have a kind of motor that combines the characteristic features of both types; with stator and rotor both non-cylindrical, and with a permanent-magnet rotor that nevertheless has a higher permeability than that of air (fig. 9c). In this motor we are concerned with energy in various forms: the applied electrical energy, which in a short time $\Delta t$ is equal to $v_i d_i \Delta t$, the energy dissipated in the resistance of the stator winding, equal to $R_{s1} v_i^2 \Delta t$, the magnetic-field energy present in the motor $W_m$, and the mechanical energy $T_e \Delta \theta$ supplied by the shaft to the load during a rotation $\Delta \theta$. The applied energy is equal to the sum of the dissipated and supplied energy and the change in the magnetic-field energy; in differential form:

$$v_i d_i d \theta = R_{s1} v_i^2 d \theta + d W_m + T_e d \theta.$$  

Here $W_m$ is a function of the stator current $i_s$ and the angular position $\theta$ of the rotor.

To derive the torque from this energy balance we make use of an expression for the voltage $v_s$ across the stator winding:

$$v_s = R_{s1} i_s + \frac{d \Phi_s}{dt}.$$  

where $\Phi_s$ is the total magnetic flux linked with the stator winding. To a first approximation this linked flux can be equated to the product of the total magnetic flux through a stator pole and the total number of turns. $\Phi_s$ is also a function of both $i_s$ and $\theta$.

Substituting in the energy balance it follows that

$$T_e = i_s \frac{d \Phi_s}{d \theta} - \frac{d W_m}{d \theta}. \quad (8)$$

Both terms on the right-hand side are connected with
The angles \( \alpha \) and \( \beta \) depend on expression (9) for the torque therefore disappearsbecause of their simple construction. In the design equipment, for which such motors are ideally suited a great handicap in applications such as small domestic the pulsating torque peculiar to this type. This is no however full account must be taken of the notable instabilities that can appear in this motor.

Single-phase synchronous motor with non-cylindrical stator

We now confine ourselves to the case shown in fig. 9a, of a motor with a cylindrical rotor magnetized along a diameter and a non-cylindrical stator. This is a single-phase synchronous motor, and therefore gives the pulsating torque peculiar to this type. This is no great handicap in applications such as small domestic equipment, for which such motors are ideally suited because of their simple construction. In the design however full account must be taken of the notable instabilities that can appear in this motor.[14]

Since the motor has a cylindrical rotor the inductance \( L_s \) of the stator winding is not dependent on the angular position \( \theta \) of the rotor, and the last term in expression (9) for the torque therefore disappears. It can be shown that the quantities appearing in the other two terms depend on \( \theta \) as follows:

\[
\Phi_{sl}(\theta) = \Phi_{sr} \cos (\theta + \alpha), \\
W_{mr}(\theta) = W_1 - W_2 \cos 2(\theta + \beta), \quad W_1 > W_2.
\]

The angles \( \alpha \) and \( \beta \) are determined by the geometry of the stator poles; they are both zero if the stator is symmetrical about the plane \( \phi_s = 0 \), as in fig. 9a. In general:

\[
T_e = -i_s \Phi_{sr} \sin (\theta + \alpha) - 2W_2 \sin (\theta + \beta). \tag{10}
\]

The second term represents the torque for zero stator current; this reluctance or 'detent' torque can be felt on turning the shaft by hand. It pulls the rotor to one of the two stable rest positions (\( \theta = -\beta \) or \( \theta = -\beta + \pi \)) where it remains. If the current is then switched on, a torque is only produced if \( \alpha \) is not equal to \( \beta \). In the configuration shown in fig. 9a (\( \alpha = \beta = 0 \)) this torque does not arise and the motor will not run. By giving the pole arcs of the pole pieces an asymmetrical shape \( \alpha \) can be made unequal to \( \beta \), thus giving a self-starting motor.

The torque pulsations found at twice the mains frequency in this motor originate from two sources. The first is the backward-running rotating field already mentioned in the single-phase motor with cylindrical stator; the second is the stator reluctance. This appears in the expression for the torque when we express the time dependence of \( i_s = i_s \cos \omega t \) and \( \theta = \omega t + \theta_0 \) in (10):

\[
T_e = -\frac{1}{2} i_s \Phi_{sr} \{ \sin (2\omega t + \theta_0 + \alpha) + \sin (\theta_0 + \alpha) \} + \frac{1}{2} i_s^2 \frac{dL_s(\theta)}{d\theta}.
\]

Reluctance motor

Reluctance motors are characterized by a non-cylindrical rotor of high-permeability material. The stator bore can be either cylindrical or non-cylindrical. Fig. 10 shows a single-phase two-pole reluctance motor.

