Stability of small single-phase synchronous motors

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Introduction

Electrical equipment nowadays does much of the work in and about the house that was formerly done manually. Electric power is used on a large scale for washing machines, vacuum cleaners and shavers, and also for tape recorders and record players. This has all become possible because such household machines have certain features in common: they are all simple to use, cheap to produce, use little power and are relatively cheap to service. In almost all cases the devices are required to operate from the ordinary single-phase mains supply.

For household use in the broadest sense, electrical energy is transformed into mechanical energy mainly by single-phase electric motors. Well known types of motors are synchronous and induction motors with their starting circuits, and hysteresis and series motors [1]. Their reliability, cost and efficiency must of course match the requirements of the devices in which they are used.

The development of oriented permanent-magnet materials with a high coercivity, such as ferroxdure, has led to the suggestion that synchronous motors with permanent-magnet rotors might offer advantages with regard to design, characteristics or efficiency. Such motors have no need for a commutator (which is subject to wear), they cause no electrical interference and they have a constant speed. Moreover, their efficiency can be high and the motor can be constructed from a small number of relatively simple components. Fig. 1 shows the principle of such a motor [2]. A laminated stator is provided with a coil that can be directly connected to the mains. A permanently magnetized rotor of ferroxdure with two diametrically opposed poles is mounted on a shaft in the stator field and can be used to drive some external load.

When the voltage is switched on, the motor should be self-starting and it should then rotate steadily in the same direction. Our permanent-magnet synchronous motors, like synchronous motors in general, do not meet these requirements unless special measures are taken. A well known and generally used method of starting a single-phase synchronous motor is the use of an auxiliary coil to give a rotating field, in combination with an induction cage or a hysteresis cylinder on the rotor. In this way the motor can start to rotate asynchronously in one direction. This solution, however, requires modifications to both the rotor and the stator and should therefore be avoided if possible.

Another difficulty in purely single-phase motors is that while the stator field does in fact change sign it has a fixed direction: it does not rotate, as in rotating-field motors. The direction of rotation of such a single-phase motor is therefore indeterminate.

Furthermore, a synchronous motor can behave as an oscillating system because there is an elastic coupling between the rotor and the rotating stator field. If the damping in this system is too small, oscillations can be set up that allow the motor to fall out of step with the field. With a non-conducting rotor, as in the present discussion, there is no electrical damping in the rotor at all. The stability of the rotary motion therefore requires special attention in such motors.

The advantages mentioned above were however sufficient to indicate a further study of these problems. In this article particular attention will be paid to the starting and stability of the motor.

Fig. 1. Principle of a single-phase synchronous motor with a permanent-magnet rotor. R rotor, magnetized in the direction $B_r$. S stator. $\theta$ angular coordinate defining the rotor position.


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Starting

Normal polyphase synchronous motors with no damping or starting cage will not in general start to rotate unaided when the stator winding of the stationary motor is connected to the supply. The polyphase supply produces a synchronously rotating field immediately but owing to its inertia the rotor cannot usually follow this field. There are now two ways of remedying this situation. The motor can be brought up to speed mechanically from outside or the frequency of the supply voltage can be reduced until the rotor is in step with the field; the frequency is then increased slowly until the rotor reaches the desired speed.

In single-phase synchronous motors an auxiliary field is usually excited, as mentioned above, which combines with the main field to give a rotating magnetic field. This can be produced by means of an auxiliary winding on the stator, connected to the supply via a capacitor, or by including one or two short-circuited turns around part of the stator poles (shaded pole). Electric motors with a large moment of inertia or high speed usually run asynchronously during starting, that is to say, starting takes place by virtue of currents induced in the rotor by the stator field. Starting of hysteresis motors will not be discussed here.

