Introduction

A two-pole induction motor that runs from the a.c. mains has a speed of slightly less than 50 revolutions per second (i.e. 3000 revolutions per minute) \(^{1}\). The speed of motors with four, six or more poles is lower by a corresponding factor. Some applications, however, require much higher speeds; filament-coiling machines require speeds up to 30 000 rev/min, ultracentrifuges for uranium enrichment spin at 60 000 rev/min, and nylon-thread spinning machines require speeds as high as 100 000 rev/min. Speeds as high as this are best obtained with a direct drive from high-speed motors supplied by a 'high-frequency' a.c. supply. Other examples are high-speed hand tools (grinding machines, 25 000 to 60 000 rev/min) and machines for processing diamond dies (80 000 rev/min). In vacuum cleaners, which require speeds of up to 20 000 rev/min, the drive is provided by an a.c. commutator motor.

Apart from the technical requirements of the application, there is another argument for using high-speed motors as the drive, and this is the improvement in the power-to-weight ratio of the motor. The torque that can be delivered is limited by the dimensions of the motor and by the scope for dissipating the heat generated; the delivered power, however, is determined by the product of torque and speed, and the power limit is only reached when the mechanical construction of the motor does not allow a higher speed. Increasing the power-to-weight ratio by raising the speed is of particular advantage when weight is an important consideration, as it is in aeronautics and space technology and also in some electrical automobiles, as yet in prototype; in many cases the advantages of a high-speed motor are not cancelled out by having to use a reduction gear.

High-speed electric motors can be of either the synchronous or the asynchronous type. In applications where a synchronous motor is required a hysteresis motor or a reluctance motor (with starting cage) is usually suitable; a motor with a permanent-magnet rotor is not so suitable because many magnetic materials cannot withstand very high centrifugal forces. If some variation of speed with load is permissible, the obvious solution is an induction motor. This is the type we shall be concerned with in this article, which describes an investigation in which the objective was an optimum design for a high-speed induction motor with a solid iron rotor \(^{2}\). In this sturdy construction the rotor serves as both a magnetic and an electric conductor. The iron losses are considerably reduced when the stator windings are located in the air gap instead of in slots. Various types of coil have been specially designed for mounting in the air gap.

Fig. 1 shows two prototypes, both designed for speeds of between 36 000 and 40 000 rev/min and a power of 300 W. The supply for both models can be provided without too much difficulty in the form of a 'square-wave' voltage of the required frequency, which is easier to generate electronically than a sine wave. The small model could for example have been used as a design basis for a vacuum-cleaner motor smaller than the present commutator motors but delivering the same power.

To design the motors it was necessary to calculate the torque. This was no easy matter, because there is no simple way of determining the behaviour of the currents in the rotor; this can only be done by solving Maxwell's equations for the given configuration of stator and rotor. The best way of finding solutions was to start by neglecting the effects at the rotor ends, and then to add later corrections for these unavoidable end effects on a rotor of finite length. A correction was also needed for the effect of the magnetic saturation of the iron.

A more detailed account of our study now follows, in which the rotor, the stator and the calculations of the field and current distribution in the rotor and of the torque are discussed in turn.

Solid rotor

In designing a high-speed induction motor we had to choose between a squirrel-cage motor and a motor with a solid iron rotor. The important factors here were the shape desired for the torque-speed characteristic, and also whether a squirrel-cage motor would have been strong enough. Typical torque-speed characteristics for both motors are shown in fig. 2. The characteristic for the squirrel-cage motor (curve \(a\)) can be changed by altering the resistance of the cage; the smaller the resistance the steeper the curve near the synchronous speed \(n_0\) and the smaller the starting torque \(T_0\).
Fig. 1. Two prototypes of high-speed induction motors with a solid iron rotor. The stator coils are not in slots but are situated in the air gap. The motor on the left has thin coils, the one on the right has toroidal coils wound around the stator iron. Both motors were designed for speeds between 36,000 and 40,000 rev/min and a power of about 300 W.

In a solid rotor the currents are forced out towards the surface as the frequency increases. The rotor resistance is therefore a function of the slip. This results in a torque-speed characteristic that is almost flat until the synchronous speed is approached, when it drops steeply to zero. The ratio of the starting torque to the rated torque is therefore better than that of a cage rotor.