The rotor is not a permanent magnet and the quantities \( \Phi_{sr} \) and \( W_{mr} \) in expression (9) for the torque now disappear. The simple expression remaining is:

\[
T_e = \frac{1}{2} i_s^2 \frac{dL_s(\theta)}{d\theta}.
\]

The variation of the stator inductance \( L_s \) with the rotor position \( \theta \) is not in general easy to calculate. As a first

[14] These have been investigated at the Philips Aachen laboratories; see: H. Schemmann, Stability of small single-phase synchronous motors; this issue, p. 235.
approximation for the case shown here let us assume that

\[ L_a = L_1 + L_2 \cos 2\theta, \quad L_1 > L_2. \]

From this we have for the torque:

\[ T_e = -k^2L_2 \sin 2\theta. \]

This torque is zero for four positions of the rotor; the two positions perpendicular to the stator field \((\theta = \pi/2\) and \(\theta = 3\pi/2)\) are unstable and the two parallel to the stator field \((\theta = 0\) and \(\theta = \pi)\) are stable. When the stator winding is energized with d.c. or a.c. current the rotor takes up one of the last two positions; the motor thus has a positioning effect but is not self-starting, even with a.c. current.

Motor operation will only occur if the motor runs synchronously with the stator current \(i_s = i_r \cos \omega t\); the torque is then given by

\[ T_e = -\frac{1}{2} k^2L_2 \{2 \sin (2\omega t + 2\theta_0) + 2 \sin 4\omega t + 2\theta_0 + \sin 2\theta_0 \}. \]

This is a pulsating torque with the average value:

\[ \langle T_e \rangle = -\frac{1}{2} k^2L_2 \sin 2\theta. \quad (11) \]

The positioning effect of the reluctance motor is made use of in a stepping motor of the reluctance type. An example with four stator poles and five rotor lobes is shown in fig. 11. The four stator windings are energized in succession with d.c. current; each time the next stator coil is energized the motor performs a step of \((i - 1) \times 360^\circ = 18^\circ\). The control pulses are produced electronically. Fig. 12 is a photograph of a stepping motor with eight lobes on the rotor and twelve poles on the stator. In this motor four poles spaced by \(90^\circ\) are energized at a time; the stepping angle is \(15^\circ\).

**Hysteresis motor**

The hysteresis motor is a special case of the synchronous motor, with the notable feature that its average torque can differ from zero at all speeds, and not just at the synchronous speed. This has the practical advantage that no special provisions have to be made for starting. If the load torque remains less than the maximum motor torque the motor runs synchronously.

We shall now look more closely at the hysteresis motor, starting with fig. 13, which shows a two-phase hysteresis motor. The two stator windings are distributed sinusoidally around the circumferences of the stator bore; the rotor consists of a homogeneous cylinder of a material such as cobalt steel. Let us first assume that one of the windings is energized with a d.c. current. A stationary stator field is then produced, which magnetizes the rotor homogeneously. If the rotor is now made to rotate by external means at an angular velocity \(\omega r\), every element of the rotor is exposed to a rotating magnetic field. The field thus rotates \(\omega r/2\pi\) times per second.

---

**Fig. 10.** Reluctance motor. The non-cylindrical rotor has a permeability much greater than that of air.

**Fig. 11.** Principle of the reluctance stepping motor. The four stator coils 1 to 4 are successively energized with d.c. current pulses; after a cycle of four pulses the rotor has rotated one lobe, and hence by a fifth of a revolution.

**Fig. 12.** Reluctance stepping motor. There are eight lobes on the rotor and twelve poles on the stator. Four stator poles spaced by \(90^\circ\) are energized at a time; the stepping angle is \(15^\circ\).
In most hysteresis motors the rotor consists of a cylinder with marked hysteresis properties, mounted around a core that can be ferromagnetic or not. Here it is more difficult to describe the hysteresis, since it can no longer be assumed that each rotor element is subjected to a rotating field of constant amplitude and angular velocity. We shall not go further into this here.

The existence of hysteresis implies that the magnetization of each element is delayed in direction with respect to the inducing stator field. The angular difference is independent of the speed of rotation, and is determined only by the magnitude of the stator field and the hysteresis properties of the rotor material. Because of this angular difference a braking torque $T_e$ is experienced when the rotor is driven; this braking torque is also independent of the speed of rotation. Fig. 14a shows the torque as a function of the speed of rotation: the torque-speed characteristic. Mechanical power $-T_e\omega_r$ is thus applied to the motor; this is completely converted into the hysteresis losses $P_{\text{hyst}}$ associated with the rotational magnetization of the rotor material:

$$-T_e\omega_r = P_{\text{hyst}}.$$ 

The hysteresis losses are proportional to the speed, since in each rotor element the direction of magnetization rotates once per revolution of the rotor; it follows from this that the braking torque is independent of the speed.

In motor operation the stator is energized in such a way as to produce a stator field that rotates once for each period of the supply current. If the angular speed $\omega_r$ of the rotor is smaller than that of this rotating field, then the rotor will rotate in the opposite direction to the field and a similar situation arises as described above for the stationary stator field. If $\omega$ is the angular velocity of the rotating field, the direction of magnetization rotates in the rotor material $(\omega - \omega_r)/2\pi$ times per second and we have:

$$T_e(\omega - \omega_r) = P_{\text{hyst}}.$$ (12)

The torque-speed characteristic is now that of fig. 14b.