Single-phase synchronous motors with a nonconducting permanent-magnet rotor cannot be started by any of the methods just mentioned. When the supply is switched on the rotor is subject only to an alternating field that only changes sign; it does not change direction. This field exerts a torque on the rotor only when the rotor field and the stator field do not lie exactly in the same direction. Their directions coincide twice per revolution; the rotor must be prevented from coming to rest in either of these two positions since it would then remain there.

The rotor of the motor shown in fig. 1 is subject not only to a torque exerted by the stator field but also to a magnetic torque (the reluctance torque) even when there is no current in the stator winding. This is because the total magnetic energy of the system depends on the orientation of the rotor magnet between the stator poles. This reluctance torque can be used to prevent the rotor from coming to rest with its field parallel to the stator field. This is just what does happen in a symmetrical configuration like that of fig. 1. If however the space between the stator poles is made asymmetrical so that the air gaps between rotor and stator along one diameter are smaller than those along another diameter (fig. 2) then the rotor will assume a rest position in which the magnetic resistance is at a minimum. The stator field is hardly affected by the above asymmetry: to the stator the rotor appears to be part of the air gap because of the relatively low permeability of the rotor material. If \( \tilde{T}_0 \) is the amplitude of the reluctance torque and \( \gamma \) the angle between the rest position (determined by the reluctance torque and the direction of the stator field) then the torque acting on the rotor in the undesired 'parallel-field' position is \( T_0 = \tilde{T}_0 \sin 2\gamma \). If the rotor is in fact to be moved from this undesired position, then \( T_0 \) must be larger than the total frictional torque \( T_{fr} \) acting on the shaft of the rotor.

In the electric motors considered here, the critical frictional torque is much smaller than the maximum torque of the motor. These motors are therefore only suitable in applications where the external frictional torque at the instant of starting is not too large.

Now let us consider the starting process in more detail. The behaviour encountered here is different from the well known starting effects in asynchronous or d.c. motors; in these motors the rotor begins to rotate in one particular direction and then rotates faster and faster in that direction. In the single-phase synchronous motors considered here there is in general a complicated aperiodic transient behaviour. As fig. 3 shows, the angular velocity is alternately positive and negative: the motor thus changes its direction continually. This irregular movement continues until the variables of the motion (speed, rotor position with respect to stator field, stator current and phase angle of this current) have adjusted themselves to the values corresponding to uniform rotation. The final direction of rotation depends on the initial conditions: in a given situation the phase of the voltage at the instant of switching on is particularly important.
A motor behaves in the manner shown in fig. 3 only when the moment of inertia of the rotor and the load
attached to it is not too great. If the combined moment
of inertia is too great the rotor will only oscillate about
an equilibrium position. In general it is not possible
to start a single-phase synchronous motor by a gradual
increase in the frequency of the supply voltage.

This rather summary discussion of methods for self-
starting shows that a single-phase synchronous motor with a permanent magnet rotor can only be used to
drive devices with a low static friction or which are
coupled to the motor only after it has started. More-
ever, the device to be driven must have a small mo-
ment of inertia and the direction of rotation must be
unimportant.

The limitations imposed by the requirements of a
small frictional torque and a low moment of inertia can largely be overcome by the use of a mechanical
coupling that reduces the mechanical load on the
motor to an acceptable value during starting. A num-
ber of schemes have been investigated, all depending
on frictional effects. In the design of such couplings it
is especially important that there should be no slip in
the coupling after starting, for this would mean that
the mechanical power of the motor could not be fully
used.

If necessary the direction of rotation can be predeter-
mined by means of a mechanical device that prevents
rotation in one direction. In each case these extra
complications must be weighed against the likely ad-
vantages.