To obtain a torque-speed characteristic of this shape with large motors, the rotor is either provided with a double cage — two concentric cages, the outer one with the highest resistance and the lowest leakage inductance — or the bars of the cage are made very deep (skin-effect cage). A double cage is not used for small motors because it takes up too much room. A skin-effect cage is not used in small motors either, because the skin depth is greater than the depth of the bars.

It is also clear that in applications where a high-speed rotor of high mechanical strength is required a squirrel-cage rotor will present more constructional problems than a solid rotor. For this reason and for the reasons mentioned earlier, a solid rotor is to be preferred — even though a squirrel-cage rotor of the same volume delivers a greater pull-out torque (the maximum torque).

A solid rotor also has various incidental advantages. Firstly, since the currents only penetrate into the outermost layer, because of the skin effect, the rotor can take the form of a hollow cylinder, which reduces the moment of inertia. Secondly, the starting current is only about twice as high as the rated current, which is an advantage in the design of the electronic converter that supplies its high-frequency current. Finally, the electrical impedance of the motor does not vary much with the motor speed. Consequently a simple single-phase electronic source can be used for the supply; the auxiliary capacitor included in series with the auxiliary winding of the motor can have the same value for starting and for the rated speed.

A disadvantage of the motor with a solid rotor is its sensitivity to higher harmonics in the stator-field distribution. These 'spatial' harmonics should be distinguished from the time harmonics that appear when the motor is run from a non-sinusoidal voltage supply. Spatial harmonics give rise to slowly rotating stator fields and are the cause of high losses in the rotor. The

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1. 60 revolutions per second in North America (3600 revolutions per minute).
2. A more appropriate term would be homogeneous iron, since what is referred to is the absence of copper bars; in any case the rotor is sometimes not solid but hollow. The term solid iron will be used here, however, to be consistent with current usage.
method for ‘filtering out’ the most undesirable higher harmonics in a squirrel-cage rotor by choosing the appropriate number of rotor bars cannot be used in a solid-rotor motor. This means that when a solid rotor is used it is of paramount importance to have a stator field that is as purely sinusoidal as possible.

**Stator without slots**

The windings that generate the stator field are usually accommodated in slots in the inside wall of the stator and parallel to the axis. The stator consists of a stack of laminations that can take the form illustrated in fig. 3a. The turns are distributed among the slots in such a way as to approximate as closely as possible to the required sinusoidal copper distribution around the circumference, but the approximation is necessarily rather poor. Furthermore, because of the slots the air gap is not completely uniform, but varies periodically around the circumference. When a solid rotor is used this circumferential variation introduces even larger deviations from a sinusoidal stator field than those due to putting the coils in slots. All these variations have a considerable effect (see fig. 7a).

A solution to this difficulty can be found by making the bore of the stator smooth instead of slotting it (fig. 3b). The stator windings then have to be introduced into the air gap and distributed in the best possible way. Even though they are made as thin as possible, this still inevitably means a considerable widening of the air gap, resulting in a smaller flux density and hence a smaller pull-out torque for the same rotor radius; at the same time the pull-out slip becomes greater. For the same outside diameter of the stator, however, the rotor can be given a larger diameter (fig. 3), which in turn increases the torque.

Apart from this, there are other reasons why a motor with a solid rotor and stator coils in the air gap should have characteristics at least as good as those for a motor with the stator coils in slots — even when the rotor volume is the same. First of all, the effective air gap with a slotted stator is greater than the geometrical air gap, since the flux is concentrated at the stator teeth, where it saturates the iron, making the permeability appreciably lower. The difference in magnitude of the air gap for the two arrangements is therefore not as great as it might seem. Secondly, air-gap coils can take greater current densities since their shape gives a large surface of contact with the air and hence better cooling than for coils in slots. Thirdly, in the case of a motor with a conventional stator some 20% of the calculated fundamental torque must be subtracted for the torque produced by the higher harmonics of the field distribution, which is negative near the nominal speed. There are hardly any higher harmonics in a motor with good air-gap coils. Finally, a stator with a smooth bore has lower iron losses. The flux concentrations at the teeth of a slotted stator contribute substantially to the iron losses because both the eddy-current losses and the hysteresis losses are approximately proportional to the

![Fig. 3](image-url) Fig. 3. a) A stator with slots for the windings. b) A stator with a smooth bore. The windings now have to be accommodated in the air gap.