When the motor is running synchronously the torque can take any value between the extreme values shown in fig. 14b. The hysteresis motor then behaves rather like a synchronous motor with a permanent-magnet rotor. This means that the angle at which the rotor magnetization follows the rotating stator field now depends upon the load. As soon as this angle tends to become larger than the angle resulting from the hysteresis, the rotor starts to 'slip' and falls below the synchronous speed. The angle between the magnetization and the rotating field then remains at the fixed value determined by the hysteresis.

In practice there are deviations from the rectangular curve shown in fig. 14b. These are due to the slots in the stator bore for the winding, the associated higher harmonics in the current-distribution pattern, and the eddy currents in the rotor body.

A hysteresis motor wound for two-phase operation as in fig. 13 can easily be modified for operation from the ordinary single-phase mains. Usually one of the windings (the main winding) is connected directly to the mains, while the other (the auxiliary winding) is connected through a capacitor. The effect of the capacitor is to advance the phase of the current in the auxiliary winding with respect to the current in the main winding. This phase difference depends to some extent on the speed of the motor. By carefully choosing the element values in the auxiliary circuit the phase differences can be made equal to $\pi/2$ at for example zero or nominal speed. The motor will then operate like a true two-phase motor at this operating point. In the first case we have a 'starting capacitor', in the second a 'running capacitor'.
II. Asynchronous motors (induction motors)

So far we have been concerned with motors whose operation does not depend upon currents in the rotor. Now we shall look at motors that do depend on such currents, and we shall begin with the motors in which these rotor currents are not supplied by an external source but are induced by the stator field. These are the induction motors. An induction motor has a rotor with closed current circuits. If these are not accessible from outside it is a squirrel-cage rotor machine. Sometimes the current circuits are given external access via slip rings for control purposes; then we have a slip-ring-rotor (or ‘wound-rotor’) machine.

When an induction motor runs synchronously with the rotating stator field, the rotor is stationary with respect to the field and no rotor currents are induced; the torque is then zero. In normal operation the motor ‘slips’ with respect to the rotating field; it is because of this slip that currents are induced in the rotor and a torque is produced. For this reason these motors are also known as asynchronous motors. Within certain limits they can suit their speed to the torque required, which in many applications is an advantage over the rather inflexible behaviour of the synchronous motor. There is also a torque when the rotor is stationary, so that induction motors are self-starting. These characteristics, and the little maintenance required — since current does not have to be supplied to the rotor via sliding contacts subject to wear — make the induction motor a tough and reliable drive unit that gives useful service in many applications.

To give a simple treatment, the torque will first be derived for known stator currents, and then for a known terminal voltage. The torque is a function of the speed; in practice the torque-speed characteristic is very useful in assessing the performance of a motor.

Derivation of the torque

With stator currents known

We shall assume a two-phase, two-pole induction motor whose rotor carries two short-circuited diametral coils (fig. 15). The stator windings are distributed around the stator circumference in such a way that higher harmonics in the stator field can be neglected. No account will be taken here of iron losses due to eddy currents and hysteresis. There are two ways of calculating the torque from the Lorentz forces. One is based on the forces experienced by the stator conductors as a result of the magnetic field excited by the rotor currents, and the other is based on the forces experienced by the rotor conductors as a result of the stator field. We shall use the second method.

The stator field is similar to the rotating stator field in the synchronous motor and is given by

\[
B_s(\phi_s) = B_s \cos(\omega t - \phi_s).
\]

The new feature is the calculation of the rotor currents. These have a frequency proportional to the difference between the angular velocities of the stator field and the rotor, and hence proportional to the slip. We define the slip \( s \) as

\[
s = \frac{\omega - \omega_r}{\omega}.
\]

The rotor currents \( i_{r(1)} \) and \( i_{r(2)} \) can be calculated with the aid of the voltage equation for the closed rotor circuit. For rotor coil 1 the equation becomes:

\[
0 = R_i i_{r(1)} + L_r \frac{d i_{r(1)}}{dt} + \frac{d \Phi_{rs(1)}}{dt}.
\]

Here \( R_i \) is the resistance of the coil, \( L_r \) is its inductance and \( \Phi_{rs(1)} \) is the part of the stator flux linked with rotor coil 1. The magnitude of \( \Phi_{rs} \) is proportional to the mutual inductance \( M \).