Experimental investigation of the motor behaviour

We shall now examine the behaviour of a single-
phase synchronous motor with a permanent-magnet
rotor that has run up to speed after switching on and
does not, as described above, remain stationary or just
perform small-amplitude oscillations. An electric mo-
tor is only suitable for practical applications if once it
has started it continues to rotate in the same direction
with little variation in speed. It can be shown that to
achieve this, the parameters of the motor, the supply
voltage and the load must be matched to each other
between fairly narrow limits. If this is not the case, a
perturbed motion will ensue that differs considerably
from the desired unperturbed motion [6]. A good
overall picture of the possible rotor motions can be
derived from measurements of angular velocity as a
function of time for various amplitudes of the supply
voltage. Table I (p. 239) shows the results of such meas-
urements on a motor designed for a voltage of 220 V,
which therefore rotates with an unperturbed motion at
this voltage. There are, however, at this voltage speed
variations at twice the mains frequency. This modu-
luation of the rotational speed is due to periodic varia-
tions in the electromagnetic torque. In a purely single-
phase synchronous motor, there is a field that rotates
in the opposite direction to the rotor at twice the mains
frequency and thus gives rise to an oscillating torque
at this frequency [7]. For a motor of other dimensions
or with another load, however, the various perturbed
motions may take place at the nominal voltage.

The angular velocity is measured by means of a
unipolar generator, which has negligible effect on the
behaviour of the motor. In the diagrams in Table I the

[3] See for example R. Richter, Elektrische Maschinen, part I,
[4] Further details are given on p. 229 of the article by E. M. H.
Kamerbeek [1].
[5] The angle \( \gamma \) is the difference between the angles \( \alpha \) and \( \beta \)
defined by Kamerbeek [1].
[6] Definitions of the terms used here are given in J. G. Malkin,
Theorie der Stabilität einer Bewegung, Oldenbourg, München
1959, p. 3.
angular velocity is plotted as a function of time. The graphs refer to the motion after starting transients have died away.

The phase of the voltage at the instant of switching on usually has no effect on the final rotation of the rotor; the situations \( \delta \) and \( 7 \) in Table I are exceptions.

The experiments show that the single-phase synchronous motor with the permanent-magnet rotor described has a useful motion only within a limited voltage range or, more correctly, in a limited range of motor parameters. The extent of this range and its limits are of course of interest. It should be remembered here that the motors must retain their unperturbed motion in spite of the spread in properties that normally occurs in industrial materials or as a result of the production process. The motors must also be able to take a load. Another interesting point is the extent to which the behaviour of the motor may be affected by nonlinear effects such as the saturation of the stator iron.

An attempt will be made to examine these questions using the equations of motion discussed in the next section, which describe the dynamic behaviour of the motor.

Equations of motion

A theoretical model of the situation must always satisfy two conditions. Firstly, it should allow the various relevant characteristics to be represented with sufficient accuracy. Secondly, the model must give a clear physical picture and it must be easy to manipulate. This second requirement means that simplifications have to be introduced and certain things neglected, in the hope, however, that nothing essential is lost. In the model of our motor it has been assumed that the magnetic state of the motor is uniquely determined by the stator current and the rotor position, in other words that hysteresis effects are completely neglected. Furthermore, saturation effects and eddy currents in the iron of the stator are neglected and the friction is assumed to depend linearly on the normal forces and to be independent of velocity (Coulomb friction). Finally it is assumed that the magnetic flux and the reluctance torque vary sinusoidally with the rotor position.

On the basis of these assumptions it is not difficult to write down the equations of motion. The first equation states that the sum of the voltages across the stator winding must be equal to the mains voltage. The sum is made up of the \( IR \) drop due to the current \( i \) through the coil of resistance \( R \), the voltage \( L\frac{di}{dt} \) due to inductance \( L \) of the coil and finally the voltage \( d\Phi_{st}/dt \) induced in the stator coil as a result of the change in the flux \( \Phi_{st} \) of the rotor linked by the stator coil. The latter contribution is proportional to the angular velocity \( \dot{\theta} \) of the rotor and is a sinusoidal function of the rotor position. The equation for the voltage across the stator coil is therefore:

\[
v \sin (\omega_m t + \epsilon) = iR + L\frac{di}{dt} + \Phi_{st} \dot{\theta} \sin \theta,
\]

where \( \omega_m \) is the angular frequency of the mains voltage and \( \epsilon \) its phase angle.