![Fig. 4](image-url) Fig. 4. Air-gap coils that produce an approximately sinusoidal field distribution and at the same time make the maximum use of the space in the air gap. The stator is wound for a two-phase supply. a) Cross-section of the (developed) stator coils. The upper coil is for one phase, the lower coil for the other one. b) The field pattern for the two phases (solid and dashed curves). c) Plan view and cross-section of a coil.
square of the magnetic flux density. In a cylindrical stator stack such flux concentrations are not encountered; furthermore the iron volume is smaller, which also helps to reduce iron losses. This is especially important in high-speed motors, since the iron losses increase approximately as the 1.5th power of the frequency and can reach considerable values at high frequencies. If an upper limit is set for the dissipation from a given stator volume, any reduction in the iron losses allows more heat to be generated in the coils, so that the air-gap coils can carry a higher current, which will produce a higher torque.

**Air-gap coils**

If it is desired to keep the gap as small as possible when using air-gap coils, and to make the maximum use of the space available for the windings, the coils cannot be arranged to give a purely sinusoidal copper distribution \(^\text{[4]}\). An approximation must then be made. It appears, however, that a reasonably good sinusoidal field distribution can be obtained with a very simple copper distribution. A distribution of this type is illustrated in fig. 4a, which shows a cross-section of the stator coils, developed along a straight line. The motor is assumed to be a two-phase machine (an obvious choice with electronic supply). The solid symbols indicate the sense of the current in the coils of one of the phases, and the dashed symbols indicate the sense of the current in the other. The resultant field distribution can be seen in fig. 4b, and fig. 4c shows a shape of coil that will give the current distribution illustrated. The thickness of this coil is stepped; if coils of the same thickness everywhere are required, the same current distribution can be obtained with a combination of the two coil shapes illustrated in fig. 5.

The large overhang of the coils in figs. 4 and 5 is a disadvantage because a substantial proportion of the energy is uselessly dissipated there, and also because they make the motors unnecessarily long. This disadvantage is not found with the toroidal arrangement shown in fig. 6. Here the stator-coil turns are completed outside the stator iron, fig. 6a shows the current directions for a stator with eight coils, each consisting of an inner winding and an outer winding with twice as many turns as the inner one. The solid arrows and symbols indicate the magnetic flux and current directions for one of the windings, and the dashed symbols those of the other. Fig. 6b shows a side view and cross-section of a coil, and fig. 6c shows the arrangement of the eight coils in the motor (see also fig. 1).

\(^\text{[4]}\) A sinusoidal copper distribution is used in the 'printed' air-gap coils described in the article by E. M. H. Kamerbeek, Torque measurements on induction motors using Hall generators or measuring windings, this issue, page 153. Here, however, the filling factor is very low.
The superiority of the field distribution obtained with air-gap coils, even with the simple copper distribution discussed here, can be seen from fig. 7. Fig. 7a shows the result of a field-distribution measurement in a stator with 12 slots, and fig. 7b shows the field distribution when air-gap coils like those in fig. 6 are used. The approximation to a sine curve here is a remarkably good one.

Calculation of the torque

The torque of an electric motor can be calculated if the radial and tangential components of the magnetic field are known at every point of a surface situated in the air gap and enclosing the rotor. In our case the calculation of these components requires the direct solution of Maxwell's equations for the air gap and for the iron of the rotor and the stator. This is possible if we introduce a number of simplifying assumptions. The principal ones are that the stator iron has infinitely high permeability so that the magnetic field-strength in it is zero, and also has zero electrical conductivity. Another assumption is that the solution is independent of the coordinate along the motor shaft, which implies that there are no perturbing effects from the end faces of the rotor (we treat the problem as if these end faces were infinitely far away, so that all the currents in the rotor run parallel to the shaft). Furthermore the permeability of the rotor iron is assumed to be constant, magnetic saturation does not therefore enter the argument. In the next subsection the expressions thus found will be corrected for the end effects of the rotor and for magnetic saturation.