When the rotor currents are known, the torque can be found as the sum of the torques on the two coils:

\[
T_e = 2bN_i \{ i_{r(1)}B_s(\theta + \pi/2) + i_{r(2)}B_s(\theta + \pi) \},
\]

where \( b \) is the radius of the rotor, \( l \) is its length and \( N_r \) is the number of turns on the rotor coil.
The stator flux linked with the rotor coil 1 is
\[ \Phi_{\text{st}(1)} = 2biLrC_1 \cos(\omega t - \theta). \]
We can also write:
\[ \Phi_{\text{st}(1)} = i_1M \cos(\omega t - \theta), \]
where
\[ M = 2biLrC_1 \]
is a quantity entirely determined by the construction of the motor. If we put
\[ \theta = \omega t + \theta_0 = (1 - s)\omega t + \theta_0, \]
then we have
\[ \Phi_{\text{st}(1)} = i_1M \cos(\omega t - \theta_0). \]
When this is substituted in the differential equation (13), we obtain a solution
\[ i_{\text{r}(2)} = \frac{s\omega i_1M \sin(\omega t - \theta_0 - \chi)}{\sqrt{R_r^2 + (s\omega L_r)^2}}, \quad (14) \]
where \( \chi = \arctan(s\omega L_r/R_r) \) is the phase angle of the impedance of the rotor coil at the angular frequency \( s\omega \) of the rotor currents. Since the current in rotor coil 2 lags behind the current in coil 1 by \( \pi/2 \) radians,
\[ i_{\text{r}(2)} = -s\omega i_1M \cos(\omega t - \theta_0 - \chi). \quad (15) \]

The following expression can be shown to apply for the torque of the induction motor for known stator currents \( i_s \):
\[ T_0 = \frac{i_s^2\omega M^2}{L_T} \frac{1}{R_T/s\omega L_T + s\omega L_T/R_T}. \quad (16) \]
The first factor of the right-hand side contains only motor constants besides the stator current; the second factor varies with the slip and has a maximum value at \( s\omega L_T = R_T \). This maximum torque
\[ T_{\text{max}} = i_s^2\omega M^2/2L_T, \]
is known as the pull-out torque and the corresponding slip \( s_{\text{max}} = R_T/s\omega L_T \) as the pull-out slip. If the load torque rises to a value higher than the pull-out torque the speed suddenly falls and the motor comes to rest — this is the reason for the name 'pull-out torque'. On altering the resistance \( R_T \) of the rotor circuit, the pull-out slip changes, but not the pull-out torque. This effect can be applied in an induction motor that will give the maximum torque at a desired speed. In this kind of motor the rotor windings are brought out to slip rings, and an external control resistor completes the circuit, via brushes. This is a slip-ring-rotor or wound-rotor machine. Varying the control resistor gives different shapes of torque-speed characteristics; see fig. 16. The starting behaviour can also be varied in this way.

With terminal voltage known

In the derivation above of the torque of the synchronous motor supplied from a constant-voltage source, the quickest method was to set up a power balance: the electrical power supplied was set equal to the mechanical power supplied plus the heat produced in the windings. This is again the quickest method here. It will also be convenient to make use of the similarity between the induction motor and a transformer. The stator windings can be compared with the primary winding of a transformer connected to the mains, and the rotor with the secondary winding. If the rotor is stationary the induction motor is effectively a transformer with a short-circuited secondary. Once the motor starts to rotate this situation changes; a load resistance appears in the secondary circuit to represent the mechanical load (fig. 17). This resistance varies with the slip and is equal to \( R_T(1 - s)/s \).

This expression can be derived by starting from the voltage equation for the stator windings. For winding 1:
\[ v_{\text{st}(1)} = R_{\text{st}(1)}i_1 + \frac{d\Phi_{\text{st}(1)}}{dt} + \frac{d\Phi_{\text{st}(2)}}{dt}. \]
Here \( \Phi_{\text{st}(1)} \) is the magnetic flux originating from the rotor curr-
The magnetic field of the rotor is a rotating field; its distribution around the circumference of the rotor depends on the construction of the motor and is usually not sinusoidal. However, for an approximately sinusoidal current-density wave around the stator the only component of this distribution of any importance in calculating the torque is the fundamental. This is given by:

\[ B_r = \hat{B}_r \sin(\omega t - \theta_0 - \phi_0), \]

as can be shown from equations (14) and (15). The field therefore rotates around the rotor at an angular frequency \( \omega_0 \). Since the rotor is itself rotating at an angular velocity \( (1 - \alpha) \omega \), the angular velocity of the rotor field with respect to the stator is equal to \( \omega_0 \):

\[ B_r = \hat{B}_r \sin(\omega t - \theta_0 - \phi_0). \]

This is also the angular velocity of the stator current-density wave, and in fact the two angular velocities must be equal if the average value of the torque is to differ from zero.