The second equation for the motor behaviour comes from the condition that the sum of the torques acting on the rotor must be zero. Apart from the frictional torque \( T_f \) and the reluctance torque \( T_0 \) there is also an electromagnetic torque acting on the rotor, which is proportional to the stator current and to the rotor flux.

The values of these torques must together be equal to the rate of change \( \ddot{\theta} \) of the angular momentum:

\[
i\Phi_{st} \sin \theta - T_f - T_0 = I \ddot{\theta}.
\]

The initial conditions of this equation are that at the instant the voltage is switched on (\( t = 0 \)) the current is zero and the rotor is in the position \( \theta = \gamma \), i.e. the position for static equilibrium.

In spite of the relatively simple form of the equations there is no general method for solving them. This is because of the nonlinearities that occur in the equations. (In the first equation the product \( \theta \sin \theta \), in the second \( i \sin \theta \).) Attempts to linearize the equations yield no clear picture of the real situation; indeed the observed instabilities are probably directly due to these nonlinearities.

The equations have therefore been solved by means of an analog computer. This method gives a direct physical picture of the behaviour of the model. A more accurate solution on a digital computer is less attractive since programs for the solution of nonlinear differential equations require a great deal of computing time.

In the analog calculation a model with the same differential equations as the electric motor under investigation is set up in the computer. The operations such as summation, integration and multiplication are carried out by operational amplifiers. Important quantities can be read off during the calculations from instruments connected at appropriate points in the circuit \(^8\).

The form of the theoretical model is determined by the equations of motion. To simulate a given motor, the various parameters of the motor must be determined and from these the coefficients of the equations must be derived. It should be noted that the inductance

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1. At low voltages the motor does not start. The rotor vibrates about the equilibrium position defined by the reluctance torque. (The same situation arises at the nominal voltage if the moment of inertia is too large.)

2. The rotor rotates but the motion is very irregular. The motion is not unidirectional even though the synchronous angular velocity is sometimes momentarily exceeded. The rotor changes its direction of rotation in a quite irregular manner.

3. The rotation has now become unidirectional. However, there are quite large periodic variations superimposed on the synchronous speed. Stroboscopic observations show that a steady-state motion is achieved with a repetition period of two revolutions of the rotor (25 Hz perturbation).

4. As the voltage is increased further the irregularities of the motion do not decrease. On the contrary, large new irregular and aperiodic motions occur in which the rotor again keeps reversing its direction of rotation. Apart from the voltage at which it occurs, this situation is no different from that described under 2.

5. When the voltage is increased still further, the motion becomes unidirectional again. Strong periodic variations are still present (hunting) but the motor runs with a mean speed equal to the synchronous value.

6. The amplitude of the hunting now becomes smaller; above a particular voltage the oscillations disappear entirely. For the first time the motor assumes its state of unperturbed rotation as at the nominal voltage (see 8).

7. At the same voltage at which this unperturbed motion occurs, however, the rotor can also run with very large speed variations. The rotor periodically comes almost to rest; the mean rotor speed is, however, the synchronous value. These variations are repeated every $1\frac{1}{2}$ periods ($33\frac{1}{2}$ Hz perturbation). Which of the two states of motion, 6 or 7, takes place is determined by the phase of the supply voltage at the instant the motor is switched on.

8. At the nominal voltage the rotation is again unperturbed. The angular velocity is now modulated only at twice the mains frequency. The modulation in speed is 20-40% of the mean synchronous value. If a large moment of inertia is coupled to the motor, the modulation depth becomes smaller. The unperturbed motion is stable for voltage variations that are not too large, and the motor can drive a load.