The magnetic fields calculated for this simplified model depend on the slip because the skin depth is different at different speeds. These fields are used for calculating the torque; the expression found is complicated, and will not be given here. A simplification is possible, however, if the values of the slip are not too small. This becomes apparent when we derive the elements of the equivalent circuit of the motor from our knowledge of the magnetic field. This equivalent circuit is shown in fig. 8; the form in fig. 8b can be derived from fig. 8a, and the accented quantities are known as referred quantities. We shall use them several times in the following calculations. In our case the rotor resistance \( R_f \) and the leakage inductance of the rotor \( L_{ro} \), unlike those of the squirrel-cage motor, are functions of the rotor speed and hence of the slip \( s \). If the slip is greater than a minimum value \( s_{min} \), it is found that both \( R_f \) and \( L_{ro} \) are simply related to \( s \). This can be seen in fig. 9, where it is shown that \( R_f \) is proportional and \( L_{ro} \) inversely proportional to \( 1/s \). The proportionality constant \( R_f/\sqrt{s} \) will be referred to as \( r_f \) and \( R_f'/\sqrt{s} \) as \( r'_f \).
The complicated expressions that we derive for $R_{r'}$ and $L_{r'}$ can then be simplified, as can also the expression for the torque.

To derive the rotor resistance $R_{r'}$ and the various inductances we use another model of the infinitely long rotor. We assume in this new model that the currents flow in a thin sinusoidal copper layer applied to the surface of the rotor; the rotor iron is assumed to be non-conducting and to have the same permeability as in the previous model. In this way we can obtain a simple definition for one value of the rotor resistance. This modification must not cause any change in the rotor dissipation, the magnetic field in the air gap or the total magnetic field energy of the system. The currents in the hypothetical copper layer can be derived from the calculated magnetic field. Since for a given torque $T_e$ and an angular frequency $\omega$ of the stator currents the total dissipation $P_r$ in the rotor is known from the relation

$$P_r = s\omega T_e,$$

we can now calculate the rotor resistance.

The distributed inductance of this rotor model with its copper layer is very low. The reason is that the sinusoidal stator and rotor windings in this model are both assumed to be in the air gap, so that the coupling is virtually ideal. To allow for the magnetic field energy present in the real motor we must include a hypothetical leakage inductance $L_{ro}$ in the rotor circuit, compared with which the leakage inductance of the model rotor is negligible. The $L_{ro}$ plotted in fig. 9 is this hypothetical value. Calculation shows that

$$R_{r'} = s\omega L_{ro}$$

for $s > s_{\text{min}}$.

The forms which the referred values $R_{r'}$ and $L_{r'}$ take when $s > s_{\text{min}}$ are:

$$R_{r'} = \pi a\sigma \sqrt[3]{s\omega/2\sigma},$$
$$L_{r'} = \pi a\sigma \sqrt[3]{s/2s\omega}.$$

Here $a$ is the radius, $l$ the length, $\mu$ the permeability and $\sigma$ the conductivity of the rotor; $I_s$ is the peak value of the copper-distribution function for the stator.

The mutual inductance between stator and rotor ($M$ in the equivalent circuit) can be calculated in the usual way from the dimensions and other characteristics of the model. At a given amplitude $I_s$ of the a.c. currents in the stator the torque is now given by:

$$T_e = I_s^2 M/(\sqrt{3}/s_s + \sqrt{s_{\text{max}}/s + \sqrt{s}})/2,$$

and the pull-out torque by:

$$T_{\text{max}} = I_s^2 M/(2 + \sqrt{s_{\text{max}}}/2) = I_s^2 M/4.83,$$

while

$$s_{\text{max}} = 2r_e^2/(\omega M)^2$$

is the slip at which the motor develops the pull-out torque.

**Effect of finite rotor length**

As mentioned earlier, the results found need to be corrected because in a real rotor the currents are not axial everywhere but flow along closed paths, so that particularly near the end faces there are tangential and radial current components, which affect the operation of the motor [61].

To establish the magnitude of the correction these current components and the magnetic fields associated with them must be known. We are able to determine these approximately on the basis of a model different from those used previously. From the currents and fields we are then able to calculate the equivalent resistance and leakage inductance of the rotor as shown in the equivalent circuit in fig. 8. By comparing these quantities with the corresponding ones for a rotor with no end effects we arrive at a correction factor.

The model used here is shown in fig. 10. The stator is assumed to be infinitely long, whereas the rotor has a length $l$ and moves in the $x$-direction relative to the stator.

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[61] See E. M. H. Kamerbeek, Torque measurements on induction motors using Hall generators or measuring windings, this issue, page 153.


a finite length \( l \) along the \( z \)-axis. The air gap is neglected \([7]\). The rotor moves along the \( x \)-axis at a velocity \( v \) with respect to the stator field. In this configuration a solution for Maxwell's equations can be found that gives the magnitude and direction of the magnetic flux density and the current density at every point of the rotor.