The flux \( \Phi_{st}(t) \) is proportional to the amplitude of \( B_r \) at the stator coil \( j \), where \( \phi_0 = 0 \). Making use again of the mutual inductance \( M \) we have:

\[ \Phi_{st}(t) = \hat{I}_r M \sin(\omega t - \chi_0), \]

and for the voltage equation:

\[ V_{st}(t) = R_{st}(t) + L_s \frac{dI_r(t)}{dt} + M \frac{d}{dt} \int \hat{I}_r \sin(\omega t - \chi_0) dt, \]

From (14) and (15) we see that

\[ \hat{I}_r = \frac{\omega I_0 M}{\sqrt{(R_{st} - s)^2 + (\omega L_s)^2}}, \]

which can be substituted in the voltage equation to eliminate \( I_r \). The equation then becomes, in complex notation:

\[ V_{st}(t) = (R_s + j\omega L_s - \frac{\omega^2 M^2}{R_s(s + j\omega L_s)}) I_{st}(t), \]

where as before the modulus of \( V_{st}(t) \) and \( I_{st}(t) \) indicates the r.m.s. value.

This equation corresponds to the equation for a transformer with a resistance \( R_{st} \) in the secondary circuit. Since the resistance of the rotor circuit is \( R_r \) there is a load resistance equal to

\[ \frac{R_r - R_s}{s}. \]

The power taken up in this resistance, multiplied by 2 since there are two rotor coils, is equal to the mechanical power supplied:

\[ T_o(1 - s) \omega = 2 \frac{1 - s}{s^2} R_s I_r^2. \]

\( (I_r \) is the r.m.s. value of the rotor current.) From this it follows that the torque is equal to

\[ T_o = \frac{2 R_s I_r^2}{(1 - s) \omega}. \]

The rotor current \( I_r \) can be expressed in terms of the stator voltage. This gives an expression for the torque that is fairly complicated; it can be simplified if we assume that the stator resistance is negligible compared with the stator reactance, as we did for the synchronous motor. We also introduce the leakage coefficient \( \sigma \), frequently used in transformer analysis:

\[ \sigma = 1 - \frac{M^2}{L_s L_T}. \]

This is a measure of the coupling between primary and secondary: it approaches zero for tight coupling and unity for weak coupling. In practice \( \sigma \) has a value between 0 and 0.2 for induction motors and is a constant of the motor. We now obtain for the pull-out torque and slip:
Three kinds of rotor

Several kinds of rotor can be used for an induction motor. The slip-ring rotor (or wound rotor) was discussed earlier; other important types are the squirrel-cage rotor and the solid rotor.

In the squirrel-cage rotor, conducting bars embedded in slots in the rotor iron are short-circuited together at their ends by two conducting rings (fig. 18). This sturdy and relatively simple type of motor is widely used.

The solid rotor consists of a homogeneous steel cylinder or ring. Here the rotor material acts at the same time as an electrical and a magnetic conductor. It is obviously the simplest and sturdiest kind of rotor imaginable. This kind of rotor is sometimes used in high-speed induction motors operated from a high-frequency supply [5].

If the skin effect is present, as in the solid rotor and in some types of squirrel-cage rotor, the rotor resistance $R_r$ and the rotor inductance $L_r$ become functions of the slip. The torque-speed characteristic of these motors then differs from the curves shown in fig. 16.

Single-phase induction motor

An induction motor will also run with just one stator winding; however, such a motor has no starting torque. This can be seen as follows. The current-density distribution of the stator winding can be resolved into two waves, one rotating clockwise, and the other rotating anticlockwise. Each wave produces a torque on the rotor. Fig. 19 shows the corresponding torque-speed characteristics. The resultant torque-speed characteristic passes through the origin. Once the motor is rotating motor operation is possible, in either direction; the torque pulsates at twice the mains frequency.

Most single-phase induction motors are provided with an auxiliary winding, which is connected in either temporarily for starting or permanently to improve normal operation. The required phase difference between the current in the main winding and the current in the auxiliary winding can be obtained by including a capacitance in series with the auxiliary winding [5] (see also p. 225), or giving the auxiliary winding a relatively high resistance.

A special type of single-phase induction motor with an auxiliary winding is the 'shaded-pole' motor. The construction of this motor is shown in the schematic diagram of fig. 20. The main winding consists of two coils wound on the stator poles and connected in series or parallel. The auxiliary winding merely consists of one or more short-circuited turns or shading coils $SC$ surrounding a portion of the pole. Currents flow in these coils even when the rotor is stationary, since they are magnetically coupled to the main winding. These currents lag behind the main current and their fields

\[
T_{\text{max}} = \frac{1 - \sigma}{\sigma} \frac{V_i^2}{\sigma \omega^2 L_s},
\]

\[
s_{\text{max}} = \frac{R_f}{\sigma \omega L_s},
\]

and for the torque in general:

\[
T_e = \frac{2T_{\text{max}}}{s/s_{\text{max}} + s_{\text{max}}/s}.
\]

![Fig. 19. Torque-speed characteristic of a single-phase induction motor. This has a pulsating stator field, which can be resolved into two rotating components, one clockwise and the other anticlockwise. The two rotating components produce two opposing torque-speed characteristics (dashed curves); the resultant curve is the torque-speed characteristic of the motor (solid curve).](image)

![Fig. 20. Single-phase induction motor of the 'shaded-pole' type. This kind of motor is fitted with auxiliary windings $SC$, the 'shading coils'. The currents induced in these produce a local magnetic field that lags behind the main field; this gives rise to a primitive kind of rotating field in the air gap, so that the motor is self-starting. $S_q$ squirrel-cage rotor.](image)
are displaced in position with respect to the main field. The arrangement forms in effect a primitive kind of rotating-field motor, which is not only self-starting but has a better running characteristic than it would have without the shaded poles. Fig. 21 is a photograph of a shaded-pole motor with three shading coils.