9. Increasing the voltage above the nominal value again gives rise to large variations in the angular velocity. The rotor comes to rest once per revolution and the direction of motion may even be momentarily reversed. Within milliseconds the rotor then assumes velocities far above the synchronous value and is then immediately slowed down (50 Hz perturbation). If still higher voltages are applied, the accelerating torques are so large that any regular motion is quite impossible.
of the stator coil(s) and the value of the rotor flux must be determined under conditions that approach the actual working conditions as nearly as possible. This is necessary because in the equations of motion a linear relation has been assumed between stator current and stator field and in practice this is not the case.

The 50 Hz perturbation, the unperturbed range and also the 33 1/3 Hz perturbation and its adjacent transition region occur in the model at almost the same voltages at which they are observed in practice. At voltages less than 160 V it is noticeable that the model exhibits less tendency to oscillation than is observed in practice. This difference is connected with the fact that the curvature was neglected in the magnetization characteristic of the stator iron. Since voltages below 160 V are of little practical importance, it is not necessary to revise the model in this connection. Apart from this the choice of the model is seen to be justified by the results.

**Periodic solutions of the equations of motion**

We have seen that the motor can rotate with a constant mean speed. Superimposed on this mean value are large periodic variations with frequencies that are simply related to the supply frequency.

In general a synchronous motor, once started, must run at a constant mean speed; for a rotor with two poles the mean speed must correspond to the frequency of the supply. This means that the mean of the speed variations over a long time must be zero, or

\[
\frac{1}{\tau} \int_{0}^{\tau} (d\theta - \omega_m) \, dt = 0, \tag{1}
\]

where the integration time \(\tau\) extends over an integral
number of periods of the speed variations. For steady-state motion, i.e. if the speed variations are periodic, then these variations can be written as the sum of a number of simple harmonic motions:

\[
\theta - \omega_m = \sum_{v=1}^{n} A_v \sin(a_v \omega_m t + \phi_v),
\]

where the factors \( a_v \) are provisionally arbitrary real numbers. Inserting (2) in (1) gives:

\[
\frac{1}{T} \int_{0}^{T} \sum_{v=1}^{n} A_v \sin(a_v \omega_m t + \phi_v) dt = 0.
\]

Either the integration or the summation may be performed first. The expression is identically zero, if the separate terms of the sum are each equal to zero:

\[
\frac{1}{T} \int_{0}^{T} \sin(a_v \omega_m t + \phi_v) dt = 0.
\]

The instants in time \( t = \tau, 2\tau, 3\tau, \ldots \), after which the rotor motion is repeated, must be synchronous with the supply frequency. In the stable unperturbed state of motion, the fundamental of the speed variations has a period equal to half the period \( T \) of the supply voltage. The integration time \( \tau \) can thus be taken equal to \( \tau_1 = T/2 = \pi/\omega_m \) which corresponds to a frequency of twice that of the supply voltage; it depends on the mains-frequency component of the stator current.

Other possibilities of synchronization occur when the period over which speed variations average out is an integral multiple of \( \tau_1 \); then \( \tau = k\tau_1 = k\pi/\omega_m \), where \( k = 1, 2, 3, \ldots \). The rotor then runs synchronously with the voltage supply. The condition (3) can now be written, on omitting the factors before the integral, as:

\[
\int_{0}^{\tau} \sin(a_v \omega_m t + \phi_v) dt = 0
\]

or

\[
\cos(a_v k\pi + \phi_v) = \cos \phi_v.
\]

This condition is fulfilled when \( a_v = 2l/k \), where \( l = 1, 2, 3, \ldots \). It follows that steady-state motion is obtained when \( a_v \) has the following values:

\[
\begin{align*}
   k & = 1 & a_1 & = 2, 4, 6, \ldots \\
   k & = 2 & a_2 & = 1, 2, 3, \ldots \\
   k & = 3 & a_3 & = \frac{2}{3}, \frac{4}{3}, \frac{6}{3}, \ldots \\
   k & = 4 & a_4 & = \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, \frac{9}{4}, \ldots \\
   k & = 5 & a_5 & = \frac{3}{5}, \frac{5}{5}, \frac{7}{5}, \frac{9}{5}, \frac{11}{5}, \frac{13}{5}, \ldots \\
\end{align*}
\]