The rotor moves along the \( x \)-axis at a velocity \( v \) with respect to the stator field. In this configuration a solution for Maxwell's equations can be found that gives the magnitude and direction of the magnetic flux density and the current density at every point of the rotor.

Consequently at the ends of the rotor the product of \( B_y \) and the local tangential field-strength \( H_0 \) produced by the stator is greater than in the infinitely long rotor, and this product \( B_y H_0 \) (the Maxwell stress \([8]\)) is a measure of the torque acting on the rotor. The field pattern shown in fig. 11 thus indicates that the end effects will lead to a higher torque. It also appears from fig. 11 that these end effects extend over several millimetres along the length of the rotor, and can thus make a significant contribution.

The total resistance of the current paths in the rotor will clearly be strongly affected by the route they take. If the current flows more or less directly across the rotor end face it has to cover a path \( n/2 \) times shorter than if it followed the circumference. To get some idea of the pattern of the currents we determined the current density and current direction in the end face. The lines in fig. 12 connect places of equal current density; the numbers indicate the current density in A/mm\(^2\), and the arrows the direction of the current in the end face.

It can be seen that the current densities at a distance of about 1 cm from the circumference of the rotor have decreased only very slightly; there can be no doubt that the currents do not only travel along the edge but across the end face by a shorter route.

Let us now see, in quantitative terms, what influence the end effects in the rotor have on the rotor parameters. This we can do by calculating the resistance and leakage inductance of the rotor using the model in

Some idea of the magnetic fields at and near the end faces can be obtained from fig. 11, which shows 'snapshots' of the lines of force in planes parallel to the end of the rotor and at various distances \( z \) from it; the slip velocity \( v \) is 20 m/s. The parameter for the curves is the magnetic flux (per metre length along the \( z \)-axis) lying outside a given curve; the unit is Wb/m. The difference between two neighbouring parameter values therefore gives the flux (per metre length along the \( z \)-axis) between the two curves. The numerical values are based on the assumption that the conductivity of the rotor iron is \( 5.10^6 \) A/Vm. At the end face (\( z = 0 \), fig. 11a) the field is equal to that in air and is symmetrical with respect to the abscissa 0, which corresponds to a null in the stator field. Even at a distance \( z = 1 \) mm from the end face (fig. 11b) the magnetic field can be seen to penetrate less deeply into the rotor iron along the \( y \)-axis and is also beginning to lag behind the stator field (moving to the left in fig. 11). This tendency increases with increasing distance from the end face (fig. 11c-g), and in the middle of the rotor the field is very little different from that in the infinitely long rotor.

We see clearly from these figures that the component \( B_y \) at the surface of the rotor is considerably larger near the end faces than in the middle of the rotor, where the values found are about the same as those for an infinitely long rotor.
Fig. 12. Calculated currents in the end face of a rotor like the one in fig. 10. The curves connect the points of equal current density; the numbers beside the curves give the current density in A/mm². The arrows indicate the sense of the current component in the x,y-plane; the current component along the z-axis is not shown. It can be seen that the currents do not only flow at the edge of the rotor but extend well over the end face.

Fig. 13. The quantity \( \eta \) as a function of the slip velocity \( v \) for two values of the ratio of the diameter \( d \) to the length \( l \) of the rotor. Owing to the effects at the end faces of the rotor the values found for the rotor resistance and leakage inductance have to be multiplied by a correction factor \((1 + \eta d/l)\). At greater slip velocities \( \eta \approx 0.9 \).

The air gap is included in the calculations made by W. P. A. Joosen, Finite-length effect in a solid-rotor motor, Philips Res. Repts. 28, 485-495, 1973 (No. 5).
Magnetic saturation

It now only remains to correct our results for the magnetic saturation in the rotor iron, which until now has been assumed to be of uniform permeability. When we determine the paths of the magnetic lines of force in a cross-section of the rotor in our original model (in which the end effects were neglected) we obtain results like those shown in fig. 14. It can be seen from this figure that the lines of force become denser at the surface of the rotor, and that the density increases as the slip with respect to the stator field increases. This eventually leads to magnetic saturation in the outer layer of the rotor.

If the existing nonlinear relation between permeability and field-strength is introduced into the equations, the calculations become impossibly complicated. Here again we have to make use of a simplified model.