Fig. 21. Shaded-pole motor with three shading coils; this motor is mainly used in record players. The squirrel-cage rotor has copper bars and end rings.

III. Commutator motors

The principle of the commutator motor — the type that has to be used when the supply is a d.c. source — will be explained with the aid of Fig. 22, which shows a commutator motor in its simplest form. A rectangular loop of wire can rotate in a magnetic field $B$; the axis of rotation is at right angles to the direction of $B$. If a current $I$ flows in the loop it will be subject to a torque; this torque varies as the sine of the angle of rotation $\theta$ and is therefore zero when averaged over a complete revolution. To obtain a torque whose average value is different from zero the current is reversed at the positions $\theta = -\pi/2$ and $\theta = \pi/2$; the torque then varies as shown in Fig. 23. The usual way of reversing the current is by means of a mechanical commutator on the shaft of the motor; the commutator in Fig. 22 consists of two segments and two brushes.

The average value of the torque is of course much larger if the rotor is arranged to carry a large number of coil loops which are all supplied with current in the way just described. Moreover, the variations in the torque will be proportionally reduced. A winding of this type is the lap winding. The principle of this winding is illustrated in the developed diagram shown in Fig. 24 of a simple lap winding with four diametral coils. These coils are arranged in practice in slots in the circumference of a laminated cylindrical core. Each slot carries two coil sides, one above the other in the slot. All the coils are connected in series, while the four connecting points are each connected to a segment of the commutator. The result is that the circuit appearing between the brushes is an armature winding consisting of two paths in parallel, the ‘armature paths’.

When a brush goes from one segment to the next, one of the coils is short-circuited by this brush. The commutation is said to be ideal if the current in the short-circuited coil varies linearly from $I/2$ to $-I/2$ or vice versa (straight-line commutation) during the short-circuiting.

The attainment of good commutation requires much careful attention, particularly in the larger motors, and special provisions such as brush-shift, commutating poles and perhaps compensating windings are quite usual. However, since we are mainly concerned here with small motors we shall not discuss these matters further.
If a d.c. current is applied to the brushes, then each coil of the armature winding takes an a.c. current whose waveform is approximately a square wave. The commutator ensures that the angular frequency of this square wave always corresponds to the angular velocity of the rotor; it functions effectively as a frequency converter. In this case it converts the zero frequency of the d.c. current into a frequency corresponding to the rotor speed.

**Fig. 23.** The torque exerted on the loop of fig. 22, as a function of the angle θ.

![Diagram of torque exerted on the loop](image)

**Fig. 24.** Development of a lap winding consisting of four diametral coils. Each coil is uppermost in a slot on one side of the rotor (solid lines), and at the bottom of a slot on the other side (dashed lines). The connection between each pair of coils is connected to a commutator segment Seg. The rotor winding gives two armature paths, which appear in parallel between the brushes; half the rotor current $I_r$ flows in each path. $N$ and $S$ indicate the north and south poles.

**Torque and speed voltage**

*Fig. 25a* is a schematic diagram of a commutator motor with a field winding on salient poles. In a motor of this construction the main field around the circumference of the rotor is approximately trapezoidal. If the current distribution around the rotor is uniform — which means that there must be a large number of rotor windings and hence a large number of segments — then the ‘current packet’ present in the stator field will always be the same and the torque will be practically constant. The general expression for this torque is

$$T_e = -b^2 l \int_0^{2\pi} S_r B_s d\phi_r,$$

where as before $b$ is the radius, $l$ the length and $S_r$ is the current density of the rotor. For the present it will be sufficient to derive from this the relation

$$T_e = C I_r \Phi_{rs},$$

(17)

Here $\Phi_{rs}$ is the magnetic flux penetrating the rotor from a stator pole and $C$ is a constant determined by the construction of the motor.