For the various values of \( a_v \), the motional states are represented by the sum of a number of sine terms. We thus find, for \( k = 1 \):

\[
\theta = \omega_m + A_1 \sin(2\omega_m t + \alpha_1) + A_2 \sin(4\omega_m t + \alpha_2) + A_3 \sin(6\omega_m t + \alpha_3) + \ldots
\]
and, in accordance with the definition given above, this equation describes an unperturbed motion. Larger values of \( k \) give the various perturbations indicated above by the fundamental frequency of the appropriate variation. For a 50 Hz supply frequency there is therefore, for \( k = 2 \), a 50 Hz perturbation:

\[
\theta = \omega_m + B_1 \sin (\omega_m t + \beta_1) + B_2 \sin (2\omega_m t + \beta_2) + B_3 \sin (3\omega_m t + \beta_3) + \ldots
\]

and for \( k = 3 \), a 33⅓ Hz perturbation:

\[
\theta = \omega_m + C_1 \sin (\frac{2}{3}\omega_m t + \gamma_1) + C_2 \sin (\frac{4}{3}\omega_m t + \gamma_2) + C_3 \sin (\frac{6}{3}\omega_m t + \gamma_3) + \ldots
\]

Perturbations are also possible for larger values of \( k \). The longer the period of a perturbation, the more complicated the resulting motion.

A periodically perturbed motion in which the accelerations are very large and the frequencies only slightly different from those mentioned here is not possible. The motor would then fall out of step because the angular positions of the rotor would exhibit steadily increasing deviations from the synchronous values.

The perturbed motions found can all be considered as subharmonic resonances of a periodic driving torque with a frequency equal to twice the supply frequency. Such a torque is always present in a single-phase motor, owing to the reverse field that rotates around the surface of the rotor at this frequency.

**Stable regions**

Experimental and theoretical investigations of the behaviour of our motor have yielded the following result. The perturbed motions that remain after the starting transients have died away are characteristic of the motor. The unperturbed motions can be regarded as transition states between the upper limit of the one perturbed region and the lower limit of the next. To be useful, an unperturbed region must be at least wide enough to cover the spread in supply voltage that may be expected. There must be no chance that any perturbations should arise as a result of the spread in material properties, dimensions or load.

The effect of the spread in the various parameters can be very conveniently investigated with the analog computer. Using the model discussed earlier, it has been found that the inductance of the stator coils, the moment of inertia and the flux linkage of the rotor are particularly important in determining the behaviour. The effect of a 10% variation in these parameters is shown in the stability diagram of fig. 6. The following conclusions can be drawn from this diagram.

Firstly, increasing the flux linkage of the rotor displaces the perturbed regions towards lower voltages. It can also be seen that an increase in the inductance of the stator coils results in a displacement of the perturbed regions to higher voltages. Finally, any increase in the moment of inertia also displaces the perturbed regions towards higher voltages. The value of the inductance of the stator coils also has a strong effect on the region mentioned earlier of oscillation below the 33⅓ Hz perturbation region. Unperturbed motion takes place only with small inductances. If the inductance increases, the unperturbed region becomes narrower, and eventually disappears altogether. Increasing the resistance of the stator coils has just the opposite effect.

It can be seen that changes in the parameter values only lead to displacement of the limits between the various stability and instability regions. No new effects are observed. In designing a motor for a given application care must be taken to obtain a sufficiently wide stability region around the nominal supply voltage and to ensure that this stability region continues to exist for all possible values of the mechanical load.