Applying the corrections both for the end effects and for the magnetic saturation we find the following expressions for the referred values of the rotor resistance and leakage inductance:

\[ R_{r,\text{corr}}' = \frac{3}{5} a l_{s}^2 \sqrt{s_{\text{max}}/f_{s}^2} \left(1 + \eta d/l\right), \]
\[ L_{\text{corr}}' = \frac{4}{5} a l_{s}^2 \sqrt{a/2\pi\sigma} \left(1 + \eta d/l\right). \]

This yields a different value for the calculated torque:

\[ T_{e} = i_{s}^2 \bar{M}/(\sqrt{s_{\text{max}}/f_{s}^2} + 1/\sqrt{1.25}) \sqrt{1.25}. \]

The pull-out torque is now greater:

\[ T_{\text{max}} = i_{s}^2 \bar{M} / 3.24, \]

a value which agrees very well with measurements. Unfortunately this is not the case with the pull-out slip,

\[ s_{\text{max}} = 1.25 r_{r,\text{corr}}' / (\omega \bar{M})^2, \]

which follows from the same assumption \( r_{r,\text{corr}}' \) is the corrected value of the proportionality constant \( r_{r}' \) mentioned earlier to allow for the end effects and the magnetic saturation. The reason for this is that \( r_{r,\text{corr}}' \) is proportional to \( \sqrt{\mu} \), and hence \( s_{\text{max}} \) to \( \mu \); now, however, the permeability of the unsaturated rotor iron can no longer be taken into account, and instead an average permeability must be used for the whole rotor. At the surface the rotor iron is saturated; the permeability there (the saturation induction divided by the tangential field-strength at the surface) is therefore smaller than in the bulk of the rotor. It appears empirically that a known that an exact calculation of magnetic fields and current densities can be made for the case of an infinite half-space of ferromagnetic material characterized by a 'rectangular' \( B-H \) curve as shown in fig. 15, at whose surface there is a sinusoidally alternating tangential magnetic field \( B \). This model shows some resemblance with the solid rotor in which also, as can be seen in fig. 14, the lines of force soon begin to bend over parallel to the surface.

As in the case of the rotor, we can again define an equivalent resistance and a leakage inductance for the model. The impedances of both are identical in modulus if we assume constant permeability, as we also found in the case of the solid rotor. If we use the \( B-H \) curve in fig. 15, however, we find that the equivalent resistance is twice as large as the modulus of the impedance of the leakage inductance. If we can now assume that the occurrence of magnetic saturation in the solid rotor leads to the same change, then for the rotor with magnetic saturation, we have \( R_{r}' = 2 s_{\omega} L_{r} \).

Fig. 14. Lines of force in a cross-section of the rotor for different frequencies of the a.c. current in the rotor. The higher the frequency the more the lines of force concentrate at the surface of the rotor; the outer layer of the rotor finally becomes magnetically saturated, which affects the performance of the motor.

Fig. 15. The 'rectangular' \( B-H \) curve used for an approximate calculation of the effects of magnetic saturation in the rotor.

\[ \text{W. MacLean, Theory of strong electromagnetic waves in massive iron, J. appl. Phys. 25, 1267-1270, 1954.} \]
Fig. 16. Measured and calculated torque-speed characteristic of a high-speed induction motor with a solid rotor; the dashed curve is the calculated characteristic. The approximate description of the end effects in the rotor and the magnetic saturation of the rotor iron is accurate enough to bring the calculated characteristic close to the measured curve.

value of twice this surface permeability gives a satisfactory average value. When this is used in calculating the pull-out slip a theoretical torque-speed characteristic is obtained that agrees well with the results of the measurements (fig. 16).

Summary. Several prototype induction motors have been developed for speeds of up to 40 000 rev/min. Square-wave voltages at frequencies up to 700 Hz are used for the supply. The iron losses have been reduced and the stator-field pattern improved by making the stator windings in the form of thin coils in the air gap rather than putting them in slots. The motors have a 'solid iron' rotor without copper bars; the rotor currents give an increasing skin effect with increasing slip, which increases the starting torque in relation to the pull-out torque. The torque-speed characteristic is calculated with the aid of simplified models; it is necessary to go back to Maxwell's equations and to solve these for the models. This is done in steps; an infinitely long rotor of homogeneous permeability is first assumed, and corrections are then made to allow for rotor end effects and for the magnetic saturation at the surface.