As soon as the rotor is in motion a voltage is induced in its conductors. The sign and magnitude of this voltage is given by the general vector relation $E = v \times B$ where $v$ is the velocity of the conductor. In our case the magnitude is equal to $\omega r b l B_s$, since the rotor speed at the circumference is $\omega r b$. This voltage can be measured at the rotor terminals when there is no current in the rotor and it is externally driven. It is called the speed voltage $E_r$, and from the above it is proportional to $\omega r \Phi_{rs}$:

$$E_r = C \omega r \Phi_{rs}.$$ 

The proportionality constant is the same as in (17); this follows from the power balance

$$T_e \omega r = E_r I_r,$$

which is the same as that discussed on p. 222 for the synchronous motor, since $E_r I_r$ is equal to the electrical power supplied less the copper losses

$$E_r I_r = V_r I_r - R_t I_r^2.$$

Here $R_t$ is the rotor-circuit resistance, including the brush-contact resistance (see fig. 25b).

![Equivalent circuit for a d.c. motor](image)

**Fig. 25.** a) D.C. commutator motor with a field winding on salient poles. $V_r$ voltage at rotor terminals. $I_r$ rotor current. b) Equivalent circuit for a d.c. motor. $R_t$ resistance of rotor winding and commutator. $E_r$ speed voltage.

To say more about the behaviour of the torque for varying supply current, voltage or speed, we need to know more about the motor circuit. This is because, unlike the types discussed earlier, this kind of motor has two windings that have to be connected to the source. There are three ways of doing this.
Three types of commutator motor

Commutator motors can be divided into three types with different circuits: the separately excited motor, the shunt motor and the series motor (fig. 26). In the separately excited motor the field winding and the rotor each have a separate supply that can be independently operated. Motors with permanent-magnet excitation can be included in this class, in which there is of course no question of controlling the main field. Permanent-magnet excitation is usually only employed in small motors.

Even though the motor with independent energization of the field winding already has appreciable scope for control, a starting resistance $R_{\text{ext}}$ is often used in the rotor circuit, as with the other types. In larger motors the starting resistance is in fact necessary to limit the current when starting at full mains voltage; at the instant when the motor is connected to the supply no speed voltage has as yet appeared to oppose the supply voltage and with no extra resistance there would be a very large surge of current. From the equations given earlier it follows that the general relation between torque and angular velocity for the d.c. motor is

$$T_e = C\phi_{rs} \frac{V_r - C_{\omega_e} \phi_{rs}}{R_e + R_{\text{ext}}}.$$  \hfill (18)

This relation can be altered by varying the three parameters $\phi_{rs}$, $V_r$ and $R_{\text{ext}}$; independent variation is only possible for the separately excited motor with field winding. The effect of such independent variation of the three parameters on the torque-speed characteristic is shown diagrammatically in fig. 27. It is clear from this figure that the d.c. motor is basically very adaptable; it is therefore very suitable for control systems.

In the shunt and series motors the three parameters cannot be varied independently. In the shunt motor the field winding and the rotor are connected in parallel across the supply voltage $V_r$ (fig. 26b). If this voltage has a constant value, $\phi_{rs}$ is constant and the torque-speed characteristic is a straight line with a negative slope that can be varied by means of $R_{\text{ext}}$ (fig. 27c).

In the series motor the field winding is in series with the rotor (fig. 26c); the main-field flux $\phi_{rs}$ now varies with the rotor current $I_r$. When the loading torque is increased the speed decreases and the rotor current increases; consequently $\phi_{rs}$ also increases at lower speeds. Curve $Se$ in fig. 28 is an example of the torque-speed characteristic of a series motor; it shows the same variation of the slope when $\phi_{rs}$ is varied as in fig. 27a.

To determine the torque-speed characteristic of a shunt or series motor completely we must know the relation between the main-field flux $\phi_{rs}$ and the excitation current $I_e$. This relation is only linear for small values; at higher values it is affected by magnetic saturation. The relation can be measured by driving the motor at constant speed and measuring the voltage produced at the rotor terminals as a function of the excitation current; this gives the relation since the speed voltage is proportional to the main field. The curve obtained in this way is called the

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Fig. 26. The three circuit arrangements that can be used with a commutator motor. a) Separately excited motor; field winding and rotor are connected to two independent sources of supply. A starting resistance $R_{\text{ext}}$ is connected in series with the rotor. Motors with permanent-magnet energization are also included under this type. b) Shunt motor; field winding and rotor are connected in parallel across the same supply voltage. c) Series motor; field winding and rotor carry the same current.
open-circuit characteristic; fig. 29 gives a typical example. D.C. motors are usually designed to allow a small amount of magnetic saturation under nominal operating conditions.

![Graph of torque-speed characteristics for a shunt motor (curve Sh) and a series motor (curve Se) with the same nominal operating point P. When the load on the motors is greater than the nominal torque the operating point moves upwards along the curves, which in the figure are continued to the point at which the rotor current has increased to the maximum permissible value (assumed to be the same for both motors). The diagram shows that the shunt motor reaches this limit at a lower torque; the shunt motor is therefore less able to withstand overload than the series motor.]