Both theory and experiment show that the single-phase synchronous motor with permanent-magnet rotor remains stable at loads equal to the maximum

![Fig. 6. Behaviour of the motor for a 10% variation in each of the following parameters: the rotor flux \( \Phi_r \) linked by the stator, the inductance \( L \) of the stator coil or coils and the moment of inertia \( I \) of the rotor. In each diagram the moment of inertia is plotted vertically and the supply voltage \( V \) horizontally. In the central diagram + indicates the situation for a motor operating correctly.]
theoretical value, provided that a small part of the load consists of a velocity-dependent damping. This is the case in many applications of these motors. A reduction of the $Q$ (quality factor) $\omega_m L/R$ of the stator coil has, as in the unloaded oscillation of the rotor, a stabilizing effect. The motor can in any case be used for applications in which the load is of a frictional nature (i.e. both Coulomb and velocity-dependent friction).

Applications

The single-phase synchronous motor with permanent-magnet rotor as described is particularly suitable for driving small mass-produced articles such as domestic appliances. As we have seen, it is not a universally applicable motor characterized simply by voltage, power and speed. Because of problems with starting, rotation direction and the stability, the motor is only suitable as a drive for certain types of load. The motor therefore has to be specially designed for the load it must drive in practice and it is because of the development work required that it is only really suitable for mass-production applications.

For loads with a small moment of inertia and low bearing friction there are few difficulties — certainly not when the load is coupled to the motor after starting and when the direction of rotation is not important. The motor is therefore very suitable for driving a reciprocating device where the direction of rotation is immaterial. Moreover, the frictional load is then dependent on the position of the driven components; it is therefore possible to couple the motor and load in such a way that the friction is least in the most critical rotor position (the position with the rotor field parallel to the stator field). Also, with a reciprocating motion, the inertia of the reciprocating parts is much reduced in the transmission to the rotor. Compared with a vibrator drive, there is the advantage that the amplitude of the motion does not depend on the load or on the supply voltage; the motor can also be designed so that the whole device still functions properly at various supply frequencies. Furthermore, the loading of a reciprocating device has no effect on the speed of synchronous motors, unlike induction and series motors.

Fig. 7 shows an electric hair/massage brush in which the attractive features of the synchronous motor are put to use. The maximum available power developed by this motor, depicted in fig. 8, is about 7 W and the motor can be run from either a 220 V/50 Hz supply or a 120 V/60 Hz supply. The stator coils are encapsulated in plastic so that the motor is protected against moisture. A similar type of motor can be used for the drive of a trimmer.

Fig. 7. Hair and massage brush driven by a single-phase synchronous motor with a permanent-magnet rotor. The mechanical coupling between the rotary motion and the reciprocating motion can be seen between the motor and the brush.

Fig. 8. The motor used in the brush of fig. 7.

For other applications a suitable coupling is required that restricts the moment of inertia during starting and ensures unidirectional rotation. With these measures, the possibility of applications in rotary shavers and similar devices can be considered.

Summary. The small single-phase motor described here with a permanent-magnet rotor is a small and efficient motor of very simple construction: a magnetized cylindrical rotor of ferroxdure and a laminated stator with a field winding. To make the motor self-starting without any special extra provisions (auxiliary coil with capacitor, or 'shaded poles') the air gap of the motor is made asymmetrical, so that the rotor field is not parallel to the stator field in the rest position. Once the motor has started, then depending on the motor parameters and the load regions of perturbation can affect the movement; the perturbations consist of very strong periodic variations in the speed of rotation. Studies made on the motor with an analog computer show that the existence of perturbed and unperturbed regions is an essential feature of this type of motor, which can be traced back to nonlinearities in the equations of motion that describe the behaviour of the motor. It is shown that these perturbed movements also satisfy these simplified equations. It is relatively easy to design a motor in such a way that a sufficiently wide unperturbed region is obtained around the nominal values of the supply voltage and the load. Finally, a number of possible applications for the motor are described. Since a special version of the design has to be produced for each application the motor is only of interest for driving devices that are to be produced in quantity.