Fig. 28. Torque-speed characteristics for a shunt motor (curve Sh) and a series motor (curve Se) with the same nominal operating point P. When the load on the motors is greater than the nominal torque the operating point moves upwards along the curves, which in the figure are continued to the point at which the rotor current has increased to the maximum permissible value (assumed to be the same for both motors). The diagram shows that the shunt motor reaches this limit at a lower torque; the shunt motor is therefore less able to withstand overload than the series motor.

Fig. 29. The main-field flux $\Phi_{rs}$ as a function of the energizing current $I_s$ (the 'open-circuit characteristic' of a d.c. motor). The magnetic saturation of the stator iron for increasing energizing current affects the shape of this characteristic.

Shunt and series motors behave quite differently when the load is varied. When the load is increased, the speed of a shunt motor falls only slightly. The extra power then delivered requires a proportional increase in the rotor current. The heavy portion of the curve Sh in fig. 28 indicates how the operating point moves in this case. The heavy portion of the curve Se indicates the shift of the operating point for a series motor when its rotor current increases by the same factor. This increase in the rotor current also results in a higher value for the main field, which contributes to an increase in the delivered torque; if the torque is doubled the rotor current only increases by about 40%, as against 100% for the shunt motor. This means that the series motor is more able to withstand overload than the shunt motor. Since the applied electrical power increases less for a series motor, the greater torque required can only be supplied at a significantly lower speed. A particular point to note is that if the loading torque is removed the motor speed rises sharply and will generally 'run away' to a speed above the maximum rated value.

If the series motor is supplied with a.c. current it gives a pulsating torque; with the field and rotor windings connected in series the main field and the rotor current are always in phase, so that their product always has the same sign. It is more difficult in this case to achieve good commutation, since the time-varying main field induces an additional voltage in the rotor coils. The stator core, and not just the rotor, also has to be laminated to prevent eddy-current losses. A series motor designed for use on either d.c. or a.c. current is known as a universal motor. Such motors are widely used in domestic equipment.

**Brushless d.c. motor**

A mechanical commutator requires periodic maintenance, is subject to wear, is noisy in operation and causes vibration and electrical interference. In many applications these effects are a nuisance or prevent the use of a mechanical commutator. For these reasons brushless d.c. motors have been under development for several years [*]. In these motors the commutation is obtained with electronic devices.

The rotor of a small brushless d.c. motor consists of a permanent magnet, while the stator has a multiphase winding. The windings can be wound on salient poles or in slots in the stator circumference. In small motors that have to provide a pulsation-free torque, i.e. no 'cogging', the windings are attached to a smooth stator bore in the form of 'air-gap coils'. Fig. 30 gives a

[*] We intend to include an article on brushless d.c. motors in the second issue on electric motors. (Ed.)
schematic diagram of a brushless d.c. motor with a two-pole rotor and four stator windings; this arrangement of stator and rotor shows considerable similarity to a four-phase synchronous motor with a permanent-magnet rotor. However, in the motor of fig. 30 the four stator windings are supplied from a d.c. source, in a way that recalls the supply arrangements for the stepping motor. The notable difference, however, is that the opening and closing of the switches between the windings and the source are determined only by the position of the rotor and the desired direction of rotation. This is shown schematically in fig. 31 for the case of clockwise rotation. Apart from brief transient effects, only two of the four windings are in use at any one moment. In small motors the switches consist of transistors, in large motors the rather less easily quenched thyristors are used.

There are various types of brushless d.c. motor, which can be classified by the nature of the elements for detecting the position of the rotor. First of all there are the Hall elements or field-dependent resistors. These are located in the rotor field, which varies with the position of the rotor \[8\]. The position of the rotor can also be established with the aid of photoelectric devices (photocells), magnetic \[7\] or electrostatic sensing elements. The electronic circuit is usually provided with a speed control and current limiting, giving a very adaptable d.c. motor that is particularly suitable for control purposes. Brushless motors are very suitable for applications where torque has to be uniform and efficiency high; they can be used for the drive in battery-operated tape recorders and record players without the need for an intermediate transmission system.

Summary. The most important types of electric motor are described and the main characteristics are derived. Electric motors can be classified as a.c. or d.c. motors. A.C. motors are either synchronous or asynchronous. In synchronous motors the number of revolutions per second is equal to the frequency of the supply divided by the number of pairs of poles. The stator windings maintain a rotating stator field, which the rotor follows; the rotor can be either a permanent-magnet rotor or made from a material of high permeability (and non-cylindrical) in the reluctance motor, or of material with considerable hysteresis (hysteresis motor). In the asynchronous or induction motor the rotor has closed current circuits in which currents are induced; the rotor 'slips' with respect to the rotating stator field. O.C. motors have a commutator, which reverses the current in the windings after every half-revolution. The commutator usually consists of brushes and sliding-contact segments, but electronic commutation is sometimes employed (brushless d.c. motor). If the commutator motor is series wound it can also be run on a.c. current (universal motor).

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\[7\] W. Radziwill, A highly efficient small brushless d.c. motor, Philips tech. Rev. 30, 7-12, 1969.